Two-Dimensional Magnetohydrodynamic Simulations of Time-Dependent Poloidal Flow.

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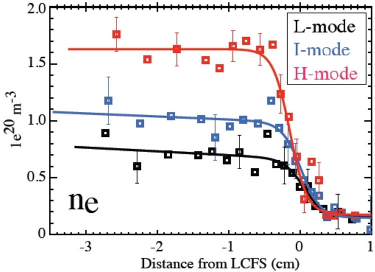
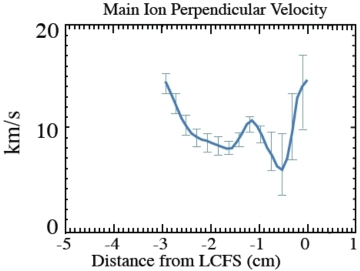
Motivation: Experimental Measurements Show Poloidal Flows of Tens km/s Near the Edge Pedestal.

- Poloidal flows in tokamaks are receiving an increasing attention, as newer and better flow measurements enhance the amount of available experimental information.[1]



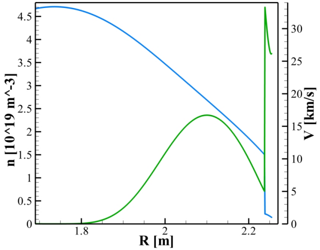


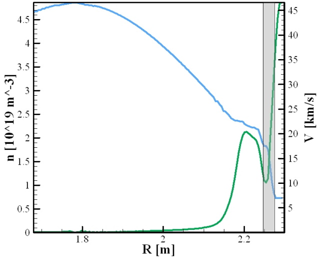
[1] Figures courtesy of R. McDermott

Motivation: MHD Equilibrium Theory Predicts the Formation of a Pedestal in the Presence of Transonic Poloidal Flows

- When the poloidal velocity is transonic with respect to the poloidal sound speed ($C_{sp} \equiv C_s B_p / B$), at equilibrium a discontinuity/pedestal is present at the transonic surface[a].
- This discontinuity is a contact discontinuity, NOT a shock.

[a] R. Betti and J. P. Freidberg, Phys. Plasmas **7**, 2439 (2000)





Motivation: MHD Equilibrium Theory Predicts the Formation of a Pedestal in the Presence of Transonic Poloidal Flows

- Equilibrium calculations confirm the prediction of theory[2]
- Using time-dependent simulations, we want to verify that the transonic equilibrium is dynamically accessible.

Transonic

Density profile in a transonic equilibrium.

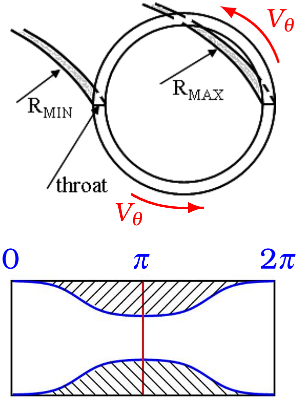
[2] L. Guazzotto, R. Betti, J. Manickam and S. Kaye, Phys. Plasmas, **11**, 604 (2004)

Outline

- Time-dependent ideal MHD simulations in the presence of transonic poloidal flow are presented.
- Time-dependent (SIM2D) and equilibrium (FLOW) results are compared and found in qualitative agreement.
- Calculations use the ideal MHD model:
  - Transients show the formation of shocks, which are likely not physical;
  - The shock-less steady state is believed to be accurate.
- The steady-state has contact discontinuities / pedestals.

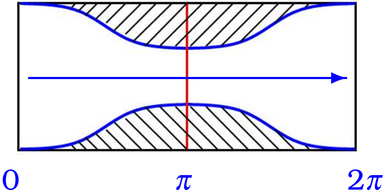
The Magnetic Field Creates a De Laval Nozzle for the Poloidal Flow

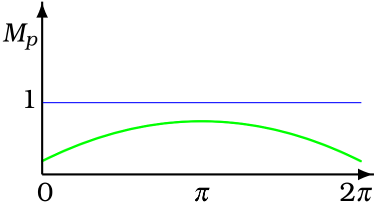
- In ideal MHD, the plasma cannot flow across magnetic surfaces.
- Due to toroidal geometry, in a tokamak the flow area between any two nested magnetic surfaces varies with the poloidal angle.
- For the poloidal flow, nested magnetic surfaces act as a de Laval nozzle.

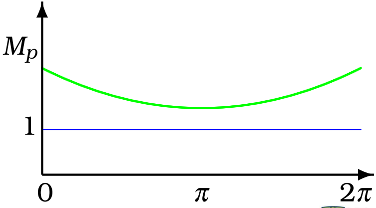


Flow Characteristics Are Determined by Nozzle Geometry.

On a magnetic surface, Mach number profiles are determined by 1-D gasdynamics.

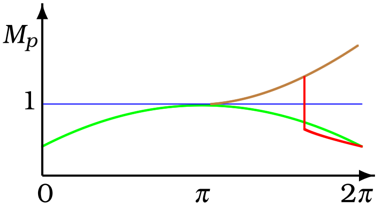


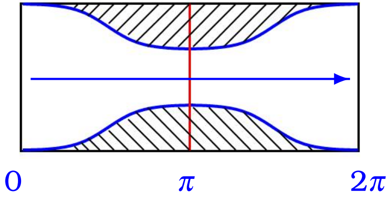




Flow Characteristics Are Determined by Nozzle Geometry.

Periodicity conditions require the formation of shocks if the flow switches to a different regime on a magnetic surface.

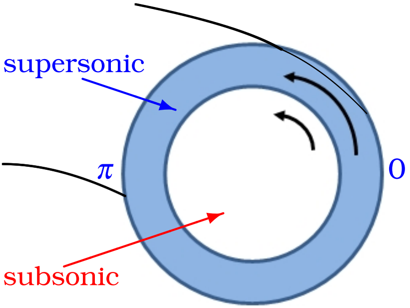


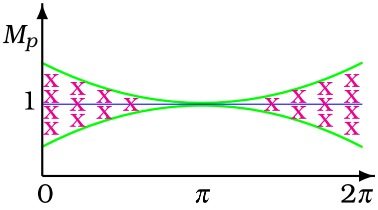


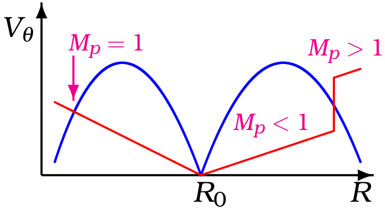
The traditional sub-supersonic flow of de Laval nozzles is not allowed at steady state.

Flow Characteristics Are Determined by Nozzle Geometry.

Radial discontinuities are present at steady state, due to the θ-dependent prohibited region for the Mach number.







For Transonic Flows, the Mach Number Discontinuity Causes a Density Discontinuity.

- If the poloidal flow is transonic, Mach numbers are **radially** discontinuous.
- Similarly, other physical quantities (e.g. density, poloidal velocity) are also discontinuous.
- Detailed analysis shows that:

$$\frac{\delta\rho}{\rho} \sim K_1(\Psi)\cos\left(\frac{\theta}{2}\right) \qquad \frac{\delta T}{T} \sim (\gamma-1)K_2(\Psi)\cos\left(\frac{\theta}{2}\right)$$

- In the isothermal case ($\gamma = 1$), there is no temperature discontinuity!

SIM2D Simulations Are Based on the ideal MHD Model.

We solve the standard ideal-MHD model time-dependent equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{V}) = 0, \qquad \text{(Continuity)}$$

$$\frac{\partial \rho \underline{V}}{\partial t} + \nabla \cdot \left(\rho \underline{V}\,\underline{V} - \underline{B}\,\underline{B} + P\underline{\underline{I}}\right) = 0, \qquad \text{(Momentum)}$$

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{V} \times \underline{B}), \qquad \text{(Faraday's Law)}$$

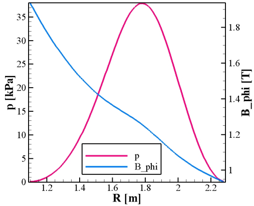
$$\frac{\partial \mathscr{E}}{\partial t} + \nabla \cdot [(\mathscr{E} + P)\,\underline{V} - \underline{B}(\underline{V} \cdot \underline{B})] = 0. \qquad \text{(Energy)}$$

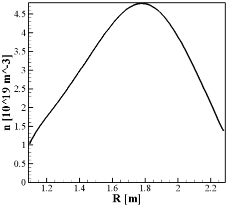
$$P \equiv p + \frac{B^2}{2}, \qquad \mathscr{E} = \frac{p}{\gamma - 1} + \rho \frac{V^2}{2} + \frac{B^2}{2}. \qquad \text{(Definitions)}$$

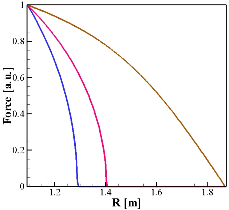
The equations are written in conservative form to capture the shocks.

Initial and Boundary Conditions

- The initial conditions are assigned using an equilibrium calculated with FLOW
- The equilibrium can be static or have <u>subsonic</u> flow.
- A momentum source is turned on at $t = 0$.

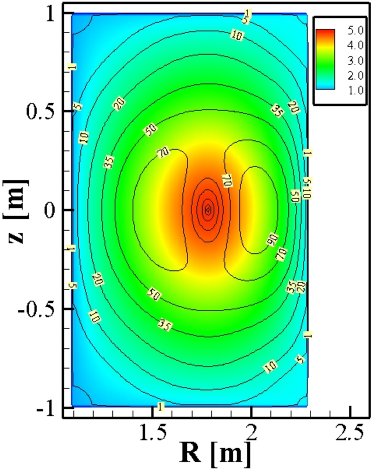






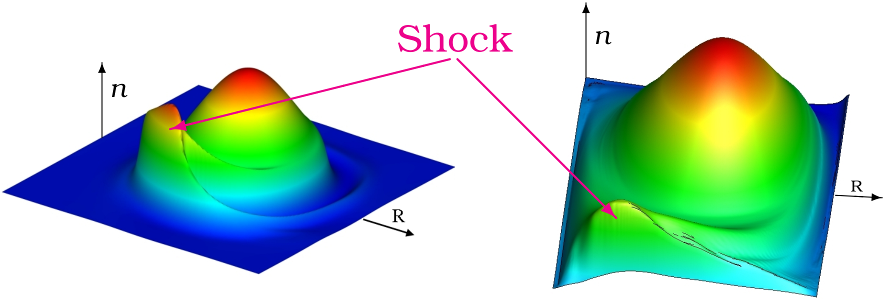
Initial and Boundary Conditions (Continued)

- The boundary of the computational domain corresponds to the plasma edge (no vacuum region).
- Rigid-wall boundary conditions are used at the (superconductive) wall.
- Initial poloidal sound speed is small at the plasma/computational edge.



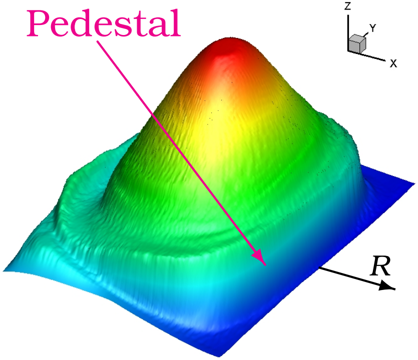
Poloidal sound speed [km/s] (lines) and density $\left[10^{19} m^{-3}\right]$ (colormap)

Poloidal Shocks Form Due to Poloidal Flow



- A shock is observed at the transonic surface.
- The shock travels in the poloidal direction from the outboard to the inboard part of the plasma.
- The shock vanishes at the inner midplane, where the flow is sonic.
- The shock is an MHD feature.

Simulations Show that Density Pedestals Form due to Poloidal Flow

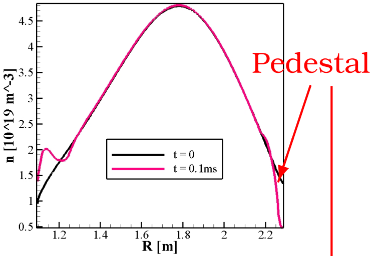


Density Pedestal

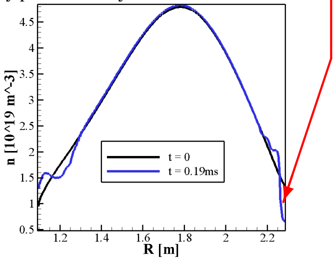
Density Pedestal

Density Profiles Develop a Pedestal Structure.

- A subsonic equilibrium is perturbed by turning on a poloidal momentum source.
- Momentum is inserted with a source localized at the plasma edge.
- After the flow becomes supersonic ($V_\theta > C_{sp}$) a shock forms and travels in the poloidal direction.
- Pedestal formation corresponds to a strong steepening of the density profile.



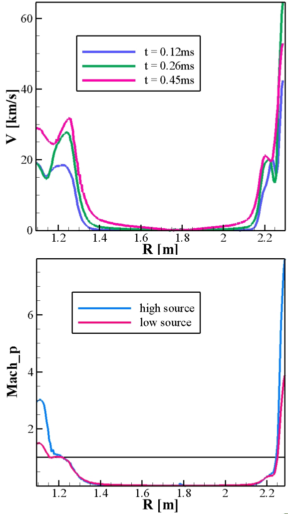
Density profile shortly after the shock formation.



Density profile after the flow has completed a poloidal revolution.

Velocity and Pressure Profiles Are Also Discontinuous.

- The velocity profile is discontinuous across the transonic surface.
- Profiles are smooth at the inboard side, sharply discontinuous at the outboard side of the plasma.
- The pressure profile also develops a pedestal structure. Since the edge pressure is small, its pedestal is not as visible as the density pedestal.



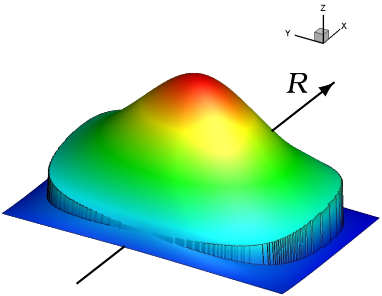
Time-Dependent Simulations Show an MHD Pedestal Formation

- Time-dependent simulations only reach an *approximate* steady state.
- At near-steady state, the pedestal structure is clearly visible.

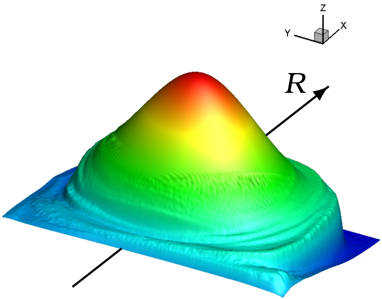
Transonic

The MHD Pedestal Structure Is as Expected From Equilibrium Theory

FLOW and SIM2D results are in **qualitative** agreement, as shown by density profiles:



FLOW equilibrium



SIM2D quasi-steady state

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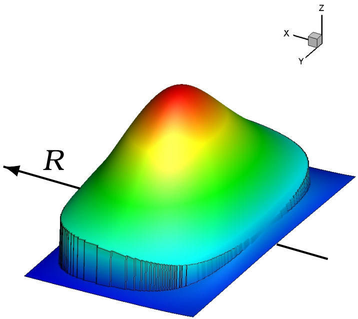

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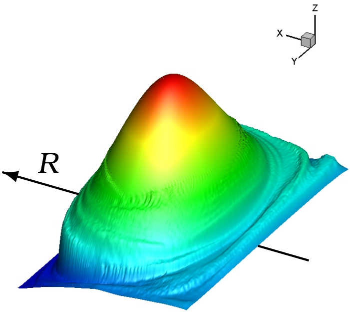

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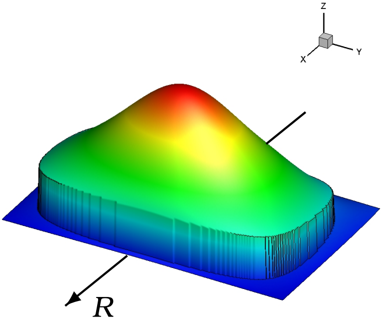

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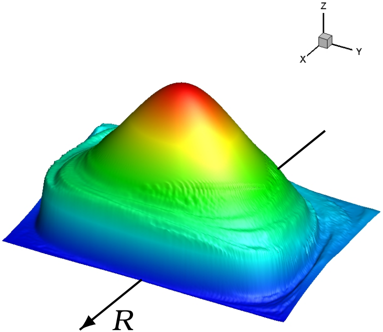

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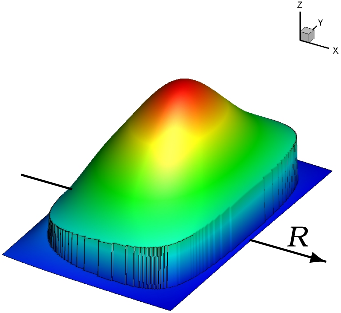
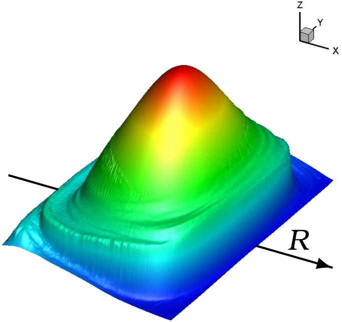


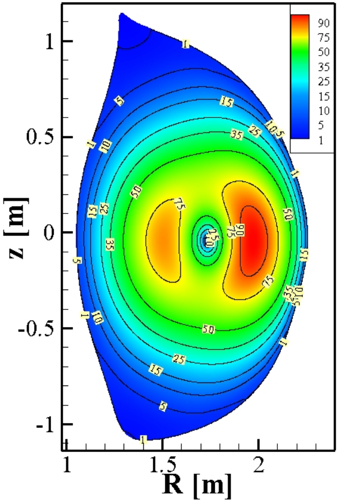
FLOW equilibrium



SIM2D quasi-steady state

Small Velocities Are Sufficient to Create Transonic Flows.

- The poloidal sound speed $C_{sp} = C_s B_p / B$ is small at the edge:
  1. C_s is small because temperature is low.
  2. $B_p << B$
- FLOW transonic equilibrium results show vanishing C_{sp} at the edge.



Poloidal sound speed [km/s]

Conclusions

- Time-dependent ideal-MHD simulations show the formation of transient shocks when the poloidal flow becomes supersonic ($V_\theta > C_{sp}$).
- Shocks move along flux surfaces and vanish.
- The steady-state solution is shock-free. Radial contact discontinuities (pedestals) develop at steady-state.
- Results are in agreement with the predictions of theory.

Can the MHD Pedestal Be Related to the Observed Edge Pedestal and L-H Transition?

Possible scenario of edge pedestal formation:

1. A momentum source makes the poloidal flow transonic (with respect to the poloidal sound speed)[3].
2. An MHD transonic *density* pedestal forms on short time scales ($\sim 2\pi a/C_{sp} \sim ms$):
   - The MHD pedestal is modulated in the poloidal angle.
   - No temperature pedestal is formed (unless $\gamma \neq 1$).
   - The velocity shear is large (∞ in ideal MHD) across the MHD pedestal.
3. Due to velocity shear turbulence suppression, a temperature pedestal develops on longer transport time scales. The density pedestal is also modified on longer time scales.

[3] e.g., K. C. Shaing and E. C. Crume, *Phys. Rev. Lett.*, **63** 2369 (1989)