

Title: Extremal vectors and the invariant subspace problem for the Hilbert space  
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Abstract.

It is known, that on some Banach spaces, there are continuous linear operators, for which the only closed invariant subspaces are the trivial ones - the whole space and the zero space. However, for reflexive Banach spaces, and in particular for the Hilbert space, it is still an open problem, whether every continuous linear operator has a closed, non-trivial invariant subspace.

In my talk, I will show that various types of extremal vectors can be used to construct invariant subspaces for new, fairly large classes of operators on the Hilbert space (and also on other Banach spaces). Examples of such extremal vectors are best approximate inverses of non-invertible operators. As an illustration of this technique, I will present a short proof of Lomonosov's famous result that operators commuting with compact operators have invariant subspaces. This proof is more constructive than the original one.

When developing this technique, one is led to consider new concepts and problems in operator theory - concepts and problems which may be of interest in themselves.