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Analytical Results for the Stark-Zeeman Broadening of the Lyman-alpha Line in Plasmas

EUGENE OKS[*]

Physics Department, 380 Duncan Drive, Auburn University, Auburn, AL 36849, USA
** Email: oksevgu@auburn.edu*

ABSTRACT: The π-component of the Ly-alpha line of hydrogenic atoms has the most remarkable, practically important properties. Previously it was found that under the conditions typical for the edge plasmas of tokamaks, the Stark width of the Ly-alpha π-component can be used for the experimental determination of the effective charge of ions, while the Ly-alpha σ-component or any component of any other hydrogenic spectral line cannot be used for this purpose. It was also found previously that the Stark width of the Ly-alpha π-component, emitted by atoms of the hydrogen or deuterium neutral beam injected for heating tokamak plasmas, can be employed for the simultaneous experimental determination of both the effective charge of ions and of the pitch angle (the angle between the beam velocity and the magnetic field). In the present paper we show another remarkable feature of the Ly-alpha π-component. Namely, in the situation where the Zeeman effect dominates over the Stark effects, the broadening of the Ly-alpha π-component tunes out to be controlled by the linear Stark effect – practically without depending on the magnetic field (while this is not the case for the Ly-alpha σ-component or any component of any other hydrogenic spectral line). Therefore, the Stark width of the Ly-alpha π-component can be used in this situation for the experimental determination of the ion density or of the root-mean-square field of a low-frequency electrostatic plasma turbulence, such as, e.g., ion-acoustic waves, or Bernstein modes, or low-hybrid waves.

Kewords: Ly-alpha line; π-component; Stark-Zeeman broadening; strongly magnetized plasmas; low-frequency electrostatic plasma turbulence

1. INTRODUCTION

Broadening of spectral lines is an important tool for spectroscopic diagnostics of various laboratory and astrophysical plasmas – see, e.g., books published in the last three decades [1-7] (listed in the reverse chronological order) and references therein. For providing the diagnostic information about variety of fields inside plasmas, especially sensitive are hydrogenic spectral lines. The hydrogenic spectral line Ly-alpha has the simplest structure and frequently used for illustrating theoretical approaches.

In particular, for strongly magnetized plasmas it was found that the π-component of the Ly-alpha line has the most remarkable, practically important properties. In paper [8], by using an advanced analytical theory, it was shown that under the conditions typical for the edge plasmas of tokamaks, the Stark width of the Ly-alpha π-component can be used for the experimental determination of the effective charge of ions, while the Ly-alpha σ-component or any component of any other hydrogenic spectral line cannot be used for this purpose. In the subsequent paper [9], by using the same advanced analytical theory as in paper [8], it was demonstrated that the Stark width of the Ly-alpha π-component, emitted by atoms of the hydrogen or deuterium neutral beam injected for heating tokamak plasmas, can be employed for the simultaneous experimental determination of both the effective charge of ions and of the pitch angle (the angle between the beam velocity and the magnetic field).

In the present paper we show another remarkable feature of the Ly-alpha π-component. Namely, in the situation where the Zeeman effect dominates over the Stark effects, the broadening of the Ly-alpha π-component tunes out to be controlled by the linear Stark effect – practically without depending on the magnetic field (while this is not the case for the Ly-alpha σ-component or any component of any other hydrogenic spectral line). Therefore, the Stark width of the Ly-alpha π-component can be used in this situation for the experimental determination of the ion density or of the root-mean-square field of a Low-frequency Electrostatic Plasma Turbulence (LEPT), such as, e.g., ion-acoustic waves, or Bernstein modes, or low-hybrid waves.

2. ANALYTICAL RESULTS

We consider the splitting/broadening of the Ly-alpha line of a hydrogenic atom of the nuclear charge Z under a magnetic field **B** (the direction of which we choose as the z-axis) and an electric field **F** at the angle θ with respect to **B**. We use atomic units: $\hbar = m_e = e = 1$. For the absolute values of these fields, we introduce the following scaled notations:

$$M = \alpha B/2,\ E = 3F/Z. \tag{1}$$

In Eq. (1), α is the fine structure constant; "M" stands for "magnetic" and "E" stands for "electric". Then the matrix elements of the interaction term in the Hamiltonian can be represented in the form (see, e.g., paper [10]):

0	0	E	0
0	$-M\cos\theta$	$2^{-1/2}M\sin\theta$	0
E	$2^{-1/2}M\sin\theta$	0	$2^{-1/2}M\sin\theta$
0	0	$2^{-1/2}M\sin\theta$	$M\cos\theta$

The eigenvalues of this matrix satisfy the following equation

$$(\Delta\omega)^2 = (E^2 + M^2)/2 \pm [(E^2 + M^2)^2/4 - E^2M^2\cos^2\theta]^{1/2}. \tag{2}$$

(We note that in atomic units, the splitting has the same expression both in terms of the energy and in terms of the frequency.)

In the case where the electric field dominates, i.e., E >> M, Eq. (2) yields

$$\Delta\omega \approx \pm M\cos\theta \tag{3}$$

for the σ-component and

$$\Delta\omega \approx \pm[E + (M^2\sin^2\theta)/(2E] \tag{4}$$

for the π-component.

In the opposite case where the magnetic field dominates, i.e., E << M, Eq. (2) yields

$$\Delta\omega \approx \pm[B + (E^2\sin^2\theta)/(2B] \tag{5}$$

for the σ-component and

$$\Delta\omega \approx \pm E\cos\theta \tag{6}$$

for the π-component.

Usually, the value and the direction of the magnetic field in a plasma is known. As for the electric field E, it can be represented by the LEPT and/or by the quasistatic part of the ion microfield. Therefore, the most interesting situation is posed via Eq. (6): even despite the magnetic field dominates, the splitting/broadening of the π-component is linear with respect to the projection of the electric field on the direction of the magnetic field. Thus, the Stark broadening of the π-component (that we calculate analytically below) offers the opportunity for the experimental determination of either the root-mean-square field F_t of the LEPT or of the ion density in strongly magnetized plasmas.

The Stark profile of the π-component at the fixed absolute value of the electric field is obtained by averaging over the spherically-symmetric angular distribution as follows:

$$S(\Delta\omega, E) = \int_0^1 d(\cos\theta)\, \delta(|\Delta\omega| - E\cos\theta) = (1/E)\, \Theta(E - |\Delta\omega|). \tag{7}$$

In Eq. (7), δ(…) is the delta-function and Θ(…) is the theta-function.

The next step is the averaging over the distribution of the absolute value of the LEPT, given by the Rayleigh distribution (see works [11, 12, 2]). The latter is

$$W(f)df = 3(6/\pi)^{1/2} f^2 \exp(-3f^2/2)df, \quad f = F/F_t. \tag{8}$$

Since

$$1/E = Z/(3F) = Z/(3F_t f), \tag{9}$$

then the Stark profile of the π-component becomes

$$S(\Delta\omega) = [Z/(3F_t] \int_{f_{min}}^{\infty} df\, W(f)/f, \qquad f_{min}(\Delta\omega) = Z|\Delta\omega|/(3F_t). \tag{10}$$

After calculating the integral, we obtain the following final expression for the Stark profile of the π-component:

$$S(\Delta\omega) = [Z/(3F_t] (6/ \pi)^{1/2} \exp\{-3[f_{min}(\Delta\omega)]^2/2\}. \tag{11}$$

Figure 1 shows the scaled Stark profile

$$s(D) = 3F_t\, S(D)/Z \tag{12}$$

versus the scaled detuning

$$D = Z\, \Delta\omega/(3F_t). \tag{13}$$

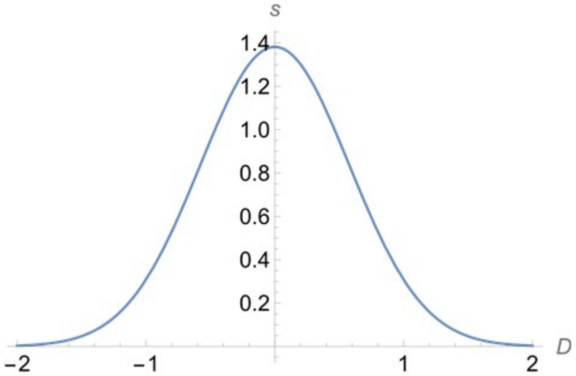

Fig. 1. The scaled Stark profile s(D) of the Ly-alpha π-component (given by Eq. (12)) versus the scaled detuning D (given by Eq. (13)) for the situation where the scaled electric field E (represented by a low-frequency electrostatic plasma turbulence) is much smaller than the magnetic field M (both fields defined in Eq. (1)).

The Full Width at Half Maximum (FWHM) of the above Stark profile is 1.36 in terms of the scaled detuning D or $4.08F_t/Z$ in terms of $\Delta\omega$. Thus, by measuring the experimental FWHM of the Ly-alpha π-component one can determine the root-mean-square field F_t of the LEPT in strongly magnetized plasmas.

In Appendix A we provide a model example for the situation where the electric field is represented by the quasistatic part of the ion microfield.

3. CONCLUSIONS

We discovered yet another unique, remarkable feature of the π-component of the Ly-alpha line – in addition to its remarkable features revealed in papers [8, 9]. Namely, in the situation where the Zeeman effect dominates over the Stark effects, the broadening of the Ly-alpha π-component tunes out to be controlled by the linear Stark effect – practically without depending on the magnetic field (while this is not the case for the Ly-alpha σ-component or any component of any other hydrogenic spectral line). Therefore, the Stark width of the Ly-alpha π-component can be used in this situation for the experimental determination of the ion density (and thus, of the electron density) or of the root-mean-square field of a Low-frequency Electrostatic Plasma Turbulence (LEPT), such as, e.g., ion-acoustic waves, or Bernstein modes, or low-hybrid waves.

We note that the LEPT had been detected by spectroscopic methods in various laboratory plasmas, for instance, in tokamaks, θ-pinches, current sheet plasmas (see, e.g., book [2] and references therein), in relativistic laser-plasma interactions (see, e.g., book [1] and references therein), as well as in solar and stellar flares (see, e.g., book [13] and references therein).

Appendix A. Model example for the situation where the electric field is represented by the quasistatic part of the ion microfield

Here we average the profile $S(\Delta\omega, E)$ from Eq. (7) over the binary distribution of the ion microfield. The latter distribution corresponds to the following distribution of the distance R of the nearest neighbor ion from the radiating atom

$$P(r)dr = 3r^2\exp(-r^3), \quad r = R/R_0, \tag{A.1}$$

where the mean interionic distance is

$$R_0 = [3/(4\pi N_i)]^{1/3}, \tag{A.2}$$

N_i being the ion density. The relation between E and r is the following:

$$1/E = ZR_0^2 r^2/(3Z_i), \tag{A.3}$$

where Z_i is the charge of the perturbing ions. Then the averaging can be expressed as

$$S(\Delta\omega) = (ZR_0^2/Z_i) \int_0^{r_{max}} dr\, r^4 \exp(-r^3) = [ZR_0^2/(3Z_i)]\, \{\Gamma(5/3) - [r_{max}(\Delta\omega)]^5\, \varphi_{2/3}[(r_{max}(\Delta\omega))^3]\}, \tag{A.4}$$

where $\Gamma(5/3)$ is the gamma function, $\varphi_{2/3}(r_{max}^3)$ is the Misra function (which is equal to the exponential integral function of the same argument, but of the negative index –2/3), and

$$r_{max}(\Delta\omega) = 3Z_i/(ZR_0^2|\Delta\omega|). \tag{A.5}$$

Figure A.1 shows the corresponding scaled Stark profile

$$s(D) = 3Z_i S(D)/(ZR_0^2) \tag{A.6}$$

versus the scaled detuning

$$D = ZR_0^2|\Delta\omega|/(3Z_i). \tag{A.7}$$

(We note that the definition of s(D) and D in Eqs. (A.6) and (A.7) differ from the analogous notations from Eqs. (12) and (13).)

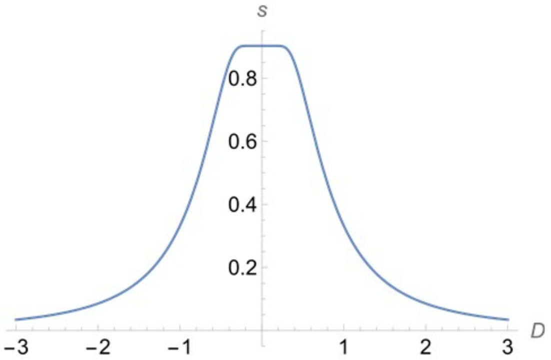

Fig. A.1. The scaled Stark profile s(D) of the Ly-alpha π-component (given by Eq. (A.6)) versus the scaled detuning D (given by Eq. (A.7)) for the situation where the scaled electric field E (represented by the quasistatic part of the ion microfield) is much smaller than the magnetic field M (both fields defined in Eq. (1)).

It is seen that the Stark profile of the Ly-alpha π-component has a unique feature: the flat top. This is a *counterintuitive* result: among the garden variety of Stark profiles of hydrogenic spectral lines previously presented in the vast amount of the literature on the Stark broadening, the flat top profile was never encountered previously.

The FWHM of the above Stark profile is 1.64 in terms of the scaled detuning D or $4.92Z_i/(ZR_0^2)$ in terms of $\Delta\omega$. We note that the latter FWHM is proportional to $N_i^{2/3}$ – according to the definition of R_0 from Eq. (A.2). Thus, by measuring the experimental FWHM of the Ly-alpha π-component in this situation one can determine the ion density (and therefore, the electron density) in strongly magnetized plasmas.

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