Examining the Feasibility of Computational Methods for Inverting Gravity and Magnetic Data: Numerical Methods and Joint Inversion

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In this presentation, I will provide a brief overview to the general, and standard question of inversion of data for an ill-posed model. I consider the case in which the data are insufficient to define the solution, the overall model is ill-conditioned, and to find the object that defines the data requires the solution of a large scale problem. My main examples are focused on the inversion of large scale magnetic and gravity potential data that are used practically in geophysical prospecting for identification of subsurface structures. Generally, for the solution of such problems it is necessary to provide some kind of prior information which can be imposed in a regularization term. For situations with more than one data set, a coupling of models can also be applied, so that the resulting formulation is even more computationally challenging for solution, but can significantly benefit determination of the underlying structures. There are multiple computational issues that arise when solving these problems, that include multiple parameters that should be determined, the efficient solution of multiple linear systems of equations, and huge demands on memory. In the talk I will address all of these concerns in relation to the solution of the joint inversion of the geophysics data sets, to include also the use of structure for the models to take advantage of $2D$ fast Fourier transforms, for speed and reduced memory demand.

Historically, Krylov iterative methods were used for the solution of large scale ill-posed linear problems, [Paige & Saunders, ACM Trans. Math. Software (1982)]. Recently, the randomized singular value decomposition (RSVD) has been proposed for improved computational efficiency, [Halko, Martinsson & Tropp, SIAM Review (2011)]. The obtained approximate models inherit ill-conditioning of the original model. Their regularization depends on desired model properties and introduces a number of model dependent tuning parameters. These include (i) size of the dominant spectral space k obtained in the Krylov iteration or by the RSVD, (ii) an associated oversampling parameter p which determines the accuracy for the spectral space, (iii) $\|\cdot\|_q$ for the fit to data term, typically $q = 2$, (iv) regularizing term $\|D\cdot\|_r, 0 < r \leq 2$, (v) operator D, usually approximating a derivative of order $\ell \geq 0$, and (vi) weighting regularization parameter α on $\|D\cdot\|_r$.

We will discuss the impact of the choices $(k, p, r, q, \ell, \alpha)$ on inversion of large scale gravity and magnetic data sets. We demonstrate the significance of the degree of ill-posedness on the selection of the parameters, and emphasize the differing properties of the magnetic and gravity problems. Numerical results for large scale problems will be presented to support our conclusions.