Poloidal Flow Simulations and Pedestal Formation.

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In tokamak experiments, a spontaneous transition from low to high confinement mode (L-H transition) is observed when the input heating power exceeds a machine-dependent threshold. The L-H transition is followed by the formation of a pedestal (sharp radial increase) in plasma properties, i.e. density, pressure and temperature. Experimentally, it is observed that L and H modes have a rather different radial electric field profile, which corresponds to different plasma rotation profiles, in particular in the poloidal (the short way around the torus) direction. That is because under the zero-resistivity assumption $\underline{E}+\underline{V}\times\underline{B}=0$ and $B_\varphi >> B_\theta$, where $\underline{V}$ is the plasma macroscopic velocity, $\underline{E}$ the electric field, $\underline{B}_{(\varphi,\theta)}$ the (toroidal, poloidal) magnetic field. Magnetohydrodynamics (MHD) theoretical work [1] predicts that in the presence of transonic flows (subsonic in the plasma core, supersonic at the edge with respect to the poloidal sound speed $C_s B_\theta/B$) a pedestal will form, with a contact radial discontinuity between a low-density, high-flow region at the plasma edge and a high-density, low-flow region in the core. Pressure, toroidal flow and temperature profiles are similarly discontinuous. It is important to stress that the poloidal sound speed is rather low at the plasma edge, both because the temperature (and therefore the sound speed) is low, and because the poloidal magnetic field is in general much lower than the total field, in particular at the separatrix. The theoretical prediction of pedestal formation has been verified with equilibrium numerical calculations. [2]. In this work, we prove with time-dependent simulations that the MHD pedestal is naturally obtained when the poloidal rotation at the plasma edge is accelerated to supersonic values. Moreover, we show that the pedestal is maintained as long as the edge rotation remains supersonic, and that the pedestal can be formed with realistic velocities for existing tokamak experiments.

The system is modeled in the MHD framework. The standard ideal-MHD model time-dependent equations are used:

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\underline{V})=0, \tag{1}$$

$$\frac{\partial\rho\underline{V}}{\partial t}+\nabla\cdot\left(\rho\underline{V}\,\underline{V}-\underline{B}\,\underline{B}+P\underline{\underline{I}}+\underline{\underline{\Pi}}\right)=0, \tag{2}$$

$$\frac{\partial\underline{B}}{\partial t}=\nabla\times(\underline{V}\times\underline{B}), \tag{3}$$

$$\frac{\partial\mathscr{E}}{\partial t}+\nabla\cdot[(\mathscr{E}+P)\underline{V}-\underline{B}(\underline{V}\cdot\underline{B})]=0. \tag{4}$$

The equations correspond, in that order, to continuity equation, momentum equation, Faraday's law, energy conservation. An ideal plasma Ohm's law and the $\nabla \cdot \underline{B} = 0$ constraint are also implied. The equations are written in conservative form to ensure conservation of physical quantities. The standard definitions are used, with $P \equiv p + B^2/2$ being the total pressure, $\mathscr{E} = \frac{p}{\gamma-1} + \rho \frac{V^2}{2} + \frac{B^2}{2}$ the total energy, γ the adiabatic index, p the pressure, ρ the density, $\underline{V}$ the total plasma velocity, $\underline{B}$ the magnetic field. We set $\mu_0 = 1$ for ease of notation. The pressure tensor $\underline{\underline{\Pi}}$ represents numerical viscosity.

Simulations are carried out using a predictor-corrector finite difference approach in Cartesian coordinates. Artificial numerical dissipation is introduced for numerical reasons. Since the time integration method we are using is prone to spurious overshoots, numerical diffusion is needed around sharp discontinuities, in particular at shock fronts. Moreover, it is necessary to ensure that the $\nabla \cdot \underline{B} = 0$ constraint is satisfied at all times. As shown in Ref. [3], this must be enforced explicitly for a numerical approach like the one used in this work. A standard projection scheme was used to enforce the condition $\nabla \cdot \underline{B} = 0$. Numerical treatment of the boundaries makes it preferable to use a square or rectangular boundary for the domain. The effect of curved boundaries on the dynamics of the pedestal formation will be investigated in future work.

In order to study the effect of transonic poloidal flow on tokamaks, we start our simulations from an equilibrium, either static or with subsonic poloidal flow, then introduce poloidal rotation with an *ad hoc* source. Equilibria with or without flow are computed in high resolution (2000x2000 points) with the equilibrium code FLOW. [2] Qualitatively similar evolutions are observed regardless of how the poloidal rotation is introduced in the system, whether as an arbitrary initial profile or as a space and time dependent momentum source. The only requirements for the creation of a transonic pedestal are that 1) the poloidal flow becomes transonic and 2) the poloidal momentum must be large enough to let the (transient) shock reach the point of minimum cross section. The second requirement refers to both the small artificial dissipative terms and the physical energy dissipation occurring at the shock front.

In the remainder of this work, we will show simulation results based on a DIII-D equilibrium, with an edge temperature of 30 eV. The machine geometry (major and minor radius, elongation) is used in the simulations. The initial equilibrium has a smooth density profile and subsonic poloidal flow, with maximum poloidal velocity $\sim 5\ km/s$ and maximum Mach number ~ 0.75. At the edge, the maximum poloidal sound speed is $C_{sp}^{MAX} \sim 13\ km/s$. In principle, the poloidal velocity only needs to exceed C_{sp}^{MAX} in order to form a transonic equilibrium. In the simulations, the value $\gamma = 1.05$ is used, corresponding to the isothermal assumptions for the energy of state (the choice $\gamma = 1$ is not allowed by our equations).

A poloidal momentum source is turned on at time $t = 0$. The source has a spatial profile peaked at the plasma edge, with no momentum input in the central region of the plasma. In general, equilibrium and time-dependent evolution of the system are rather insensitive to the poloidal velocity profile in the inner region of the plasma, as long as the flow remains subsonic in that region. After the flow becomes supersonic (with respect to the poloidal sound speed) at the edge, a shock forms in the outboard region of the plasma. The shock then travels in the poloidal direction, and disappears after reaching the inboard midplane. After the shock disappearance, a discontinuity (pedestal) is left at the plasma edge. As expected from theory, the pedestal height is modulated with the poloidal angle, being maximum at the outer midplane, and minimum on the inner side of the plasma. It is important to stress that the pedestal is *not* created by the shock. Rather, a discontinuous (shock-less) solution is the only equilibrium solution with transonic poloidal flow allowed by the MHD time-independent equations [1].

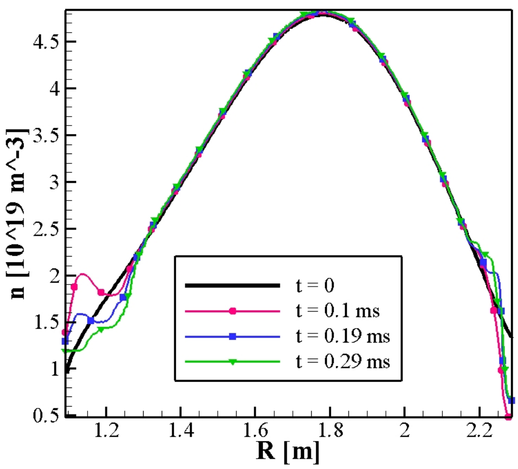

Figure 1: Density profile at different times, showing the formation of a sharp pedestal structure in the outboard side of the plasma.

The density profile on the midplane at four different times is shown in Fig. 1. From the figure, it is easy to recognize the presence of a sharp gradient in the outer region of the plasma at later times during the simulation. No density discontinuity is present in the inner region of the plasma, in agreement with theory predictions. Moreover, the poloidal velocity and Mach number are discontinuous in the outer section of the plasma, but continuous on the inner midplane (not shown). The figure also shows that the density profile is still evolving after the initial transient (which ends in approximately 0.19 *ms*). That is due to the lack of physical dissipation in the numerical simulations, which allows oscillations to persist for very long simulation times. This is confirmed by plots at later simulation times, which are not shown in Fig. 1. Similarly, also the poloidal velocity and Mach number do not reach a perfect steady state, but the flow remains supersonic in the edge region after the initial transient, with the maximum Mach number oscillating between the values $3 \lesssim M_p \lesssim 4$. This corresponds to a maximum poloidal velocity of $40 \lesssim V_p^{MAX} \lesssim 80$ $[km/s]$. Notice that the plasma temperature, and therefore poloidal sound speed, is also not exactly constant after the initial transient. The magnetic field is instead essentially constant during the simulation, due to the low plasma β.

Additional details on the density profile are visible in a three-dimensional plot of the plasma at late simulation time, which is shown in Fig. 2. From the figure, it is possible to appreciate the fact that the pedestal height is approximately constant in the outboard part of the cross section, and smoothly decreases as one moves closer to the inner part of the cross section.

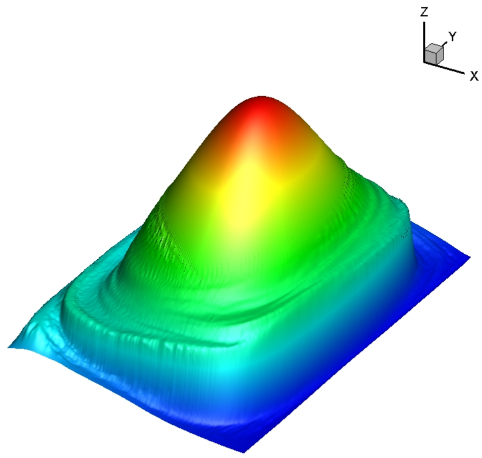

Figure 2: Three-dimensional density plot, showing the poloidal modulation of pedestal height.

In conclusion, time-dependent simulations show that a radial discontinuity (pedestal) forms in a plasma in the presence of transonic poloidal flow. The discontinuity is not a shock. Plasma velocity is parallel to the discontinuity, with the flow being supersonic on one side of the discontinuity, subsonic on the other. Due to numerical reasons (the lack of a self-consistent model for velocity dissipation) the system only reaches an approximate steady state. Nevertheless, the main characteristic of the transonic equilibrium described in detail in Refs. [1, 2] are correctly reproduced. Moreover, the transient is also consistent with the predictions of theory.

Finally, the velocity necessary to create transonic equilibria is relatively low. Velocities of the order of $\sim 40 km/s$ are sufficient for a transonic equilibrium to exist in the simulations presented in this work. In reality, even lower velocities could be sufficient to generate a transonic equilibrium, since the poloidal sound speed at the plasma edge is small, and vanishingly so at the X-point, where $B_p = 0$.

References

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