

34th Cumberland Conference Program

ACLC Building

Rooms: ACLC 108, ACLC 010, ACLC 012, and ACLC 102.

Plenary talks are one hour including Q&A. Contributed talks are 30 minutes including Q&A. Speaker names in the schedule are linked to the corresponding abstracts.

Talk Schedule

Saturday

Time	ACLC 108	ACLC 010	ACLC 012	ACLC 102
8:30–9:30	Song (012)			
9:30–10:00	<i>Coffee</i>			
10:00–10:30	Ye	Milans	Noble	Mitra
10:30–11:00	Helm	Hollars	Carr	Thornburgh
11:00–11:30	Echeverría	Nie	Goddard	Aceska
11:30–12:00	McFarland	Hawranick	Johnson	Noles/Stucky
12:00–2:00	<i>Lunch (on your own)</i>			
2:00–3:00	Cranston (012)			
3:00–3:30	Mudrock	Tian	Zhao	Baker
3:30–4:00	Singh	McCourt	Hou	Fu
4:00–4:30	<i>Coffee</i>			
4:30–5:30	Bonato (012)			
6:30	<i>Conference dinner at KPOT</i>			

Sunday

Time	ACLC 108	ACLC 010	ACLC 012	ACLC 102
8:30–9:30	Nelson (012)			
9:30–10:00	<i>Coffee</i>			
10:00–10:30	Hao	Liu	Grace	Loeb
10:30–11:00	Allred	Yip	Kim	Bruns
11:00–11:30	Sivaraman	Buchanan	Bhattacharya	Copenhaver
11:30–12:00	Harrelson	Zhang	A. Pavelescu	Nochumson
12:00–12:30	<i>Lunch (provided)</i>			
12:30–1:00	Bona	Reid	Vasquez	Desai
1:00–1:30	Wells	Lewis	Farrell	E. Pavelescu
1:30–2:00	Wash			

Abstracts

Plenary Talks

- Bonato: New directions in pursuit-evasion games and processes
- Cranston: Reconfiguration of Colorings and List Colorings
- Nelson: Two-coloured lines in finite geometry
- Song: Dominating Hadwiger's Conjecture

Contributed Talks

- Aceska: Constructing Difference Distance Magic Graphs
- Allred: Enumerative Chromatic-Choosability
- Baker: An Induced Subgraph Paradigm for Strongly Regular Graph Constructions
- Bhattacharya: The Unbreakable Quasigraphic Matroids
- Bona: Partially labeled trees
- Bruns: Intersecting Families of Spanning Trees of $K_{n,n}$
- Buchanan: Extremal problems for monotone paths
- Carr: Conflict-Free-Colorings of Complete Multipartite Graphs and Rectangular Lattices
- Copenhaver: Summing Steiner distances in ordered trees on n vertices.
- Desai: On some Asymptotic Problems involving Positive and Negative Square Energies of Graphs
- Echeverría: Coloring graphs with independence number two and no odd clique immersions
- Farrell: Results in Independent P_3 -Isolation
- Fu: Hypergraph Personalized PageRank via High Order Walks with Edge-Dependent Vertex Weight
- Goddard: Colorings of Graphs without Fall Colorings
- Grace: Matrix Patterns and Matroid Adjoints
- Hao: Strong chromatic index of bipartite graphs
- Harrelson: List-edge-coloring triangulations with maximum degree at most five.
- Hawranick: Covering complete r -partite hypergraphs with few monochromatic components
- Helm: A Quantitative Kuratowski Theorem
- Hollars: Pancyclicity in hypergraphs with large uniformity
- Hou: Minimum Spectral Gap of Trees and Graphs
- Johnson: Colorings of graphs with reference to other graphs
- Kim: Grassmann-Plücker functions for orthogonal matroids
- Lewis: Designing Graphs with a Prescribed Domination Structure
- Liu: Rainbow planar Turán numbers of cycles
- Loeb: Reconstruction of Rooted Binary Trees from Leaf-induced Subtrees
- McCourt: A topological product Tverberg theorem
- McFarland: The structure of group-labeled graphs forbidding an (edge-spanning) immersion
- Milans: Paths in hypergraph tournaments
- Mitra: $L(2,1)$ -Labeling of Cozero-Divisor graphs associated to commutative rings
- Mudrock: On Maximizing Satisfied Vertex Requests in List Coloring
- Nie: Hypergraph Ramsey numbers with quasipolynomial growth rate
- Noble: A Few Problems on the Triangular Lattice
- Nochumson: Counting Isomorphism Classes of Spanning Trees of $K_{a,b}$
- Noles/Stucky: Color Trades on Generalized Theta and Wheel Graphs
- Pavelescu: An Infinite Family of Linklessly Embeddable Tutte-4-Connected Graphs
- Pavelescu: Properties of graphs and their complements

- Reid: On the domination number of fullerenes
- Singh: Loose elements and paving matroids
- Sivaraman: On perfectly divisible graphs
- Thornburgh: On lower bounds for the distances between APN functions
- Tian: Removing a Long Cycle in Graphs
- Vasquez: Variations on Weak Saturation
- Wash: Pick's Theorem - Novel Proofs and a Novel Problem in Combinatorial Geometry
- Wells: A discrete view of Gromov's filling area conjecture
- Ye: Circuit Covering of Signed Graphs
- Yip: Arithmetic progressions in Euclidean Ramsey theory
- Zhang: On the dual of Erdős' circuit spectrum problem
- Zhao: An extremal problem for minimally k -connected graphs

Plenary Talks

New directions in pursuit-evasion games and processes

Anthony Bonato
Toronto Metropolitan University

We overview recent work on burning, cooling, liminal burning, and pursuit-evasion games on graphs, including Cops and Robbers and the Localization game.

[Back to schedule](#)

Reconfiguration of Colorings and List Colorings

Daniel Cranston
Virginia Commonwealth University

A *proper k -coloring* of a graph G assigns to each vertex v a color $\alpha(v)$, with $\alpha(v) \in \{1, \dots, k\}$ such that $\alpha(v) \neq \alpha(w)$ for every edge vw . (A *list coloring* is similar, except that distinct vertices may have distinct lists of allowable colors.)

A *recoloring step* in a graph G for a coloring α recolors some vertex v with a color allowable for v that is not used by α on any neighbor of v , yielding a new proper coloring. Given proper colorings α and β of G , we ask questions like: Can we transform α to β by a sequence of recoloring steps? And: Over all α and β , what is the longest that a shortest sequence from α to β can be?

In this talk we survey results on reconfiguration of colorings and list colorings. We end with a few conjectures.

[Back to schedule](#)

Two-coloured lines in finite geometry

Peter Nelson
University of Waterloo

Given a colouring of the points of a projective plane, when is it true that every line contains at most two colours? I will discuss recent generalizations of classical results in this area, and a surprising link with a famous question in graph theory.

[Back to schedule](#)

Dominating Hadwiger's Conjecture

Zi-Xia Song

Department of Mathematics, University of Central Florida

A dominating K_t minor in a graph G is a sequence (T_1, \dots, T_t) of pairwise disjoint non-empty connected subgraphs of G , such that for $1 \leq i < j \leq t$, every vertex in T_j has a neighbor in T_i . Replacing “every vertex in T_j ” by “some vertex in T_j ” retrieves the standard definition of a K_t minor. The strengthened notion was introduced in 2024 by Illingworth and Wood, who asked whether every graph with chromatic number t contains a dominating K_t minor. This is a substantial strengthening of the celebrated Hadwiger's Conjecture, which asserts that every graph with chromatic number t contains a K_t minor. Sergey Norin referred to this question as the “Dominating Hadwiger's Conjecture” and believes it is likely false.

In this talk, we present our recent work on the Dominating Hadwiger's Conjecture and discuss the key ideas of our results.

Joint work with Michael Scully and Thomas Tibbetts.

[Back to schedule](#)

Contributed Talks

Constructing Difference Distance Magic Graphs

Roza Aceska

Ball State University

In a distance difference magic labeling, each vertex is given a unique value from one up to the number of vertices such that for each vertex the sums of the labels of vertices in the in-neighborhood minus the sums of the labels of vertices in the out-neighborhood equals zero. We expand on this concept by showing a connected difference distance magic oriented graph on vertices exists for each integer $n \geq 5$. We also construct arbitrarily large difference distance magic oriented graphs using a new graph sum with carefully selected oriented graphs, called ornaments.

[Back to schedule](#)

Enumerative Chromatic-Choosability

Sarah Allred

University of South Alabama

Counting proper (classical) colorings of graphs is a fundamental topic in enumerative combinatorics that has been extensively studied since the early 20th century. The chromatic polynomial of a graph G , denoted $P(G, m)$, is equal to the number of proper m -colorings of G . List coloring is a well-studied generalization of classical coloring that was introduced in the 1970s. A graph G is chromatic-choosable when its list chromatic number $\chi_\ell(G)$ is equal to its chromatic number $\chi(G)$. Chromatic-choosability is a well-studied topic, and in fact, some of the most famous results and conjectures related to list coloring involve chromatic-choosability.

In 1990, Kostochka and Sidorenko introduced the list color function of a graph G , denoted $P_\ell(G, m)$. The list color function of G is the list analogue of $P(G, m)$. A graph is said to be enumeratively chromatic-choosable if $P(G, m) = P_\ell(G, m)$ whenever $m \geq \chi(G)$. In this talk, I will present some results and open questions on enumerative chromatic-choosability. In particular, I give a characterization of enumeratively 2-chromatic-choosable graphs. This is joint work with Jeff Mudrock.

[Back to schedule](#)

An Induced Subgraph Paradigm for Strongly Regular Graph Constructions

Ben Baker
Auburn University

How many disjoint copies of some H can appear in an induced subgraph of a strongly regular graph? We give upper and lower bounds that depend on the number of vertices in their neighborhood. In the equality cases, the edges leading to the subgraph must be evenly distributed. This generalizes a result of Lou-Murin which bounds the independence number of a putative Conway 99 graph.

When searching for a construction of a strongly regular graph, it is common to start with some subgraph H and add edges while respecting strongly regular conditions. This search for viable edges is often done via computer. Although any starting H could potentially work, our result suggests using an H that achieves equality on one of our bounds. Its even edge distribution simplifies the ensuing computer search. We consider the application of this approach to Conway's 99-graph problem.

[Back to schedule](#)

The Unbreakable Quasigraphic Matroids

Sayantani Bhattacharya
Auburn University

Quasi-graphic matroids are combinatorial objects that generalize graphs. A matroid which is also unbreakable has the structural property that contraction of a 'closed' set does not disconnect it, which gives additional insight on the matroid in question. We generalize and strengthen a number of results of Fife, Mayhew, Oxley, and Semple to characterize the underlying graphs of unbreakable quasi-graphic matroids. We assume no knowledge of matroid theory. This is joint work with John David Clifton and Zach Walsh.

[Back to schedule](#)

Partially labeled trees

Miklos Bona
University of Florida

We define a class of partially labeled trees which are between decreasing binary trees and unlabeled plane 1-2 trees. They are enumerated by Boolean-Catalan numbers. Refining this enumeration so that the number of right edges is taken into account leads to the Boolean-Narayana numbers. Their generating polynomials $\text{BoNa}_n(x)$ are between the Narayana polynomials and the Eulerian polynomials. We prove that accordingly, they have similar properties to those, like the real zeros only property, or that they satisfy a linear recurrence relation. We consider generalizations of the problem as well.

[Back to schedule](#)

Intersecting Families of Spanning Trees of $K_{n,n}$

Gordian Bruns
University of Memphis

Intersecting families are a central topic in extremal combinatorics, originating with the Erdős–Ko–Rado theorem, where intersecting k -sets of an n -set were considered. Recently, Frankl, Hurlbert, Ihringer, Kupavskii, Lindzey, Meagher, and Pantangi initiated the study of analogous questions for spanning trees of the complete graph, determining the largest families of spanning trees in which any two share at least t edges.

In this talk, we consider the corresponding problem for the balanced complete bipartite graph $K_{n,n}$. We determine, for sufficiently large n and $t = O(n/\log n)$, the asymptotically largest t -intersecting families of spanning trees and characterize the extremal constructions. Compared to the complete graph case, the bipartite setting introduces additional combinatorial and probabilistic difficulties. To overcome these challenges, we combine spectral techniques based on Kirchhoff's matrix-tree theorem and Laplacian eigenvalue estimates with the spread approximation method developed by Kupavskii and Zakharov. This is joint work with Dheer Noal Desai, Alexander L. Gavrilyuk, Josias Gomez, and Nathan Lindzey.

[Back to schedule](#)

Extremal problems for monotone paths

Calum Buchanan
University of Denver

A path in an edge-ordered graph is *monotone* if its edge labels are increasing from one endpoint to the other. The problem of finding the *altitude* of a graph G , the maximum length of a monotone path guaranteed in every edge-ordering of G , was introduced by Chvátal and Komlós in 1971. In 2020, Gerbner, Methuku, Nagy, Pálvölgyi, Tardos, and Vizer introduced the systematic study of the edge-ordered Turán number. We study this problem for monotone paths, proving that the maximum size of an edge-ordered graph of order n which does not contain a monotone path of length 4 is $5n/2$. We also prove results on the altitudes of complete bipartite graphs and hypercubes. This talk is based on joint work with Neal Bushaw, Van Magnan, Puck Rombach, and Per Wagenius.

[Back to schedule](#)

Conflict-Free-Colorings of Complete Multipartite Graphs and Rectangular Lattices

John Carr
University of North Alabama

A conflict-free-coloring of a graph G is a vertex-coloring of G such that for every vertex v , there is a color assigned to exactly one vertex among v and its neighbors. This idea has received much attention recently with much work done in the planar setting, along with studying the idea from a complexity standpoint. We provide explicit constructions for some variations of conflict-free-colorings of complete multipartite graphs and rectangular lattices.

[Back to schedule](#)

Summing Steiner distances in ordered trees on n vertices.

Keith Copenhaver
Eckerd College

The Wiener index of a graph is the sum of all minimal distances between pairs of vertices. The Steiner distance of a set of vertices is the size of the minimal spanning tree including that set. The Steiner k -Wiener index of a graph G mixes these two notions, we take the sum of all Steiner distances of sets of k vertices within a graph G . We find a closed form for the sum of all Steiner k -Wiener indices of ordered trees on n vertices.

[Back to schedule](#)

On some Asymptotic Problems involving Positive and Negative Square Energies of Graphs

Dheer Noal Desai
University of Memphis

For a graph G with adjacency matrix $A(G)$, we define its positive square energy $s^+(G)$ (and similarly negative square energy $s^-(G)$) to be the sum of all the squares of the positive (negative) eigenvalues of $A(G)$. The eigenvalues of the adjacency matrix of a graph provide a variety of structural information about its underlying graph. For example, in the context of square energies, it is known due to a result of Ando and Lin, that the chromatic number $\chi(G) \geq 1 + \max \left\{ \frac{s^+(G)}{s^-(G)}, \frac{s^-(G)}{s^+(G)} \right\}$.

In this talk we will discuss some open problems regarding square energies and graphs of fixed order n . In particular, we share our progress on some asymptotic questions posed by Clive Elphick and William Linz in their recent paper on the "Symmetry and asymmetry between positive and negative square energies of graphs".

This is based on joint work with Maarten De Boeck.

[Back to schedule](#)

Coloring graphs with independence number two and no odd clique immersions

Henry Echeverría
Universidad de Valparaíso

To split off a pair of adjacent edges uv, vw is to delete both these edges and to add the edge uw , if it is not already present. A graph G is said to contain another graph H as an immersion if H can be obtained from G by iteratively splitting off edges and deleting vertices and edges. In this talk, I plan to show you that every graph G such that $\alpha(G) \leq 2$ and G has no K_t as an odd immersion is $\lceil \frac{3}{2}(t-1) \rceil$ -colorable. Joint work with Jessica McDonald.

[Back to schedule](#)

Results in Independent P_3 -Isolation

Garrett Farrell
Clemson University

In this talk we consider the independent P_3 -isolation number of graphs. This is the minimum number of vertices in a set S such that if one removes S and all its neighbors, what remains contains only isolated vertices and edges. Naturally, the parameter for each family has an upper bound by the independent isolation number and the independent domination number, but we explore how the bound on independent P_3 -isolation may be further reduced. We give an upper bound for any arbitrary graphs, then proceed to subcubic graphs, trees, and maximal outerplanar graphs.

[Back to schedule](#)

Hypergraph Personalized PageRank via High Order Walks with Edge-Dependent Vertex Weight

Kayla Fu
Middle Tennessee State University

This project aims to simplify hypergraphs without compromising their structural integrity and introduces a groundbreaking approach to high-order walks for analyzing the complex structures of hypergraphs. The initiative utilizes the Laplacian operator, employing it in a novel method to calculate Personalized PageRank (PPR).

This research includes the development of sophisticated mathematical models and the creation of efficient C++ algorithms, which are applied to real-world datasets across various industries. By conducting a comparative analysis with existing methodologies, the project not only seeks to enhance the accuracy and personalization of ranking and recommendation systems for large datasets but also aims to make significant contributions to the field of hypergraph theory. This initiative promises to deepen our understanding of hypergraphs and their potential applications in data science and network analysis.

[Back to schedule](#)

Colorings of Graphs without Fall Colorings

Wayne Goddard
Clemson University

A fall coloring of a graph is a proper coloring of the vertices such that for each vertex its closed neighborhood contains all colors. The simplest example of a connected graph that doesn't have a fall coloring is the 5-cycle. We consider the question of how close to a fall coloring one can get in general. We investigate several metrics such as the number of bad vertices or how far from a vertex does one have to go to see all colors. This includes joint work with S.W. Mafunda and M.H. Rafanomezantsoa.

[Back to schedule](#)

Matrix Patterns and Matroid Adjoints

Kevin Grace
University of South Alabama

A zero-nonzero pattern matrix is a way to represent which entries of a matrix are zeros. A problem in combinatorial matrix theory is to study the minimum rank among all matrices that can come from these pattern matrices by substituting elements of some field for the nonzero entries of the pattern matrix. A lower bound on the minimum rank of a pattern matrix is its triangle number, the size of the largest upper triangular submatrix that can be obtained by permuting rows and columns. We show that whether this bound is tight for a particular pattern has somewhat surprising connections to the notion of an adjoint of a matroid. The notion of an adjoint of a matroid M arises from the attempt to attach a matroid to the lattice-theoretic dual of the lattice of flats of M . This is joint work with Louis Deaett.

[Back to schedule](#)

Strong chromatic index of bipartite graphs

Yanli Hao
Georgia Institute of Technology

An edge-coloring of a graph G is called a **strong edge-coloring** if all its color classes are induced matchings. The minimum number of colors required for such a coloring, denoted by $\chi'_s(G)$, is known as the **strong chromatic index** of G . For each vertex v of G , let $d_G(v)$ denote the degree of v in G . Let G be a bipartite graph with partite sets A and B , and let $\Delta_A = \max\{d_G(a) : a \in A\}$ and $\Delta_B = \max\{d_G(b) : b \in B\}$. A conjecture of Faudree et al. asserts that $\chi'_s(G) \leq \Delta_A \Delta_B$. With Dr. Xingxing Yu and Tianchi Yang, we show that $\chi'_s(G) \leq 1.72 \Delta_A \Delta_B$, for $\max\{\Delta_A, \Delta_B\}$ sufficiently large and $\min\{\Delta_A, \Delta_B\} \geq 34$.

[Back to schedule](#)

List-edge-coloring triangulations with maximum degree at most five.

Joshua Harrelson
Middle Georgia State University

We will skim through a proof showing such triangulations satisfy the LECC.

[Back to schedule](#)

Covering complete r -partite hypergraphs with few monochromatic components

Luke Hawranick
University of South Carolina

An edge-coloring of a hypergraph is *spanning* if every vertex sees every color used in the coloring. We prove that for $k \geq 2r \geq 6$, in any spanning k -coloring of the edges of a complete r -partite r -uniform hypergraph H , the vertices of H can be covered by a set of at most $k - r + 1$ monochromatic connected components. This proves a conjecture of Gyárfás and Király which is related to a special case of Ryser's conjecture. This is joint work with Ruth Luo.

[Back to schedule](#)

A Quantitative Kuratowski Theorem

Alec Helm
University of South Carolina

A planar graph is a graph which can be drawn in the plane such that edges do not intersect each other. This nice graph family is described by Kuratowski's celebrated 1930s theorem which states that a finite graph is planar if and only if it contains no subgraphs which are subdivisions of K_5 or $K_{3,3}$; we call such graphs Kuratowski subgraphs. The notion of crossing number of a graph allows for the quantification of a graph's failure to be planar. The crossing number of a graph is the minimum number of edge intersections needed to draw the graph in the plane, and thus a graph is planar if and only if its crossing number is zero. In this context, a finite graph has positive crossing number if and only if it contains a Kuratowski subgraph. We ask the following question: can we bound the crossing number of graphs with a fixed number of Kuratowski subgraphs? We answer this in the affirmative and answer some questions about the crossing number of graphs with exactly k Kuratowski subgraphs for various k .

[Back to schedule](#)

Pancyclicity in hypergraphs with large uniformity

Isaiah Hollars
University of South Carolina

A Berge cycle of length ℓ in a hypergraph \mathcal{H} is a sequence of alternating vertices and edges $v_0 e_0 v_1 e_1 \dots v_{\ell-1} e_{\ell-1} v_0$ such that $\{v_i, v_{i+1}\} \subseteq e_i$ for all i , with indices taken modulo ℓ . For n sufficiently large and $r \geq \lfloor \frac{n-1}{2} \rfloor - 1$ we prove exact minimum degree conditions for an n -vertex, r -uniform hypergraph to contain Berge cycles of every length between 2 and n . In conjunction with previous work, this provides sharp Dirac-type conditions for pancyclicity in r -uniform hypergraphs for all $3 \leq r \leq n$ when n is sufficiently large.

[Back to schedule](#)

Minimum Spectral Gap of Trees and Graphs

Wenxin Hou
University of South Carolina, Department of Mathematics

The spectral gap of a simple graph G is defined as the difference between the largest and the second-largest eigenvalues of its adjacency matrix. Jovovi?, Koledin, and Stani? conjectured that, among all trees with n vertices, the minimum spectral gap is attained by a double comet. Stani? further conjectured that, among all connected graphs with n vertices, the minimum spectral gap is achieved by a double kite graph. In this talk, we will present recent progress toward proving these two conjectures.

[Back to schedule](#)

Colorings of graphs with reference to other graphs

Peter Johnson
Auburn University

If G and H are graphs on the same vertex set V , a G -coloring of H is a function from V into V which assigns to each vertex v a vertex in the open neighbor set of v in G , such that if u and w are adjacent in H , then they are assigned different vertices. The work (joint with J. Briggs, A. Cook, S. Heuss, and S. Shan) is about the determination of the following. V will be the vertex set of every graph mentioned. If G has no isolated vertices,

$$\text{MC}(G) = \{H \mid H \text{ is } G\text{-colorable, and no graph } X \text{ on } V \text{ with } E(X) \subsetneq E(H) \text{ is } G\text{-colorable}\};$$

and

$$\text{mNC}(G) = \{H \mid H \text{ is not } G\text{-colorable, but every graph } Y \text{ on } V \text{ with } E(Y) \subsetneq E(H) \text{ is } G\text{-colorable}\}.$$

If H is a graph on V ,

$$\text{mC}'(H) = \{G \mid H \text{ is } G\text{-colorable, } H \text{ is not } X\text{-colorable if } X \text{ is a graph on } V \text{ with } E(X) \subsetneq E(G)\};$$

and

$$\text{MNC}'(H) = \{G \mid H \text{ is not } G\text{-colorable, } H \text{ is } Y\text{-colorable for any graph } Y \text{ on } V \text{ with } E(G) \subsetneq E(Y)\}.$$

[Back to schedule](#)

Grassmann–Plücker functions for orthogonal matroids

Donggyu Kim
Georgia Institute of Technology

We present a new cryptomorphic definition of orthogonal matroids with coefficients using Grassmann–Plücker functions. The equivalence is motivated by Cayley’s identities expressing principal and almost-principal minors of a skew-symmetric matrix in terms of its Pfaffians. As a corollary of the new cryptomorphism, we deduce that each component of the orthogonal Grassmannian is parameterized by certain part of the Plücker coordinates. This is joint work with Changxin Ding.

[Back to schedule](#)

Designing Graphs with a Prescribed Domination Structure

Tom Lewis
Furman University

Let G be a triangle-free graph. Our objective is to build an auxiliary graph H whose dominating sets are *prescribed* or *characterized* by G in the following sense:

1. G is a subgraph of \overline{H} .
2. The domination number of H is 2.
3. The vertex pair $\{x, y\} \subset V_H$ dominates H if and only if $\{x, y\}$ is an edge of G .

In effect, the dominating sets of H are prescribed by G : the line graph of G is the γ -graph of H .

In this talk, we will discuss the process by which an auxiliary graph H can be built from a given prescriptor graph G .

[Back to schedule](#)

Rainbow planar Turán numbers of cycles

Xiaonan Liu
Vanderbilt University

The rainbow Turán number of a fixed graph H , denoted by $\text{ex}^*(n, H)$, is the maximum number of edges in an n -vertex graph such that it admits a proper edge coloring with no rainbow H . We study this problem in planar setting. The rainbow planar Turán number of a graph H , denoted by $\text{ex}_{\mathcal{P}}^*(n, H)$, is the maximum number of edges in an n -vertex planar graph such that it has a proper edge coloring with no rainbow H . We consider the rainbow planar Turán number of cycles. We know that $\text{ex}_{\mathcal{P}}^*(n, C_3)$ is exactly its planar Turán number, which is $2n - 4$ for $n \geq 3$. We show that $\text{ex}_{\mathcal{P}}^*(n, C_4) = 3n - 6$ for $n = k^2 - 3k + 2$ where $k \geq 5$, and $\text{ex}_{\mathcal{P}}^*(n, C_k) = 3n - 6$ for all $k \geq 5$ and $n \geq 3$.

[Back to schedule](#)

Reconstruction of Rooted Binary Trees from Leaf-induced Subtrees

Sarah Loeb
Hampden-Sydney College

The reconstruction problem asks if we can uniquely identify a larger structure from its smaller substructures. I'll discuss this problem in the context of rooted binary trees as motivated by rooted phylogenetic tree shapes. Here, the smaller substructures are leaf-induced binary subtrees. We show such trees are reconstructable.

This is joint work with Ann Clifton, Éva Czabarka, Audace Dossou-Orly, Kevin Liu, Utku Okur, László Székely, and Kristina Wicke.

[Back to schedule](#)

A topological product Tverberg theorem

Grace McCourt
Iowa State University

In this talk, we discuss the configuration space / test map scheme, which is a useful proof technique in topological combinatorics. We use it to prove a generalization of the topological Tverberg theorem. One special instance of our general theorem is the following: Let Δ denote the 8-dimensional simplex viewed as an abstract simplicial complex, and suppose that its vertices are arranged in a 3×3 array. Then for any continuous map $f : \Delta \rightarrow \mathbb{R}^3$ it is possible to partition the rows or the columns of the vertex array into two parts, such that the disjoint faces σ and τ induced by the two parts satisfy $f(\sigma) \cap f(\tau) \neq \emptyset$.

This work is join with Andreas Holmsen, Daniel McGinnis, and Shira Zerbib.

[Back to schedule](#)

The structure of group-labeled graphs forbidding an (edge-spanning) immersion

Caleb McFarland
Georgia Tech

A Γ -labeled graph is an oriented graph with edges invertibly labeled by a group Γ . We prove a structure theorem for Γ -labeled graphs which forbid a fixed Γ -labeled graph as an immersion, for any finite Γ . Roughly, we show that such graphs admit a tree-cut decomposition in which every bag either contains few high degree vertices or is nearly signed over a proper subgroup of Γ .

We also discuss edge-spanning immersions, in which the branch trails of the immersion must partition the edge set. We show that Γ -labeled Eulerian graphs forbidding a fixed Γ -labeled edge-spanning immersion admit approximately the same structure.

Joint work with Ioannis Eleftheriadis, Rose McCarty, Evangelos Protopapas, and Paul Wollan.

[Back to schedule](#)

Paths in hypergraph tournaments

Kevin Milans
West Virginia University

It is well known that every tournament contains a spanning path. We study a generalization to *fully directed hypergraphs*, where each edge comes with a total ordering of its vertices. For $0 \leq k \leq r!$, an (r, k) -tournament is a fully directed hypergraph such that each set of r vertices spans k edges. A $(2, 1)$ -tournament is just a conventional tournament. Let $f(n, r, k)$ be the minimum, over all (r, k) -tournaments G with n vertices, of the maximum size of a (tight) path in G . In our language, the statement that every tournament has a spanning path becomes $f(n, 2, 1) = n$.

In the case $r = 3$, it is easy to show that $f(n, 3, 1) = f(n, 3, 2) = 3$ when $n \geq 3$ and $f(n, 3, 5) = f(n, 3, 6) = n$. The cases $(r, k) = (3, 3)$ and $(r, k) = (3, 4)$ are interesting. We show that $\Omega((\log n)^{1/4 - o(1)}) \leq f(n, 3, 3) \leq O(\log n)$ and that $f(n, 3, 4) \geq \Omega(n^{1/5})$. We conjecture that $f(n, 3, 4) = n$.

In general, we define the *growing path threshold* $g(r)$ to be the minimum k such that $f(n, r, k) \rightarrow \infty$ as $n \rightarrow \infty$. We show that $g(r)$ is one more than the maximum size of a set of vertices not containing a cycle in a particular directed graph, and we use this to show that $g(r)/r!$ is in the range $[1 - \frac{1}{r} - \frac{1}{r^{2-o(1)}}, 1 - \frac{1}{r} - \frac{\phi(r)}{r!}]$ where $\phi(r)$ is the Euler totient function. This is joint work with Richard C. Devine.

[Back to schedule](#)

$L(2,1)$ -Labeling of Cozero-Divisor graphs associated to commutative rings

Sarbari Mitra
Fort Hays State University

An $L(2, 1)$ -labeling of a graph Γ is a vertex mapping to non-negative integers where adjacent labels differ by at least two and distance-two vertices receive distinct labels. The minimum span of such a labeling is denoted by $\lambda(\Gamma)$. This presentation examines the $L(2, 1)$ -labeling of cozero-divisor graphs $\Gamma'(R)$ for the ring $R = \mathbb{Z}_n$. By exploiting the structural decomposition of the vertex set into independent sets, we determine exact λ -numbers for $n = p^t q^s$ and $n = p^t q r$. Furthermore, we establish general lower bounds for $\lambda(\Gamma'(\mathbb{Z}_n))$ based on prime power decompositions. These findings provide a characterization of labeling properties for this algebraic graph class.

[Back to schedule](#)

On Maximizing Satisfied Vertex Requests in List Coloring

Jeffrey Mudrock
University of South Alabama

Flexible list coloring was introduced by Dvořák, Norin, and Postle in 2019 in order to address a situation in list coloring where we still seek a proper list coloring, but each vertex may have a preferred color assigned to it, and for those vertices we wish to color as many of them as possible with their preferred colors. This can be thought of as a relaxation of the classical pre-coloring extension problem. Specifically, suppose G is a graph and L is a list assignment for G . A *request* of L is a function r with nonempty domain $D \subseteq V(G)$ such that $r(v) \in L(v)$ for each $v \in D$. We say G is (k, ϵ) -flexible if there exists a proper L -coloring f of G such that $f(v) = r(v)$ for at least $\epsilon|D|$ vertices whenever L is a k -assignment for G and r is a request of L with domain D .

Much of the literature on flexible list coloring focuses on showing that for a fixed graph G and $k \in \mathbb{N}$ there exists an $\epsilon > 0$ such that G is (k, ϵ) -flexible, but it is natural to try to find the largest possible ϵ for which G is (k, ϵ) -flexible. In this talk we present results on finding the maximum such ϵ for fixed G and k . Our results reveal a connection to the Hall ratio which was introduced in 1997 by Hilton, Johnson Jr., and Leonard under the name fractional Hall-condition number. This is joint work with Timothy Bennett, Michael Bowdoin, Haley Broadus, Daniel Hodgins, Adam Nusair, Gabriel Sharbel, and Joshua Silverman.

[Back to schedule](#)

Hypergraph Ramsey numbers with quasipolynomial growth rate

Jiayi Nie
Georgia Institute of Technology

For a 3-uniform hypergraph F , let $r(F, n)$ be the smallest N such that any N -vertex F -free 3-uniform hypergraph has an independent set of size n . We construct a family of 3-uniform hypergraphs F for which $r(F, n) = n^{\log^{\Theta(1)}(n)}$; and in particular $r(F, n) = n^{\Theta(\log n)}$ for some F . This is the first example of such a Ramsey number known to be neither polynomial nor exponential.

[Back to schedule](#)

A Few Problems on the Triangular Lattice

Matt Noble
Middle Georgia State University

Let T_n denote the triangular lattice with n rows and $\frac{n(n+1)}{2}$ points. Define a *proper coloring* to be an assignment of colors to the points of T_n such that no three points constituting the vertices of an equilateral triangle all receive the same color. Denote by $f(n)$ the smallest possible number of colors that can be used in a proper coloring of T_n . In this talk, we'll exactly determine $f(n)$ for small n and detail what we know and don't know concerning the function's asymptotic behavior. We will also describe a related and perhaps surprising enumeration result that popped up along the way.

[Back to schedule](#)

Counting Isomorphism Classes of Spanning Trees of $K_{a,b}$

Shayne Nochumson
Auburn University

Spanning trees of complete bipartite graphs exhibit a rich interaction between degree sequences and graph structure. In this paper, we obtain lower bounds on the number of isomorphism classes of spanning trees in $K_{a,b}$, $2 \leq a \leq b$ in terms of $P_a(a+b-1)$ and $P_b(a+b-1)$ where $P_k(m)$ is the number of integer partitions of m of length k . Furthermore, we obtain an upper bound in terms of a and b .

[Back to schedule](#)

Color Trades on Generalized Theta and Wheel Graphs

Kaitlin Noles and Lexie Stucky
University of North Alabama

Two proper edge-colorings of a graph G are mate-colorings if and only if every vertex of G is incident to the same set of colors under each edge-coloring while each edge receives a different color under each edge-coloring. The color-trade-spectrum (CTS) of a graph G is the set of all t for which there exists two mate-colorings of G using t colors. A generalized theta graph, denoted $\theta_{n_1, n_2, \dots, n_k}$, consists of k paths having only the starting and ending vertices in common with lengths $n_1, n_2, \dots, n_k \in \mathbb{N}$. A generalized wheel graph, denoted W_k^n , consists of a central vertex and an n -cycle with a path of length k between the central vertex and each vertex in the cycle. We determined the color-trade-spectra of generalized theta graphs and generalized wheel graphs.

[Back to schedule](#)

An Infinite Family of Linklessly Embeddable Tutte-4-Connected Graphs

Andrei Pavelescu
University of South Alabama

In 1997, Maharry produced an example of a simple graph of order 13 which was 4-connected, triangle free (which he called Tutte-4-connected), and linklessly embeddable. It was only the second known such example. Through private communication with Maharry, Robertson had conjectured that $K_{5,5}$ minus a perfect matching was the only graph with these properties, so Maharry's discovery was quite extraordinary. In this talk, we show that the class of linklessly embeddable Tutte-4-connected graphs is quite rich, as we provide an example of such a graph for every order $n \geq 14$.

[Back to schedule](#)

Properties of graphs and their complements

Elena Pavelescu
University of South Alabama

We examine several properties of graphs and ask whether it is possible for both a graph and its complement to have the same property. For example, we may ask whether a planar graph with 9 vertices can have a planar complement. The answer is no, by work of Battle, Harary, and Kodama. In this talk, we ask similar questions about the k -apex property, linkless embeddability, and knotless embeddability. This talk is based on joint work with Gregory Li and Andrei Pavelescu.

[Back to schedule](#)

On the domination number of fullerenes

James Reid
The University of Mississippi

A fullerene graph is a planar, 3-regular, and 3-connected graph with only pentagonal and hexagonal faces. There are fullerene graphs on n vertices for all even $n \geq 20$ except for $n = 22$. These graphs arise as mathematical models of fullerene molecules in chemistry, where vertices correspond to carbon atoms and edges to covalent bonds. We propose an upper bound for the domination number of a fullerene on n vertices. This bound is developed by considering the domination number of a narrow nanotube.

[Back to schedule](#)

Loose elements and paving matroids

Jagdeep Singh
Mississippi State University

An element of a rank- r matroid is called loose if it is contained in no circuit of size less than r . This concept naturally generalizes paving matroids, which are precisely the matroids where every element is loose. In this talk, we investigate the structure and size of representable matroids possessing loose elements. We provide a complete characterization of binary matroids with a loose element, and show that the size of a ternary matroid with a loose element is linear in its rank. We also discuss the generalization of these results to matroids with k -loose elements.

[Back to schedule](#)

On perfectly divisible graphs

Vaidyanathan Sivaraman
Mississippi State University

Perfectly divisible graphs are a generalization of perfect graphs. Recently Chinh Hoang proposed several conjectures about perfectly divisible graphs. We refute one of them, and prove special cases of several others. This is joint work with Mohammed A. Mutar.

[Back to schedule](#)

On lower bounds for the distances between APN functions

Darrion Thornburgh
Vanderbilt University

Whether two distinct APN functions can have a Hamming distance of 1 remains an open problem. In 2020, L. Budaghyan et al. introduced a new CCZ-invariant Π_F which can be used to provide lower bounds on the Hamming distance between a given APN function $F: \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$ and other APN functions. Lower bounds on the distance from an APN function F to any other are known for almost bent (AB) functions and when F is a 3-to-1 quadratic function with n even. In this talk, we reinterpret Π_F in terms of the exclude multiplicities of the graph $\mathcal{G}_F = \{(x, F(x)) : x \in \mathbb{F}_2^n\}$ of F as a Sidon set. We establish lower bounds on the distance for even n when F is plateaued APN, generalize the known lower bounds for quadratic 3-to-1 function to all 3-to-1 plateaued functions (e.g. Kasami functions), and derive new lower bounds for when F is the APN inverse function over \mathbb{F}_{2^n} for n odd. We also provide some secondary results such as a brief proof that the only known APN function that is not CCZ-equivalent to a power function nor quadratic function (called the Brinkmann-Leander-Edel-Pott) is also not CCZ-equivalent to a plateaued function. This work was done in collaboration with Maria Mihaila.

[Back to schedule](#)

Removing a Long Cycle in Graphs

Lin Tian
Middle Tennessee State University

Thomassen proved that every $(s + 3)$ -connected graph G contains a cycle C such that $\kappa(G - V(C)) \geq s$. Further, he proved that every $(s + t - 1)$ -connected graph G of minimum degree at least $g(s, t)$, its vertex set can be partitioned into two sets S and T such that the subgraphs $G[S]$ and $G[T]$ have connectivity at least s and t , respectively. In this paper, we prove that every $(s + 1)$ -connected graph with minimum degree at least $\frac{3s}{2} + 2m - 3$ contains a cycle of length at least m whose removal remains s -connected. We also consider the same result for triangle-free graphs. Our proof leads to a polynomial algorithm for finding such a long cycle in every $(s + 1)$ -connected graph.

[Back to schedule](#)

Variations on Weak Saturation

Roman Vasquez
Wesleyan College

Weak saturation is a process which measures the presence of certain subgraphs H inside of a given graph. A graph G is weakly H -saturated if we can obtain K_n by adding one edge to G at a time, where each additional edge creates at least one new copy of H . Several established variations of weak saturation exist, such as edge bootstrap percolation, weak saturation in hypergraphs, and weak saturation in host graphs other than K_n . We will present some results related to some of these variants.

[Back to schedule](#)

Pick's Theorem - Novel Proofs and a Novel Problem in Combinatorial Geometry

James Wash
Auburn University

Pick's theorem is a well-known geometric result that links the area of lattice polygons to the number of lattice points within their perimeter. Less well known is its equivalence to Euler's polyhedral formula, and the wide variety of proof techniques this enables. In this talk drawing from recent Master's research, three novel proofs of Pick's theorem or a Pick's theorem variant are presented. In addition, a novel combinatorial problem based on traditional Pick's theorem proofs is presented - along with preliminary counting results for bounds, four special cases, and an equivalent definition of the problem on a subclass of the well-covered graphs.

[Back to schedule](#)

A discrete view of Gromov's filling area conjecture

Chris Wells
Auburn University

In differential geometry, a metric surface M is said to be an isometric filling of a closed metric curve C if $\partial M = C$ and $d_M(x, y) = d_C(x, y)$ for all $x, y \in C$. That is, M does not introduce any "shortcuts" between points on its boundary. Gromov's filling area conjecture from 1983 asserts that among all isometric fillings of the Riemannian circle, the one with the smallest surface area is the hemisphere. Gromov's conjecture has been verified if, say, M is homeomorphic to the disk and in a few other cases, but it is still open in general. Admittedly, I'm not a differential geometer, so we consider instead a particular discrete version of Gromov's conjecture which is likely fairly natural to anyone who studies graph embeddings on arbitrary surfaces. We obtain reasonable asymptotic bounds on this discrete variant by applying standard graph theoretic results, such as Menger's theorem. These bounds can then be translated to the continuous setting to show that any isometric filling of the Riemannian circle of length 2π has surface-area at least 1.36π (the hemisphere has area 2π). This appears to be the first quantitative lower-bound on Gromov's problem that applies to an arbitrary isometric fillings. (Based on joint work with Joe Briggs)

[Back to schedule](#)

Circuit Covering of Signed Graphs

Dong Ye
Middle Tennessee State University

A signed graph is a graph associated with a signature $\sigma : E(G) \rightarrow \{-1, 1\}$. A circuit of a signed graph is either a positive cycle or a barbell (two negative cycles joined by a path). A circuit is a minimal dependent set of signed graphic matroid. A circuit cover is a family of circuits which cover all edges of the signed graph. Bouchet proved that a signed graph has a circuit cover if and only if it is flow-admissible. In this talk, we will focus on recent developments in the study of circuit covers of signed graphs.

[Back to schedule](#)

Arithmetic progressions in Euclidean Ramsey theory

Chi Hoi Yip
Georgia Institute of Technology

Euclidean Ramsey theory concerns Ramsey-type problems over Euclidean spaces. In this talk, I will discuss some recent progress towards monochromatic arithmetic progressions in Euclidean Ramsey theory. Joint work with Gabriel Carrier and Kenneth Moore.

[Back to schedule](#)

On the dual of Erdős' circuit spectrum problem

Shunzhe Zhang
University of Mississippi

The cycle spectrum of a graph is defined to be the set of cycle sizes. Erdős (1992) conjectured that every connected graph G with girth g and average degree d has a cycle spectrum of size at least $\Omega(d^{\lfloor (g-1)/2 \rfloor})$. Sudakov and Verstraë (2008) proved this conjecture in a stronger form. In this paper, we consider the dual question of Erdős: Can we find an exponential function $f(\delta, g)$ such that every graph G with girth g and minimum degree δ has a bond spectrum of size at least $f(\delta, g)$? We solve this problem by giving such an exponential function $f(\delta, g)$ when the girth of the graph is at least seven. We also show that the bond spectrum of any graph with sufficiently large number of vertices and large girth contains an arithmetic progression. This is joint work with Bing Wei and Haidong Wu.

[Back to schedule](#)

An extremal problem for minimally k -connected graphs

Yingze Zhao
University of Mississippi

Let k be a positive integer. A graph is minimally k -connected if it is k -connected, but for each edge e of G , the deletion of e from G is not k -connected. A classical result of Mader states that every minimally k -connected graph contains at least $\Delta(G)$ vertices of degree k , where $\Delta(G)$ is the maximum degree of G . In this paper, for $k = 3, 4$, we characterize all minimally k -connected graphs G having exactly $\Delta(G)$ vertices of degree k . Our results give a complete description of the extremal graphs attaining Mader's bound in these two cases.

[Back to schedule](#)