RC Circuit and Time Constant

Object: To investigate the voltages across the resistor and capacitor in a resistor-capacitor circuit (RC circuit) as the capacitor charges and discharges. We also wish to determine the capacitative time constant for the circuit.

Apparatus: Resistor (100Ω), capacitor (330 μF), two voltage sensors, power amplifier, patch-cords, computer, PASCO Interface, and Data Studio Software.

FOREWORD

Consider the circuit shown in **Figure 1**:

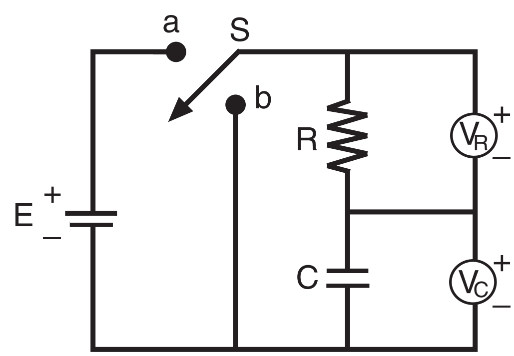


Figure 1. A resistor-capacitor series circuit.

When the switch S is thrown to position *a*, charges start to move and we have a current flow. These charges move through the resistor and begin to charge the capacitor. At any time, the sum of the voltages around the circuit loop must be zero (Kirchoff's Rule), hence we have:

(1) $$E - V_R - V_C = 0$$

Where:

$$E = \text{the applied voltage}$$

(2) $$V_R = iR = \text{the voltage drop across the resistor}$$

$$V_C = \frac{q}{C} \quad \text{the voltage drop across the capacitor}$$

When **(2)** is inserted into equation **(1)** we obtain:

(3) $$E = iR + \frac{q}{C}$$

If we include the relationship between the current and the charge, namely

(4) $$i = \frac{dq}{dt}$$

We obtain:

(5) $$E = R\left(\frac{dq}{dt}\right) + \frac{q}{C}$$

Equation (**5**) contains two variables, the charge on the capacitor (q) and the time (t). After separating these variables we have:

(6) $$\left(\frac{1}{RC}\right) dt = \frac{dq}{(EC - q)}$$

Setting equation **(6)** up for integration and using the fact that at the time $t = 0$ the charge $q = 0$ as the lower limit and the fact that at some other time t the charge is q as the upper limit we have:

(7) $$\left(\frac{1}{RC}\right)\int_0^t dt = \int_0^q \left(\frac{dq}{EC - q}\right)$$

Let's now make a change of variable by letting $u = EC - q$ and hence $du = -dq$. We will also need to change the limits of the q integration by noting that when $q = 0$ the lower limit is $u = EC$ and for the upper limit we replace q with $u = EC - q$. This allows us to rewrite (**7**) as follows:

(8) $$-\left(\frac{1}{RC}\right)\int_0^t dt = \int_{EC}^{EC-q}\left(\frac{du}{u}\right)$$

Equation (**8**) integrates to:

(9) $$-\left(\frac{1}{RC}\right) t\Big|_0^t = \ln u\Big|_{EC}^{EC-q}$$

Inserting the limits we have:

(10) $$-\left(\frac{1}{RC}\right)t = \ln(EC - q) - \ln(EC) = \ln\left(\frac{EC - q}{EC}\right)$$

Taking the antilog of equation (**10**) we obtain:

(11) $$e^{-t/RC} = (EC - q) / EC$$

Which may be solved for q to obtain:

(12) $$q = EC\,(1 - e^{-t/RC})$$

Looking at equation **(12)**, we see that after time t >> RC, the capacitor is fully charged and the charge on it is

(13) $$q = EC = Q_{max}$$

If the charge on the capacitor at any time is given by equation **(12)**, then the potential difference across the capacitor at any time is

(14) $$V_C = q / C = EC\,(1 - e^{-t/RC}) / C = E\,(1 - e^{-t/RC})$$

A plot of V_C as a function of time is shown in **Figure 2**.

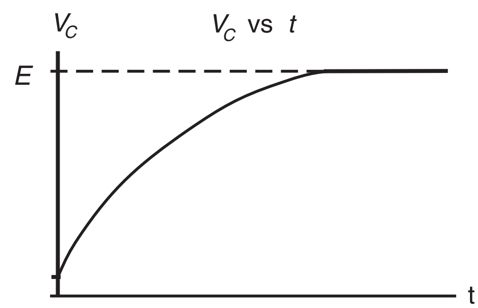


Figure 2. **Plot of the potential difference V_C across the capacitor in a RC-circuit as a function of time while the capacitor is charging**

The current in the circuit at any time may be obtained from equation **(12)** as follows:

(15)
$$i = \frac{dq}{dt} = \frac{d[EC(1 - e^{-t/RC})]}{dt} = \left(\frac{E}{R}\right) e^{-t/RC} = I_{Max} e^{-t/RC}$$

where:

(16)
$$I_{max} = E / R$$

From equation **(15)**, note that at $t = 0$ the current has the maximum value of E / R and after a long time compared to RC the current is zero.

If the current in the circuit at any time is given by equation **(15)**, the potential difference across the resistor at any time t is

(17)
$$V_R = iR = I_{\max} \mathrm{R} e^{-t/RC} = E e^{-t/RC}$$

A plot of V_R as a function of time is shown in **Figure 3**.

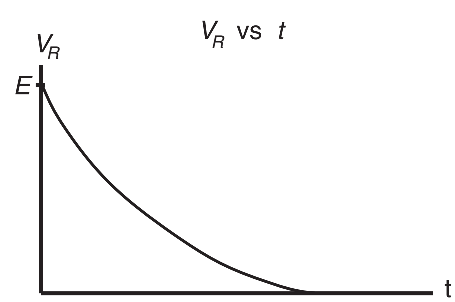


Figure 3. **Plot of the potential difference V_R across the resistor in a RC-circuit as a function of time while the capacitor is charging.**

Figure 4 gives a summary of what happens when the switch *S* is thrown to position *a* and the potential difference *E* is applied to the RC circuit charging the capacitor.

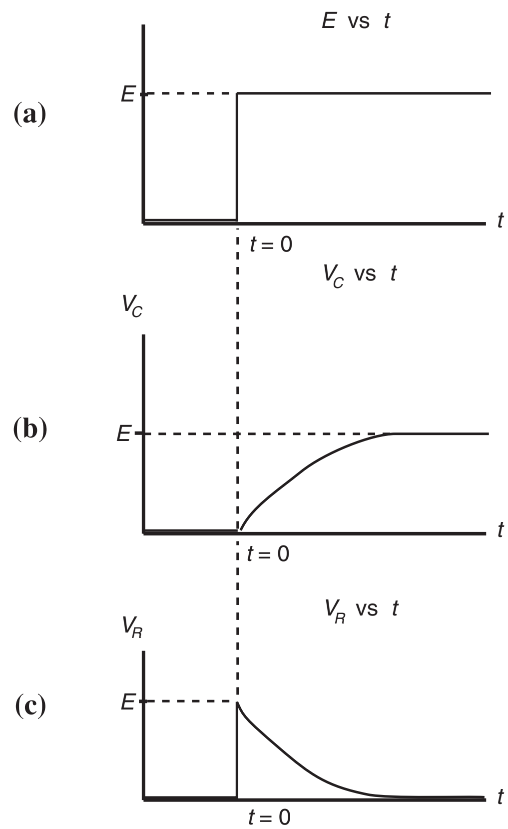


Figure 4. **(a) Voltage E applied to the RC-circuit as a function of time.**
(b) Potential difference V_C across the capacitor as a function of time.
(c) Potential difference V_R across the resistor as a function of time.
Note that at all times $V_C + V_R = E$.

Now that we have a good record of what happens when the switch is thrown to position "*a*" and the capacitor charges, let's throw the switch to position "*b*" and let the capacitor discharge.

Once again, the sum of the voltages around the loop must be zero, hence

(18) $$V_R + V_C = 0$$

where:

(19) **E = O, there is no applied voltage**
$V_R = iR$ = the voltage drop across the resistor
$V_C = q / c$ = the voltage drop across the capacitor

When **(19)** is inserted into equation **(18)** we obtain

(20) $$q / c = -iR$$

Use the relationship between i and q [equation (**4**)] to obtain

(21) $$\frac{q}{RC} = \frac{-dq}{dt}$$

Separate the variables q and t to obtain

(22) $$\frac{-dt}{RC} = \frac{dq}{q}$$

Setting equation (**22**) up for integration and making use of the fact that at $t = 0$, $q = Q_{max} = EC$ as the lower limit and at some time t later the charge on the capacitor is q as the upper limit, obtain

(23) $$-(1/RC) \int_0^t dt = \int_{Q_{\max}}^q dq / q$$

Integrate equation (**23**) to obtain

(24) $$-(1/RC)\, t\Big|_0^t = \ln q\Big|_{Q_{\max}}^q$$

Insert the upper and lower limits into equation (**24**) to obtain

(25) $$-(1/RC)\, t = \ln q - \ln Q_{\max} = \ln (q / Q_{\max})$$

Take the antilog of equation (**25**) to obtain

(26) $$e^{-t/RC} = (q / Q_{\max})$$

Solving equation (**26**) for q, obtain the following expression for the charge on the discharging capacitor as a function of time.

(27) $$q = Q_{\max} e^{-t/RC}$$

If the charge of the capacitor at any time as it discharges is given by equation (**27**) the potential difference across the capacitor at any time as it discharges is

(28) $$V_C = q / C = (Q_{\max} / C) e^{-t/RC} = \mathbf{E} e^{-t/RC}$$

Looking at equations (**27**) and (**28**) we see that at $t = 0$ the charge on the discharging capacitor has its maximum value $Q_{\max}$ and the potential difference across the capacitor has a maximum value of E. After some time $t >> RC$, there is no charge on the capacitor, the potential difference across the capacitor is zero, and the capacitor is totally discharged.

A plot of V_C as a function of time is shown in **Figure 5**.

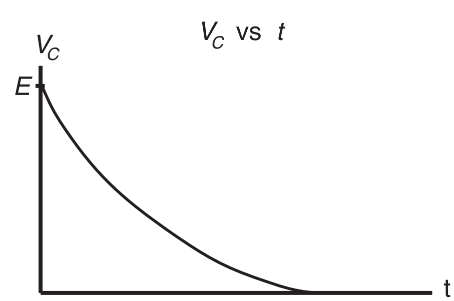


Figure 5. Plot of the potential difference V_C across the capacitor in a RC-circuit as a function of time while the capacitor is discharging.

As the capacitor discharges, the current at any time obtained from equation (**27**) is as follows:

(29) $$i = dq / dt = d(Q_{\max}\, e^{-t/RC}) / dt = -(Q_{\max} / RC) e^{-t/RC} = -(E / R) e^{-t/RC} = -I_{\max}\, e^{-t/RC}$$

If the current in the circuit of the discharging capacitor at any time is given by (**29**). the potential difference across the resistor at any time t is

(30) $$V_R = iR = -I_{max}Re^{-t/RC} = -Ee^{-t/RC}$$

where:

(31) $$E = I_{max}R$$

A plot of V_R as a function of time is shown in Figure 6.

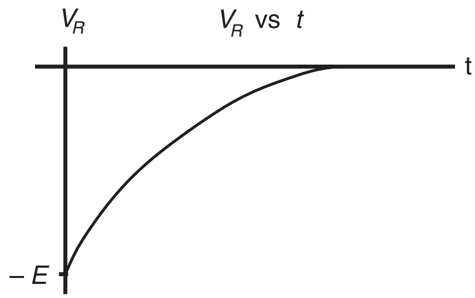


Figure 6. Plot of the potential difference V_R across the resistor in a RC-circuit as a function of time while the capacitor is discharging.

Figure 7 gives a summary of what happens when the switch S is thrown to position b, the applied voltage E is disconnected, and the capacitor is discharged.

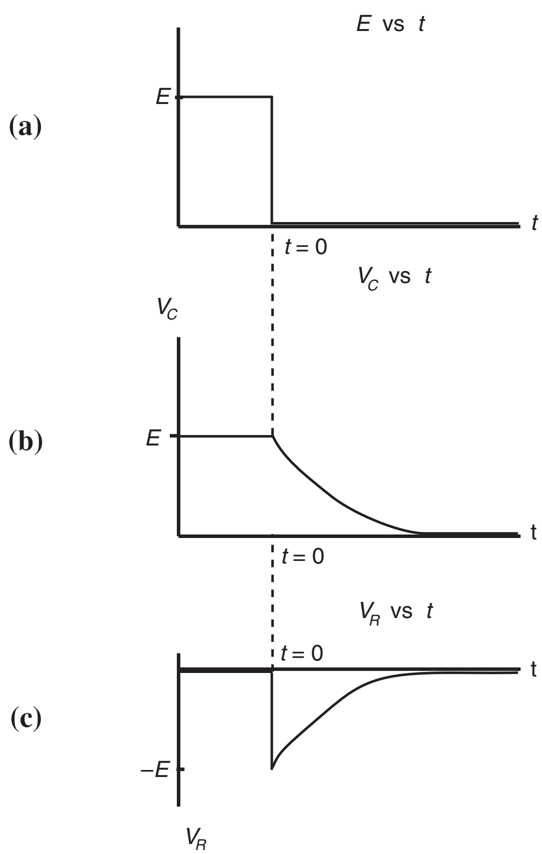


Figure 7. (a) Voltage E applied to the RC-circuit as a function of time.
(b) Potential difference V_C across the capacitor as a function of time.
(c) Potential differenceV_R across the resistor as a function of time.
Note that at all times $V_C+V_R = E$.

Now if the switch is flipped back and forth from *a* to *b* to *a* to *b* etc., the voltage is alternately applied and disconnected, and the capacitor alternately charges and discharges. All of the preceding information allows us to conclude that a record of E, V_R and V_C would look like Figure 8.

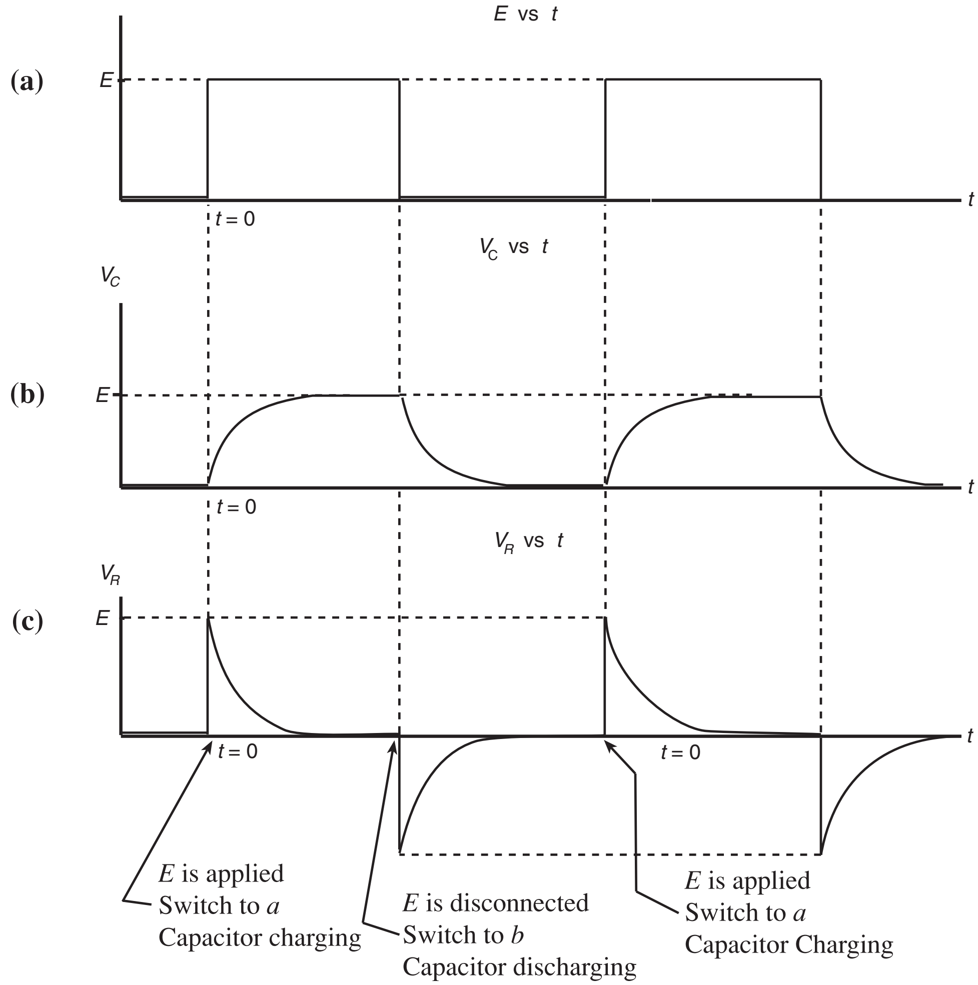


Figure 8.

For a RC-circuit, the quantity *RC* is a characteristic time called the capacitative time constant. The time constant is the time it takes the capacitor to charge to 63% of its maximum charge or alternately the time for a discharging capacitor to lose 63% of its charge. This is shown below by using the time $t = RC$ in the expression for the charge on a charging capacitor [equation (**12**)] and in the expression for the charge on a discharging capacitor [equation (**27**)].

For a charging capacitor

(12) $$q = Q_{max}\,(1 - e^{-t/RC})$$

Inserting the time $t = RC$, obtain:

$$q = Q_{max}\,(1 - e^{-RC/RC})$$

$$= Q_{max}\,(1 - e^{-1})$$

$$= (0.63)Q_{max}$$

For a discharging capacitor

(27) $$q = Q_{max}e^{-t/RC}$$

Inserting the time $t = RC$, obtain:

$$Q_{max}e^{-RC/RC}$$

$$= Q_{max}e^{-1}$$

$$= (0.37)Q_{max}$$

From the above we see that after a time $t = RC$ the charge on a capacitor is 63% of its maximum value and the discharging capacitor has a charge of 37% of its maximum value (that is it has lost 63% of its maximum charge).

As a matter of convenience, the time constant is represented by the symbol τ, hence

$$\tau = RC \tag{31}$$

Since the potential difference across the capacitor at any time is $V_C = q / C$, we may also say that the time constant is the time for the potential difference across a charging capacitor to reach 63% of the applied voltage (E) and/or it is the time for the potential difference across a discharging capacitor to fall to 37% of the applied voltage.

Another unique time for a RC circuit is the time for the charge on the capacitor or the potential difference across a charging capacitor to rise to one half its maximum value and/or to fall to one half its maximum value for a discharging capacitor. We will refer to this time as $T_{1/2}$.

For a charging capacitor

$$V_C = E(1 - e^{-t/RC}) \tag{14}$$

If $t = T_{1/2}$ then $V_C = \frac{1}{2}E$. Also, $\tau = RC$.

Inserting these values, we obtain:

$$\frac{1}{2} E = E\,(1 - e^{-T_{1/2}/\tau})$$

$$e^{-T_{1/2}/\tau} = \frac{1}{2}$$

$$\frac{T_{1/2}}{\tau} = 1\text{n}2$$

$$\tau = \frac{T_{1/2}}{1\text{n}2} \tag{32}$$

For a discharging capacitor

$$V_C = Ee^{-t/RC} \tag{28}$$

If $t = T_{1/2}$ then $V_C = \frac{1}{2}E$. Also, $\tau = RC$.

Inserting these values, we obtain:

$$\frac{1}{2} E = Ee^{-T_{1/2}/\tau}$$

$$e^{-T_{1/2}/\tau} = \frac{1}{2}$$

$$\frac{T_{1/2}}{\tau} = 1\text{n}2$$

$$\tau = \frac{T_{1/2}}{1\text{n}2} \tag{32}$$

This gives us an alternate method for finding the time constant. We simply go to the $V_C = E / 2$ position on the V_C *vs* t plot of a charging or discharging capacitor and the corresponding time is $T_{1/2}$. We can then determine the time constant using equation **(32)**.

PROCEDURE

In this activity, a power amplifier is used to produce a low frequency positive only square wave. When this wave form is applied to a RC series circuit, it has the same effect as connecting and then disconnecting a DC voltage source. When the voltage source is connected and then disconnected, the capacitor charges and then discharges. We will observe the output of the power amplifier and use voltage sensors to measure the voltage across the resistor and capacitor as the capacitor charges and discharges.

We will use a graphical display of these voltages (i.e. E, V_R and V_C) to investigate the behavior of the RC-circuit while the charge is increasing, steady at its maximum value, and decreasing. We will also determine the time constant for the RC-circuit directly and indirectly.

Part I. Initial Equipment and Software Set-up

1. Start Data Studio, and select *Create Experiment.*

2. Connect the Power Amplifier to the computer via an analog port on the PASCO interface. Plug the Power Amplifier into an AC outlet, and turn the power amplifier on. Connect the power amplifier to an RC circuit consisting of a 100Ω Resistor and a 330μF Capacitor.

3. Inform the software which analog port you plugged the Power Amplifier into by selecting the Power Amplifier icon and dragging it to the appropriate analog port.

4. A signal generator box should appear. Change the wave pattern from the default *Sine Wave* to a *Positive Square Wave*. Note that a *Positive Square Wave* and a *Square Wave* are two different types of wave forms. Since we are using the Signal generator as our switch, the wave form needs to be a *Positive Square Wave* so that the voltage will alternate between 0.00 and 4.00 volts. If a normal *Square Wave* is used, the voltage will alternate between -4.00 and 4.00 volts. Change the frequency to 0.40Hz. Change the amplitude to 4.00V. The **Auto** button should already be selected.

5. Connect a voltage sensor across the 100Ω resistor, and connect it to the computer via an analog port on the PASCO interface.

6. Inform the software which analog port you plugged the voltage sensor into by selecting the voltage sensor icon and dragging it to the appropriate analog port.

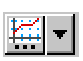
7. Connect a voltage sensor across the 330μF capacitor, and connect it to the computer via an analog port on the PASCO interface.

8. Inform the software which analog port you plugged the voltage sensor into by selecting the voltage sensor icon and dragging it to the appropriate analog port.

9. Click the **Start/Stop Options** button 

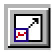
, and select the *Automatic Stop* tab. Select the time option and input 6 seconds. Click **OK**.

10. Create a graph of the Voltage across the Capacitor vs. time.

11. Double click somewhere within the body of the actual graph, or click on the **Graph Settings** button  located on the graph toolbar to open the Graph Settings window. Select the layout tab, and under Group Measurements select the *Do Not Group* option. Next select the Tools tab. Under the Smart Tools, set the *Data Point Gravity* to 0. Select **OK.**

Part II. Collection of Data and Analysis of the Time Constant

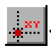
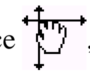
1. Click the **Start** button. The software will collect data for 6 seconds and automatically stop.

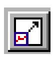
2. Click the **autoscale**  button located on the graph tool bar.

3. Maximize the graph to fill the screen.

4. Magnify a section of the graph where the capacitor is charging. Stretch the time scale if necessary.

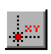
5. Determine the amount of time it takes to reach one half of the maximum voltage using the smart tool.

   Click on the **Smart Tool** button . When the smart tool curser has the following appearance , it can be moved to any location on the graph. Place the smart curser at the location where the capacitor begins to charge. By moving the mouse slightly to the 2nd quadrant of the smart tool curser, you should be able to change the curser to the delta curser . When the delta curser is present, left click and drag the curser to the point where the voltage is one half of the maximum. The difference in time between these points is the time at "half-max", or $T_{1/2}$.

6. Click the **autoscale**  button on the graph.

7. Magnify a section of the graph where the capacitor is discharging.

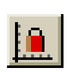
8. Determine the amount of time it takes to reach one half of the maximum voltage using the smart tool in a similar manner as step 5.

9. Click on the **smart tool** button  so that the smart tool is off.

Part III. Completion of Software Set-up and Data Analysis

1. Add a graph of the Voltage Across the Resistor vs. Time to the existing graph. Click and drag the icon that represents the Voltage Across the Resistor to the existing graph. When the entire graph is boxed in a dotted line, drop the icon.

2. Repeat step one for the Output Voltage of the signal generator.

3. Make sure that the **align x-axis lock** button  has been selected.

4. You should now have three graphs that look like Figure 8 in the Foreword of the experiment. However, the graphs will be in a different order.

5. Click on each graph one at a time and turn on the smart tool.

6. Make the Voltage Across the Capacitor vs. time graph active by clicking on it.

7. Click the **autoscale** button on the graph.

8. Magnify a section of the graph in an area where the voltage of the Capacitor is increasing.

9. Pick a point where the voltage of the Capacitor is increasing and determine the voltage of the Capacitor using the smart tool. Also determine the voltage of the Resistor and Output Voltage for the same time. Print the graph with the smart tool data visible. Comment on the result.

10. Repeat steps 6 and 9 for a point where the voltage of the Capacitor is remaining stable. Print the graph with the smart tool data visible. Make sure you comment on the results.

11. Repeat steps 6 and 9 for a point where the voltage of the Capacitor is decreasing. Print the graph with the smart tool data visible. Make sure you comment on the results.

NAME ____________________________ SECTION ____________________________ DATE ________

DATA AND CALCULATION SUMMARY

Charging Capacitor

T_{start} = ________________ s Time when the voltage across the charging capacitor starts to increase.

$T_{half\ max}$ = ________________ s Time when the voltage across the charging capacitor reaches the value $V_C = E / 2$.

$T_{1/2}$ = ________________ s Time for the voltage across the charging capacitor to rise to half its maximum value. ($T_{1/2} = \mathrm{T}_{half\ max} - \mathrm{T}_{start}$)

τ = ________________ s Time constant for the RC circuit. ($\tau = T_{1/2}$ / In 2)

R = ________________ Ω Resistance of the resistor.

C = ________________ F Capacitance of the capacitor.

τ = ________________ s Time constant for the RC circuit. ($\tau = RC$)

Discharging Capacitor

T_{start} = ________________ s Time when the voltage across the discharging capacity starts to decrease.

$T_{half\ max}$ = ________________ s Time when the voltage across the discharging capacitor reaches the value $V_C = E / 2$.

$T_{1/2}$ = ________________ s Time for the voltage across the discharging capacitor to fall to half its maximum value. ($T_{1/2} = \mathrm{T}_{half\ max} - \mathrm{T}_{start}$)

τ = ________________ s Time constant for the RC circuit. ($\tau = T_{1/2}$ / In 2)

Voltages and comments for a time when the voltage across the capacitor is increasing:

E =___________ volts V_R___________ volts V_C =___________ volts

Voltages and comments for a time when the voltage across the capacitor is stable:

E =___________ volts V_R___________ volts V_C =___________ volts

Voltages and comments for a time when the voltage across the capacitor is decreasing:

E =___________ volts V_R___________ volts V_C =___________ volts

Questions

1. The time to half-maximum voltage is how long it takes the capacitor to charge half-way. Based on your experimental results, how long does it take for the capacitor to charge to 75% of its maximum?

2. After four "half-lives", to what percentage of the maximum charge is the capacitor charged?

3. What is the maximum charge for the capacitor in this experiment?

4. What is the maximum current flowing through the resistor in this experiment?