International Workshop on
Recent Advances in Evolution Equations and Applications

Dedicated to Dr. Georg Hetzer
on the Occasion of his Upcoming 70th Birthday

Department of Mathematics and Statistics, Auburn University
July 8-9, 2016

Organizing Committee: Xiaoying Han, Paul G. Schmidt (Chair), Wenxian Shen

Theme

Evolution equations arise in the mathematical modeling of time-dependent phenomena in nature and society and, thus, have numerous applications in the sciences, engineering, economics, and other fields. Classical examples of evolution equations include ordinary and partial differential equations; but the complexity of real-world phenomena frequently renders traditional approaches inadequate. Dealing with multiple time scales, time delays, nonlocal effects, random fluctuations, noisy data and measurements, and a host of other complications requires novel and sophisticated mathematical tools. Many such tools have been developed in the recent past, but a coherent theory is just beginning to emerge. Thus, evolution equations constitute a broad and vibrant area of research at the intersection of modern mathematics and applied science. The workshop will bring a group of internationally renowned experts on evolution equations and their applications to Auburn, to share recent advances and trends in the field with our faculty and graduate students.

Dedication

The workshop will be dedicated to our colleague Dr. Georg Hetzer, on the occasion of his upcoming 70th birthday. After starting his academic career at Aachen University of Technology in Germany, Dr. Hetzer joined Auburn University in 1986. He was hired, as a Full Professor, to spearhead the development of a research and graduate program in partial differential equations. Due to his energetic leadership over three decades, Auburn now has a strong, productive, and internationally acclaimed program in partial differential equations, attracting talented faculty and researchers as well as increasing numbers of graduate students. Dr. Hetzer's impact on the program can hardly be overestimated.

Speakers

Hakima Bessaih (University of Wyoming)
Tomás Caraballo (University of Seville, Spain)
María José Garrido Atienza (University of Seville, Spain)
Jerome Goddard (Auburn University Montgomery)
David Gómez Castro (Universidad Complutense de Madrid, Spain)
Arturo Hidalgo (Polytechnic University of Madrid, Spain)
Anotida Madzvamuse (University of Sussex, UK)
A. J. Meir (Southern Methodist University)
Tung Nguyen (University of Illinois Springfield)
Juan Francisco Padial (Polytechnic University of Madrid, Spain)
Björn Schmalfuß (University of Jena, Germany)
Lourdes Tello (Polytechnic University of Madrid, Spain)
Yi Wang (University of Science and Technology of China)

Schedule of Talks

Talks will be given in 228 Parker Hall; breakfast and refreshments will be served in 244 Parker Hall.

Friday, July 8

9:00–9:30 a.m., Breakfast

9:30–9:45 a.m., Paul Schmidt (Auburn University), Opening remarks

9:45–10:30 a.m., Yi Wang (University of Science and Technology of China), Dynamics of almost periodically forced scalar parabolic equations on the circle

10:30–10:45 a.m., Refreshments

10:45–11:30 a.m., Björn Schmalfuß (University of Jena, Germany), Invariant manifolds and random evolution equations

11:30–12:00 a.m., María José Garrido Atienza (University of Seville, Spain), Stochastic lattice models with fractional noisy inputs

Lunch break

2:00–2:45 p.m., Anotida Madzvamuse (University of Sussex, UK), Stability analysis and simulations of coupled bulk-surface reaction-diffusion systems on stationary and evolving volumes

2:45–3:00 p.m., Refreshments

3:00–3:30 p.m., A. J. Meir (Southern Methodist University), On some nonstandard PDE

3:30–4:00 p.m., Hakima Bessaih (University of Wyoming), Mean field limit of interacting filaments and vector-valued nonlinear PDEs

Saturday, July 9

9:00–9:30 a.m., Breakfast

9:30–10:15 a.m., Tomás Caraballo (University of Seville, Spain), Navier-Stokes equations with delays: existence of solutions and their asymptotic behavior

10:15–10:30 a.m., Refreshments

10:30–11:15 a.m., Tung Nguyen (University of Illinois Springfield), Coexistence and extinction in time-periodic Volterra-Lotka type systems with nonlocal dispersal

11:15–11:45 a.m., Jerome Goddard (Auburn University Montgomery), Modeling the effects of U-shaped density dependent dispersal via reaction diffusion equations

Lunch break

2:00–2:30 p.m., David Gómez Castro (Universidad Complutense de Madrid, Spain), Steiner symmetrization of semilinear parabolic and elliptic equations

2:30–3:00 p.m., Juan Francisco Padial (Polytechnic University of Madrid, Spain), Existence of solutions for a Bernoulli-type evolution problem with unknown Radon measure data

3:00–3:15 p.m., Refreshments

3:15–3:45 p.m., Lourdes Tello (Polytechnic University of Madrid, Spain), On a global climate energy balance model with nonlinear diffusion

3:45–4:15 p.m., Arturo Hidalgo (Polytechnic University of Madrid, Spain), Numerical simulation of a climatological energy balance model with continents distribution

Titles and Abstracts

Titles and abstracts are listed in alphabetical order, by speaker's last name.

Hakima Bessaih (University of Wyoming)

Title: Mean field limit of interacting filaments and vector-valued nonlinear PDEs

Abstract: Families of N interacting curves are considered, with long range, mean field type, interaction. A family of curves defines a 1-current, concentrated on the curves, analog of the empirical measure of interacting point particles. This current is proved to converge, as N goes to infinity, to a mean field current, solution of a nonlinear vector valued partial differential equation. In the limit, each curve interacts with the mean field current and two different curves have an independence property if they are independent at time zero. This set-up is inspired from vortex filaments in turbulent fluids.

Tomás Caraballo (University of Seville, Spain)

Title: Navier-Stokes equations with delays: existence of solutions and their asymptotic behavior

Abstract: In this talk we will show several methods to analyze the long time behaviour of solutions to 2D Navier-Stokes models when some hereditary characteristics (constant, distributed or variable delay, memory, etc) appear in the formulation. First the local stability analysis of steady-state solutions is studied by using several methods: the theory of Lyapunov functions, the Razumikhin-Lyapunov technique, by constructing appropriate Lyapunov functionals and finally by using a method based in Gronwall-like inequalities. Then the global asymptotic behaviour of solutions can be analyzed by using the theory of attractors. As the delay terms are allowed to be very general, the statement of the problem becomes nonautonomous in general. For this reason, the theory of nonautonomous pullback attractors appears to be appropriate.

María José Garrido Atienza (University of Seville, Spain)

Title: Stochastic lattice models with fractional noisy inputs

Abstract: The aim of this talk is to analyze stochastic lattice equations driven by a non-trivial multiplicative fractional Brownian motion (fBm) with Hurst parameter in (1/2,1). We will obtain the existence of a unique solution for the model, that will rely on a fixed point argument, based on nice estimates satisfied by the stochastic integral with an fBm as integrator. Further, we will focus on investigating the long time behavior of the solution, proving that when zero is a solution of the model and the initial condition belongs to a neighborhood of zero, then the corresponding solution of the lattice equation is exponentially stable with some exponential rate.

Jerome Goddard (Auburn University Montgomery)

Title: Modeling the effects of U-shaped density dependent dispersal via reaction diffusion equations

Abstract: Dispersal is broadly defined as movement from one habitat patch to another and typically is considered to encompass three stages: 1) emigration, 2) inter-patch movement, & 3) immigration. Dispersal can have both beneficial and detrimental effects on the persistence of spatially structured systems. Recent empirical results indicate that certain organisms? emigration from a patch is dependent on their own density?known as density dependent emigration. In fact, a U-shaped relationship between density and emigration has been observed in several organisms in field studies. To date, little is known about the patch-level consequences of such a dispersal strategy. In this talk, we will discuss a population model built upon the reaction diffusion framework that is designed to model the patch-level effects of U-shaped density dependent emigration. In particular, we will discuss the existence and stability properties of positive steady state solutions to this model for one-dimensional habitat patches. A brief discussion regarding ecological conclusions of the model?s predictions will also be included. Several methods from nonlinear analysis will be employed such as time map analysis (quadrature method) and linearized stability analysis.

David Gómez Castro (Universidad Complutense de Madrid, Spain)

Title: Steiner symmetrization of semilinear parabolic and elliptic equations

Abstract: We consider parabolic reaction-diffusion mathematical model

$$\begin{cases} w_t - \Delta w + \lambda\beta(w) = f, & (0,+\infty)\times G \\ w = w_0, & \{0\}\times G \\ w = 0, & (0,+\infty)\times\partial G \end{cases}$$

and the steady-state elliptic equation. We analyze the different efficiency of the domain, which is defined

$$\eta(t,G) = \frac{1}{|G|}\int_G \beta(w(t,x))\,dx$$

We extended the Steiner symmetrization techniques for domains $G = G' \times G''$ previously used by Alvino, Díaz, P.L. Lions and Trombetti [1] and Chiacchio [4] for the elliptic linear problem coupled with techniques from Schwarz symmetrization in the nonlinear problem (see [5]). We show that, for this kind of domain, $G' = B_R$ provides the lowest effectivity for fixed G'' and $|G'|$. To obtain this result we apply fundamentally the Trotter-Kato formula. A surprising fact is that in the proof for the associated elliptic equation one must apply that the rearrangement comparison holds for the parabolic problem, while typically the results are obtained in the reversed order. No direct proof is known for the elliptic problem. The results improve some previous results in the literature (see, e.g., [2], [3]).

References:

1. A. Alvino, G. Trombetti, J. I. Díaz, and P. L. Lions, Elliptic equations and Steiner symmetrization, Communications on Pure and Applied Mathematics, XLIX (1996), pp. 217–236.
2. R. Aris and W. Strieder, Variational Methods Applied to Problems of Diffusion and Reaction, vol. 24 of Springer Tracts in Natural Philosophy, Springer-Verlag, New York, 1973.
3. C. Bandle, A note on optimal domains in a reaction-diffusion problem, Zeitschrift für Analysis und ihre Answendungen, 4 (3) (1985), pp. 207–213.
4. F. Chiacchio, Steiner symmetrization for an elliptic problem with lower-order terms, Ricerche di Matematica, 53 (2004), pp. 87–106.
5. J. I. Díaz, Nonlinear Partial Differential Equations and Free Boundaries, Vol.I.: Elliptic equations, Research Notes in Mathematics, Pitman, London, 1985.

Arturo Hidalgo (Polytechnic University of Madrid, Spain)

Title: Numerical simulation of a climatological energy balance model with continents distribution

Abstract: In this work we present some numerical results on the mathematical treatment of a global two-dimensional diffusive climate model with land-sea distribution. The model is based on a long time averaged energy balance and leads to a nonlinear parabolic equation for the averaged surface temperature which, in the oceanic areas, is coupled with a deep ocean model (see e.g. Hetzer-Tello [2], Watts-Morantine [5]). The coupling is given by a dynamic and diffusive boundary condition. The numerical scheme used is based on the finite volume method with WENO spatial reconstruction whilst time discretization is accomplished via a third order Runge-Kutta TVD scheme. Previous

numerical results without continents are in Diaz et al [1], Hidalgo-Tello [3]. Numerical results with land-sea distribution are in Hidalgo-Tello [4].

References:

1. J.I.Diaz, A.Hidalgo, L.Tello. Multiple solutions and numerical analysis to the dynamic and stationary models coupling a delayed energy balance model involving latent heat and discontinuous albedo with a deep ocean. Proceedings of the Royal Society A (2014) 470:20140376.
2. G. Hetzer, L. Tello. On a reaction diffusion system arising in Climatology. Dynamic Systems and Applications, 11 (2002) 381-402.
3. A. Hidalgo, L. Tello. A Finite Volume Scheme for simulating the coupling between deep ocean and an atmospheric energy balance model. In the book Modern Mathematical Tools and Techniques in Capturing Complexity. Springer Series in Synergetics (2011).
4. A. Hidalgo, L. Tello. On a climatological energy balance model with continents distribution. Discrete and Continuous Dynamical Systems - Series A (DCDS-A), 35(4) (2015) 1503-1519.
5. R.G. Watts, M. Morantine. Rapid climatic change and the deep ocean, Climatic Change 16 (1990), 83-97.

Anotida Madzvamuse (University of Sussex, UK)

Title: Stability analysis and simulations of coupled bulk-surface reaction-diffusion systems on stationary and evolving volumes

Abstract: In this talk I will present new models for coupled systems of bulk-surface reaction-diffusion equations on stationary and evolving volumes. The bulk reaction-diffusion equations are coupled to the surface reaction-diffusion equations through linear Robin-type boundary conditions. I will then state and prove the necessary conditions for diffusion-driven instability for the coupled system on stationary and evolving domains. Due to the nature of the coupling between bulk and surface dynamics, I am able to decouple the stability analysis of the bulk and surface dynamics. Under a suitable choice of model parameter values, the bulk reaction-diffusion system can induce patterning on the surface independent of whether the surface reaction-diffusion system produces or not, patterning. On the other hand, the surface reaction-diffusion system can not generate patterns everywhere in the bulk in the absence of patterning from the bulk reaction-diffusion system. For this case, patterns can only be induced in regions close to the surface membrane. Various numerical experiments are presented to support theoretical findings. The most revealing numerical result is that, Robin-type boundary conditions seem to introduce a boundary layer coupling the bulk and surface dynamics.

A. J. Meir (Southern Methodist University)

Title: On some nonstandard PDE

Abstract: In this talk I will briefly describe some nonstandard pde that arise from applications. In the problems of interest the pde are supplemented by constraints (e.g. interface conditions, or integral constraints). I will describe some theoretical results and the finite element approximation of solutions of these equations.

Tung Nguyen (University of Illinois Springfield)

Title: Coexistence and extinction in time-periodic Volterra-Lotka type systems with nonlocal dispersal

Abstract: We will discuss the coexistence and extinction of time periodic Volterra-Lotka type competing systems with non-local dispersal. Such issues have already been studied for time independent systems with non-local dispersal and time periodic systems with random dispersal, but have not

been studied yet for time periodic systems with non-local dispersal. In this talk, the relations between the coefficients representing Malthusian growths, self regulations and competitions of the two species have been obtained which ensure coexistence and extinction for the time periodic Volterra-Lotka type system with nonlocal dispersal. The underlying environment of the Volterra-Lotka type system under consideration has either hostile surroundings, or non-flux boundary, or is spatially periodic.

Juan Francisco Padial (Polytechnic University of Madrid, Spain)

Title: Existence of solutions for a Bernoulli-type evolution problem with unknown Radon measure data

Abstract: The classical semilinear problems $-\Delta u(x) = F(x, u(x))$, $x \in \Omega \subset \mathbf{R}^N$ (with boundary conditions) has been intensively studied in the last century when F is a given function $F : \Omega \times \mathbf{R} \to \mathbf{R}$. Nevertheless, some models in Physics or Mechanics can be expressed as $-\Delta u(x) = \mu(x, u)$ in $\mathcal{D}'(\Omega)$ where $\mu(x, u)$ is a *Radon measure* depending on the own solution u, unknown a priori. For instance, this type of problems arise in some ideal fluid dynamics or optimal design context (*Bernoulli-type problem*). This class of elliptic problems were studied in Diaz–Padial–Rakotoson [3]. Our aim is to study the existence of solutions for a nonlinear Bernoulli-type free boundary problem for the evolution case with a unknown measure data associated to the (interior) Bernoulli problem. We introduce a semi-implicit time differencing in order to obtain a family of elliptic problems like the cases studied in Diaz–Padial–Rakotoson [3]. For each one of this problems, we will apply a general *mountain pass principle* due to Ghoussoub-Preiss in order to find a weak solution for a sequence of approximate nonsingular problems. Finally, passing to the limit thanks to some a priori estimates, we obtain the solution, first for each elliptic problem that comes that semi-implicit schema, and later, passing to the limit, we obtain a weak solution for the original parabolic problem.

References:

1. H. W. Alt, L.A. Caffarelli, A. Friedman, Variational problems with two phases and their free boundaries, A.M.S., vol. 282, numb. 2 (1984), pp. 431–461.
2. L. Boccardo and T. Gallouet, Nonlinear elliptic and parabolic equation involving measure as data, J. Funct. Anal. 87, (1989) pp. 149–169.
3. J.I. Díaz, J.F. Padial and J.M. Rakotoson, On some Bernoulli free boundary type problems for general elliptic operators, Proceedings of the Royal Society of Edinburgh, 137A (2007), pp 895-911.
4. N. Ghoussoub and D. Preiss, A general mountain pass principle for location and clssifiying critical points, Annales de l'I.H.P., section C, tome 6, n. 5 (1989), pp. 321-330.

Björn Schmalfuß (University of Jena, Germany)

Title: Invariant manifolds and random evolution equations

Abstract: We introduce the random graph transform. In addition we present several tools like the Oseledets ergodic theorem to find fixed points of this transform. These fixed points give us invariant manifolds for random evolution equations.

Lourdes Tello (Polytechnic University of Madrid, Spain)

Title: On a global climate energy balance model with nonlinear diffusion

Abstract: We study a global climate energy balance model (EBM) which spatial domain is the Earth surface. Budyko-type coalbedo is included in the model as a multivalued term. This kind of models are very sensitive to small fluctuations of the parameters (see e.g Hetzer [3] and Arcoya et al. [1]). In fact, the number of solutions depends on the value of the Solar constant.

This EBM of one layer is coupled with a deep ocean model in order to show the relationship between the rapid climatic change in Glacial-Holocene transition and the past changes in deep water formation. We present some results related with the mathematical treatment of this coupled model. One of the main difficulties is the dynamic and diffusive boundary condition.

References:

1. D. Arcoya, J.I. Diaz and L. Tello. S-shaped bifurcation branch in a quasilinear multivalued model arising in Climatology, Journal of Differential Equations, 150 (1998) 215-225.
2. J.I.Diaz, A.Hidalgo, L.Tello. Multiple solutions and numerical analysis to the dynamic and stationary models coupling a delayed energy balance model involving latent heat and discontinuous albedo with a deep ocean. Proceedings of the Royal Society A (2014) 470:20140376.
3. G. Hetzer. The structure of the principal component for semilinear diffusion equations from energy balance climate models, Houston Journal of Math. 16, 203-216 (1990).
4. G. Hetzer. The number of stationary solutions for a one-dimensional Budyko-type climate models, Nonlinear Analysis, Real World Applications, 2 (2001), 259-272.
5. G. Hetzer, L. Tello. On a reaction diffusion system arising in Climatology. Dynamic Systems and Applications, 11 (2002) 381-402.
6. A. Hidalgo, L. Tello. A Finite Volume Scheme for simulating the coupling between deep ocean and an atmospheric energy balance model. In the book Modern Mathematical Tools and Techniques in Capturing Complexity. Springer Series in Synergetics (2011).
7. R.G. Watts, M. Morantine. Rapid climatic change and the deep ocean, Climatic Change 16 (1990), 83-97.

Yi Wang (University of Science and Technology of China)

Title: Dynamics of almost periodically forced scalar parabolic equations on the circle

Abstract: In this talk, we focus on the skew-product semiflow generated by an almost periodic spatially-homogeneous scalar reaction-diffusion equation on the circle. The structures of the omega-limit sets, as well as the minimal sets is investigated. We discovered certain new phenomena that reinforce the possible appearance of the almost periodically forced circle flow in infinite-dimensional dynamical systems. This is a joint series of work with Wenxian Shen and Dun Zhou.