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Experimental Studies of Molecular Hydrogen Ions Formed by Proton Collisions with Hydrogen Atoms May Yield Another Proof of the Existence of the Second Flavor of Hydrogen Atoms

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ABSTRACT: By now there are proofs of the existence of the Second Flavor of Hydrogen Atoms (SFHA) from three different kinds of atomic experiments: 1) from the experimental distribution of the linear momentum in the ground state of hydrogen atoms; 2) from the experimental cross-sections of charge exchange between hydrogen atoms and low-energy protons; 3) from the experimental cross-sections of the electron impact excitation of hydrogen atoms in the states of the principal quantum number n=2. In the present paper we analyze Molecular Hydrogen Ions (MHIs) formed by collisions of low-energy protons with the SFHA. We find that the resulting MHIs would lack a significant number of terms compared to the MHIs formed by collisions of low-energy protons with the usual hydrogen atoms. We show that in this situation, the radiative transition between the terms of such MHIs of the lowest quantum numbers would be between the terms 5fσ and 4dσ. We calculate the position of the edge of the corresponding molecular band and find it to be at the frequency 14700 cm–1 or equivalently at the wavelength of 680 nm, belonging to the visible range. It should be easier to observe this band compared to the spectral bands that are completely beyond the visible range. We emphasize that these results open up another avenue for finding an additional experimental proof of the existence of the SFHA. Namely, if the SFHA is present in the gas (in addition to the usual hydrogen atoms), on which a beam of low-energy protons is incident, then the relative intensity of the band, corresponding to the radiative transitions between the terms 5fσ and 4dσ of the MHIs, would be enhanced compared to the absence of the SFHA.

Keywords: second flavor of hydrogen atoms; molecular hydrogen ion; proton collisions; molecular spectral bands

1. INTRODUCTION

The theoretical discovery of the SFHA in paper [1] was motivated by the huge discrepancy (by several orders of magnitude) between the experimental high-energy tail f the linear momentum distribution in the ground state of hydrogen atoms and the previous theories. The allowance for the SFHA eliminated this huge discrepancy. Since then, there were found also two other proofs of the existence of the SFHA from atomic experiments: one – from the experiments on charge exchange of hydrogen atoms with incoming protons [2], another – from the experimental cross-sections of the electron impact excitation of hydrogen atoms in the states of the principal quantum number n=2 [3].

There are also two kinds of the astrophysical evidence of the existence of the SFHA. The first one is related to the puzzling observation of the redshifted 21 cm spectral line from the early Universe by Bowman et al [4]. They found that the absorption in this spectral line was about two times stronger than predicted by the standard cosmology. This meant that the primordial hydrogen gas was significantly cooler than predicted by the standard cosmology. In paper [5] it was shown that this big discrepancy between the observations by Bowman et al [4] and the standard cosmology can be eliminated – qualitatively and quantitatively – in the case where the additional cooling was due to collisions with the SFHA. These results made the SFHA a candidate for dark matter.

The second astrophysical evidence of the existence of the SFHA is the following. Jeffrey at al [6] recently found from observations that the distribution of dark matter in the Universe is smoother than predicted by Einstein's gravitation, what prompted calls for a non-Einsteinian gravity, i.e., for new physical laws. However, in paper [7] it was demonstrated that this perplexing observation can be also explained – qualitatively and quantitatively – by using the SFHA.

The SFHA-based explanations of the puzzling astrophysical observations by Bowman et al [4] and by Jeffrey et al [6] did not require any change of physical laws (and thus were favored by the Occam razor principle). The theoretical discovery of the SFHA in paper [1] was based on the standard Dirac equation of quantum mechanics.

In the present paper we offer an experimental possibility for an additional proof of the existence of the SFHA. It is about collisions of low-energy protons with the SFHA. One of the possible outcomes of such collisions is charge exchange: for the SFHA it has a slightly larger cross-section than for the usual hydrogen atoms – as shown in paper [2] where the comparison with experiments confirmed this theoretical result. In the present paper we focus on another outcome of such collisions: the capture of the incoming proton and the formation of the Molecular Hydrogen Ions (MHIs). We reveal peculiar features of the MHIs formed in this way. This opens up another avenue for finding an additional experimental proof of the existence of the SFHA.

2. ALLOWED MOLECULAR TERMS AND RADIATIVE TRANSITIONS

There have been lots of theoretical studies of the MHI – see, e.g., reviews [8-10] and references therein, as well as papers [11, 12] and references therein. There are several reasons for this. From the theoretical point of view, the MHI is the simplest stable molecule and thus represents the test-bench of molecular quantum mechanics. From the applied point of view, the MHI is important in astrophysics because it is involved in reaction chains leading to the formation of polyatomic molecules.

From the theoretical point of view, at fixed nuclei (which in this situation are protons), the MHI is a particular case of the two-Coulomb center system, the latter consisting of two fixed nuclei of charges Z_1 and Z_2 separated by a distance R, and one electron. The two-Coulomb center system allows the complete separation of variables (in elliptical coordinates) – see, e.g., the textbook [13]. This is the consequence of the higher than geometric symmetry of this system, manifested by the existence of an additional conserved quantity: the projection of the super-generalized Runge-Lenz vector on the internuclear axis [14].

As a result of the complete separation of variables, states of the system are described by sets of elliptical quantum numbers {k, q, m}. Here k is the radial quantum number, q is the angular quantum number, and m is the azimuthal quantum number: they are equal to the number of nodes of the corresponding parts of the wave function. In particular, the azimuthal quantum number m is the projection of the orbital momentum (in atomic units) on the internuclear axis.

Usually, instead of (k, q, m), there are introduced the following linear combinations: the orbital quantum number

$$l = q + m \tag{1}$$

and the principal quantum number

$$N = k + q + m + 1 = k + l + 1. \tag{2}$$

Then the states are described by the sets (N, l, m). Finally, the numerical values $l = 0, 1, 2, 3, \ldots$ are substituted by letters s, p, d, f, …, respectively, and the numerical values of m = 0, 1, 2, … are substituted by Greek letters σ, π, δ, …. For example, the state (1, 0, 0) becomes denoted as 1sσ, the state (3, 2, 1) becomes denoted as 3dπ, and so on.

When the incoming slow proton is relatively far from the hydrogen atom, i.e., for a relatively large internuclear separation R, the states are described by the sets of the parabolic quantum numbers [n_1, n_2, m] – see, e.g., the textbook [13]. For the purpose of our paper, the most important is the correspondence between these parabolic quantum numbers (relevant for relatively large R) and the elliptical quantum numbers (relevant for relatively small R). According to book [15], the correspondence is the following.

For any Z_1 and Z_2:

$$k = n_1. \tag{3}$$

As for the correspondence between the other quantum numbers, for $Z_1 = Z_2$ (i.e., for the situation we are interested in), there are two subcases. For even q, one has

$$q = 2n_2, \tag{4}$$

while for odd q, one has

$$q = 2n_2 + 1. \tag{5}$$

Here we come to the central point. The primary distinctive feature of the SFHA is that is has only states of the zero orbital momentum [1, 5, 16]: the S-states, which are spherically symmetric. In terms of the parabolic quantum numbers, the spherically symmetric states can correspond only to the situation where

$$n_1 = n_2, \qquad m = 0. \tag{6}$$

Therefore, in this situation, Eq. (1) becomes

$$l = q \tag{7}$$

and Eq. (3) becomes

$$k = n_2, \tag{8}$$

so that Eq. (2) becomes

$$N = l + n_2 + 1. \tag{9}$$

On substituting $q = l$ (according to Eq. (7)) in Eqs. (4) and (5), we arrive to the following results (taking into account also Eq. (9)). For even n_2, the possible states have the quantum numbers

$$l = 2n_2, \qquad N = 3n_2 + 1. \tag{10}$$

For odd n_2, the possible states have the quantum numbers

$$l = 2n_2 + 1, \qquad N = 3n_2 + 2. \tag{11}$$

(We remind that $n_2 = 0, 1, 2, \ldots$.)

In other words, the MHI formed by collisions with low-energy protons with the SFHA would have significantly smaller number of energy terms compared to the usual MHI. Namely, the even terms are

$$1s\sigma,\ 4d\sigma,\ 7f\sigma,\ \ldots \tag{12}$$

The odd terms are

$$2p\sigma,\ 5f\sigma,\ 8g\sigma,\ \ldots \tag{13}$$

Now let us consider the consequences of these peculiar features of the energy terms (of this kind of the MHI) in the radiation spectrum. All allowed radiative transitions are between the terms of $m = 0$, so that the radiation would have the π-polarization only ($\Delta m = 0$). Further, taking into account the selection rule $|\Delta l| = 1$, we find the following.

The radiative transitions between the terms of the lowest quantum numbers might have been between the terms 2pσ and 1sσ. However, while the term 2pσ corresponds to the stable state of the molecule, the term 1sσ does not.

As we proceed to the terms of higher quantum numbers, the next possibility is the radiative transition between the terms 5fσ and 4dσ. Figure 1, which we created using the tables from Madsen and Peek paper [17], shows these two energy terms. It is seen that both these terms correspond to the stable states of the molecule.

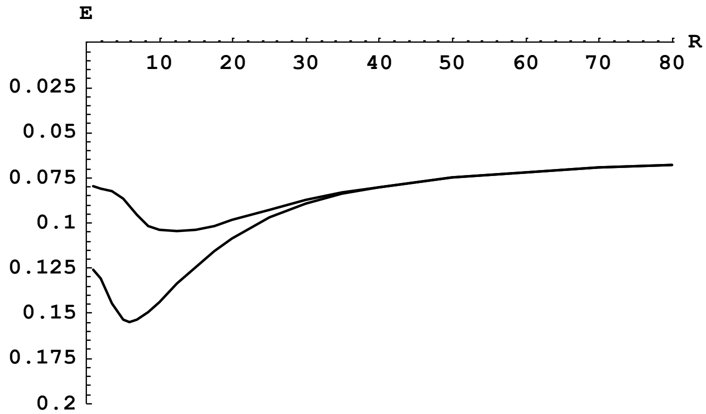

Fig. 1. Two of the terms of the MHI formed by collisions of low-energy protons with the SFHA. The lower curve: term 4dσ. The upper curve: term 5fσ. The energy E is in units of hartree, the internuclear distance R is in atomic units.

Figure 2 shows the frequency F (in units of 10^5 cm^{-1}) of the radiative transitions between these two terms versus the internuclear distance R (in atomic units). At the maximum (located at R = 5), the frequency is F = 14700 cm^{-1}. This means, that the edge of the corresponding spectral band is at the wavelength of 680 nm. So, this spectral band starts at the red part of the visible range (and continues through the red part into the infra-red range). Thus, it should be easier to observe compared to the spectral bands that are completely beyond the visible range.

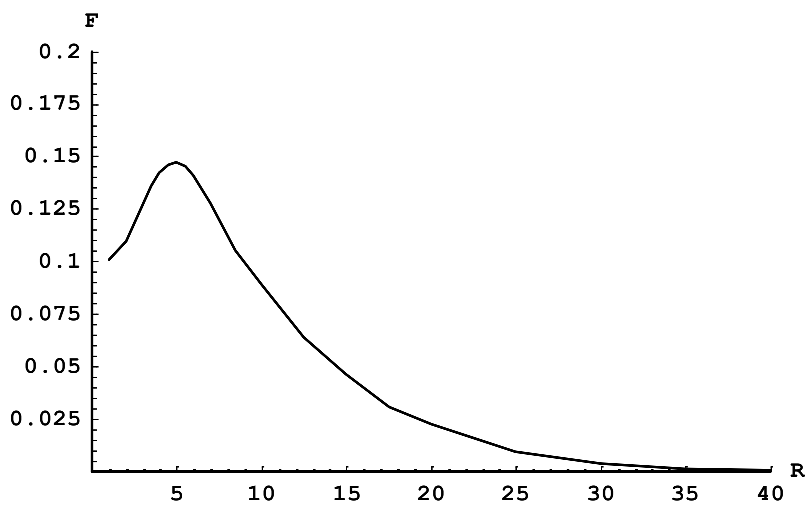

Fig. 2. The frequency F (in units of 10^5 cm^{-1}) of the radiative transitions between these two terms from Fig. 1 versus the internuclear distance R (in atomic units).

These results open up another avenue for finding an additional experimental proof of the existence of the SFHA. Indeed, let us consider an experiment where a beam of low-energy protons is incident on a gas of hydrogen atoms. If the SFHA is present in the gas (in addition to the usual hydrogen atoms), then the *relative intensity* of the band, corresponding to the radiative transitions between the terms 5fσ and 4dσ, would be enhanced compared to the absence of the SFHA. This is because the SFHA would not contribute to the usually observed bands, corresponding to the radiative transitions between the terms of lower quantum numbers.

3. CONCLUSIONS

We considered the MHIs formed by collisions of low-energy protons with the SFHA. We found that the resulting MHIs would lack a significant number of terms compared to the MHIs formed by collisions of low-energy protons with the usual hydrogen atoms.

We showed that in this situation, the radiative transition between the terms of such MHIs of the lowest quantum numbers would be between the terms 5fσ and 4dσ. We calculated the position of the edge of the corresponding molecular band and found it to be at the frequency 14700 cm^{-1} or equivalently at the wavelength of 680 nm, belonging to the visible range. So, it should be easier to observe this band compared to the spectral bands that are completely beyond the visible range.

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