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Explaining of the Proton Radius Puzzle by Using the Second Flavor of Muonic Hydrogen Atoms

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ABSTRACT: Before year 2010, the proton charge radius r_p was determined by the spectroscopic method, relying on the electron energy levels in hydrogen atoms, and by the elastic scattering of electrons on protons. In 2010 and then in 2013, two research teams determined r_p from the experiment on muonic hydrogen atoms and they claimed r_p to be by about 4% smaller than it was found from the experiments with electronic hydrogen atoms. Since then, several research groups performed the corresponding experiments with electronic hydrogen atoms and obtained contradictory results: some of them claimed that they found the same value of r_p as from the muonic hydrogen experiments, while others reconfirm the larger value of r_p. The conclusions of the latest papers (including reviews) is that the puzzle is not resolved yet. In the present paper we bring to the attention of the research community, dealing with the proton radius puzzle, the contributing factor never taken into account in any previous calculations. This factor has to do with the hydrogen atoms of the second flavor, whose existence is confirmed in four different types of atomic experiments. We present a relatively simple model illustrating the role of this factor.

We showed that disregarding the effect of even a relatively small admixture of the second flavor of muonic hydrogen atoms to the experimental gas of muonic hydrogen atoms could produce the erroneous result that the proton charge radius is by about 4% smaller than its actual value, so that the larger out of the two disputed values of the proton charge radius could be, in fact, correct.

KEYWORDS: proton radius puzzle; muonic hydrogen atoms; second flavor of hydrogen atoms

1. INTRODUCTION

Before year 2010, the proton charge radius r_p was determined by the spectroscopic method, relying on the electron energy levels in hydrogen atoms, and by the elastic scattering of electrons on protons. The mean value of the proton charge radius, recommended by CODATA (Committee on Data of the International Science Council) was $r_p = (0.8775 \pm 0.0051) x 10^{-13}$ cm – see, e.g., the reviews by Pohl et al [1] and by Gao and Vanderhaenghen [2], as well as references therein.

In 2010, Pohl et al [3], and then in 2013, Antognini et al [4] determined r_p from the experiment on muonic hydrogen atoms. Because the ratio of the muon mass m_μ to the electron mass m_e is $m_\mu/m_e \approx 207$, the average muon-proton distance in muonic hydrogen atoms is about 200 smaller than the electron-proton distance in electronic hydrogen atoms. Therefore, the shift of the energy of an S-state, caused by the finite proton size, for muonic hydrogen atoms is about 8 million times greater than for electronic hydrogen atoms. Consequently, muonic measurements should be much more sensitive to r_p than the corresponding electronic measurements. The resulting proton charge radius was claimed to be $r_p = (0.84087 \pm 0.00039) x 10^{-13}$ cm, e.g., by about 4% (or 5 standard deviations) smaller than the above CODATA value. This result prompted calls for a new physics beyond the Standard Model.

In 2019, Bezginov et al [5] remeasured the n = 2 Lamb shift for electronic hydrogen atoms. They deduced the value of r_p consistent with the muonic measurements from papers [3, 4]. The same year, Xiong et al [6] remeasured r_p in the electron scattering experiment and also found it to be consistent with the muonic measurements from papers [3, 4].

The results from papers [5, 6] favor the smaller charge radius of the proton. However, they do not explain why the experimental values of r_p, found before year 2010, yielded the larger value. Besides, Fleurbaey et al [7] reported the larger value $r_p = (0.877\pm0.013)\text{x}10^{-13}$ cm, obtained from the two-photon measurements in the electronic hydrogen (they measured the 1S - 3S two-photon transition frequency of hydrogen by using a continuous-wave excitation laser at 205 nm).

So, the puzzle is not considered to be resolved yet – see, e.g., the conclusions of Karr-Marchand paper of 2019 [8] and of Gao-Vanderhaenghen review of 2022 [2].

There are many theoretical factors contributing to the shift of S-states of muonic hydrogen atoms – see, e.g., reviews by Pohl et al [1] and by Karshenboim et al [9]. In the present paper we bring to the attention of the research community, dealing with the proton radius puzzle, the contributing factor never taken into account in any previous calculations. This factor has to do with the hydrogen atoms of the second flavor, whose existence is confirmed in four different types of atomic experiments.

There are two analytical solutions of the Dirac equation for hydrogen atoms (two coupled differential equations for the components of the Dirac bispinor have two solutions). One solution is only weakly singular at small r, while the other solution is more strongly singular at small r. The second solution is rightly rejected for the model where the proton is either point-like, as well as for the models where the charge distribution inside the proton is a uniform spherical shell or a uniformly charged sphere. However, well-known experiments on the elastic scattering of electrons on protons, performed in the previous century, revealed that the actual charge distribution has the maximum at $r = 0$, thus being significantly different from the above models (see, e.g., Simon et al (1980) [10] and Perkins (1987) [11]).

In papers [12, 13], the following was shown analytically. After taking into account the actual charge distribution inside the proton, the second solution outside the proton can be tailored with the regular solution inside the proton for any S-state. In other words, the second solution outside the proton is legitimate for all S-states. This second type of hydrogen atoms possessing only the S-states (the energies of the S-states being the same as for the usual first solution) was later named the Second Flavor of Hydrogen Atoms (SFHA) – by using analogy with the quantum chromodynamics where up and down quarks are named two flavors [14].

Outside the proton, for the S-states at small r, the radial wave function R(r) for the first solution scales as $\sim 1/r^{\beta/2}$ where

$$\beta = \alpha^2, \tag{1}$$

(α being the fine structure constant ($\alpha = e^2/(\hbar c) \approx 0.007297$), while for the SFHA R(r) scales as $\sim 1/r^{2-\beta/2}$. Consequently, for relatively large values of the linear momentum $p >> p_0 = me^2/\hbar$ (where m is the mass of the atomic lepton, whether it is electron or muon), the corresponding wave function in the momentum representation $\varphi(p)$ for the SFHA falls off much slower than for the hydrogen atoms of the first (usual) flavor. This is because $\varphi(p)$ and R(r) are interconnected by the Fourier transform, so that for the SFHA, the more rapid increase of R(r) as r decreases translates into the slower decrease of $\varphi(p)$ as p increases in the range of $p >> p_0$.

By now the existence of the SFHA is proven in four various types of atomic experiments, as follows.

A. Experimental distribution dw = F(p)dp of the linear momentum p in the ground state of electronic hydrogen atoms.

For $p_0 << p << mc$, i.e., in the non-relativistic part of the tail of the distribution (we note that $p_0/mc = \alpha \approx 0.007297$), the experimental result, deduced by Gryzinski [15] from the analysis of atomic experiments, was $F_{exper}(p) \sim (mc/p)^4$, while the corresponding theoretical result by Fock [16] was $F_{theor}(p) \sim (mc/p)^6$. (Here F(p)dp is the probability of finding the linear momentum in the interval (p, p+dp).) This means that for the ratio $F_{theor}(p)/F_{exper}(p) = (mc/p)^2$, for the values of $p \sim 10p_0$, the discrepancy $F_{theor}(p)/F_{exper}(p)$ between the experimental and theoretical results was ~ 200 times (!).

In paper [12] it was shown that with the allowance for the SFHA, this huge discrepancy was completely eliminated. No alternative explanation of this huge discrepancy was ever offered.

B. Experiments on the electron impact excitation of electronic hydrogen molecules

There was a discrepancy by at least a factor of two between the experimental and theoretical cross-sections of the excitation to the lowest triplet states, as pointed out in paper [17]. In the same paper it was shown that this large discrepancy can be eliminated if the SFHA was present in the experimental gas. Again, no alternative explanation of this significant discrepancy was ever offered.

C. Experiments on the electron impact excitation of electronic hydrogen atoms

The theoretical ratio of the cross-section for the excitation for the state 2s to the cross-section for the excitation of the state 2p was by 20% higher than the corresponding experimental ratio – well beyond the experimental error margin of 9%, as pointed out in paper [18]. In the same paper it was shown that this significant discrepancy can be eliminated if the SFHA was present in the experimental gas. Again, no alternative explanation of this significant discrepancy was ever offered.

D. Experiments on the charge exchange between electronic hydrogen atoms and protons

There was a noticeable discrepancy between the experimental and theoretical cross-sections, as pointed out in paper [19]. In the same paper it was shown that this noticeable discrepancy can be eliminated if the SFHA was present in the experimental gas. Again, no alternative explanation of this significant discrepancy was ever provided.

The present paper has two cruxes. The first crux is that muonic hydrogen atoms should also have two flavors – because all analytical results from paper [12] for the ground state and their generalization in paper [13] for any S-state are valid for muonic hydrogen atoms after replacing m_e in those calculations by m_μ. So, there should exist the Second Flavor of Muonic Hydrogen Atoms (SFMHA).

The second crux of the present paper is that since for the SFMHA the radial wave function R(r) in the vicinity of the proton – and consequently inside the proton (because both the outside and inside parts of R(r) match at the proton boundary) – is significantly different compared to the usual muonic hydrogen atoms, then even a relatively small admixture of the SFMHA to the usual muonic hydrogen atoms in the experimental gas can affect the shift of the S-states and thus modify the determination of the proton charge radius from the experimental Lamb shift of muonic hydrogen atoms.

We present a simple model illustrating that even a relatively small admixture of the SFMHA to the usual muonic hydrogen atoms in the experimental gas can lead to the false conclusion that the proton charge radius is by about 4% smaller than its actual value.

2. MODEL

For the ground state of muonic hydrogen atoms, outside the proton, the radial part of the Dirac bispinor, based on Eq. (17) from paper [12], can be represented in the form:

$$\begin{aligned} f(r) &\approx -2\beta^{5/4}\,\{1/r^{\beta/2} - \varepsilon[R_p^2/(5\beta r^2)]\}, \\ g(r) &\approx 4\beta^{3/4}\,\{1/r^{\beta/2} - \varepsilon[R_p^2/(5\beta r)]\}. \end{aligned} \tag{2}$$

In Eq. (2), ε is the relatively small share of the SFMHA in the experimental muonic hydrogen gas ($\varepsilon << 1$), R_p is the proton radius in units of the muonic Bohr radius $a_{0\mu} = \hbar^2/(m_\mu e^2)$, r is the distance from the origin in units of the muonic Bohr radius $a_{0\mu}$. Eq. (2) was simplified compared to Eq. (17) from paper [12], by using the fact that $\beta = \alpha^2 << 1$. We also note that in Eq. (17) from paper [12], the second term in f(r) and g(r) was proportional to the quantity

$$\Delta = E_0 - E, \tag{3}$$

which is the shift (with the minus sign) of the ground state energy due to the finite proton size, the shift being in units of $m_\mu c^2$. Since in our Eq. (2), the second term in f(r) and g(r) is assumed to be a relatively small correction to the first term (since $\varepsilon << 1$), then while deriving Eq. (2) we used for the shift the following approximate textbook expression (see, e.g., Flügge textbook [20]):

$$|\Delta| \approx 2\beta R_p/5. \tag{4}$$

The squared absolute value of the wave function of the ground state is

$$4\pi[f^2(r) + g^2(r)]. \tag{5}$$

From Eq. (2) it is seen that $f^2(r)/g^2(r) \sim \alpha^2 << 1$, so that

$$|\Psi_0(r)|^2/(4\pi) \approx g^2(r) \approx 16\beta^{3/2}/r^\beta - \varepsilon[32\beta^{1/2}R_p^2/(5r^{1+\beta/2})] + \varepsilon^2[16R_p^4/(25\beta^{1/2}r^2)]. \tag{6}$$

The shift of the ground state energy äE due to the proton finite size is (in analogy to Eq. (3) from review [1] or to Eq. (66) from review [2])

$$\delta E(\varepsilon, R_p) = b\,|\Psi_0(R_p)|^2\, R_p^2 = b\{16\beta^{3/2}/R_p^{\beta/2} - \varepsilon(32\beta^{1/2}R_p^{1-\beta/2}/5) + \varepsilon^2[16R_p^2/(25\beta^{1/2})]\}, \tag{7}$$

where b is a constant of no importance for the purpose of the present paper. We would like to find out whether there exist a value of $\varepsilon << 1$, such that

$$\delta E(\varepsilon, R_p) = \delta E(0, 0.96R_p), \tag{8}$$

so that while disregarding a relatively small admixture of the SFMHA to the experimental muonic hydrogen gas, one would deduce – from the experimental shift – the value of R_p that would be by 4% smaller than the actual value of R_p.

Equation (8) is quadratic with respect to ε – so, it has the following two solutions:

$$\varepsilon_1 = 1.07\text{x}10^{-5}/R_p^{1.000027} \approx 1.07\text{x}10^{-5}/R_p, \tag{9}$$

$$\varepsilon_2 = 5.22\text{x}10^{-4}/ R_p^{1.000027} \approx 5.22\text{x}10^{-4}/R_p. \tag{10}$$

The numerical value of the proton charge radius r_p (defined as the root-mean-square radius of the proton charge distribution) in units of the muonic Bohr radius $a_{0\mu}$ is 0.00343. The proton "sphere" radius R_p would be by the factor of $(5/3)^{1/2}$ greater than r_p (it would be equal to 0.00443) if the proton would be a uniformly charged sphere (what the proton is not). The actual value of R_p should be between 0.00343 and 0.00443. For further numerical estimates of ε_1 and ε_2 we adopt the value $R_p \approx 0.004$, so that

$$\varepsilon_1 \approx 0.003, \quad \varepsilon_2 \approx 0.13. \tag{11}$$

Physically, the share of the SFMHA $\varepsilon_2 = 0.13$ seems to be slightly more preferable (compared to $\varepsilon_1 = 0.003$). This is because it is of the same order of magnitude as the share of the SFHA in the experimental gas of the electronic hydrogen molecules, which (the share) was required for eliminating the large discrepancy (by at least of a factor of two) between the theoretical and experimental cross-sections of the excitation by the electron impact [17].

Since the proton charge radius r_p is proportional to R_p, then the above result about the determination of R_p from the energy shift is also true for r_p. Namely, indeed, even a relatively small admixture of the SFMHA to the usual muonic hydrogen atoms in the experimental gas can lead to the false conclusion that the proton charge radius r_p is by about 4% smaller than its actual value.

3. CONCLUSIONS

We presented a model illustrating the effect of the SFMHA on the determination of the proton charge radius from the experimental energy shift of muonic hydrogen atoms. We showed that disregarding the effect of even a relatively small admixture of the SFMHA to the experimental gas of muonic hydrogen atoms could produce the erroneous result that the proton charge radius is by about 4% smaller than its actual value, so that the larger out of the two disputed values of the proton charge radius could be, in fact, correct.

We do not claim that this model yields the final resolution of the multi-year dispute about the proton charge radius. We presented this relatively simple model just to get the message across: to up to the attention of the corresponding research community the importance of the factor disregarded in all previous theoretical works aimed on deducing the proton charge radius from the experimental data. This factor is the SFMHA – the muonic counterpart of the electronic SFHA, whose existence is proven in four different types of atomic experiments. We hope that our results would motivate further theoretical works in this very fundamental area of physics.

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