Inelastic Collisions

Object: To see if momentum and energy are conserved for an inelastic collision.

Apparatus: Ballistic pendulum, two-meter stick, tray with carbon paper, balance, and ruler.

Foreword

The momentum p of a body is defined as the product of its mass m and velocity v , or

(1) $$p = mv \ .$$

If the mass of the object does not change, then any change in momentum Δp is a consequence of a change in speed Δv, or

(2) $$\Delta p = m\Delta v \ .$$

In cgs units, the momentum is expressed in gm-cm/sec and in mks units, it is expressed in kg-m/sec. For convenience, we shall employ the mks units in this experiment.

The principle of conservation of momentum may be derived directly from Newton's Second and Third Laws of Motion. Let an object of mass m_1 and velocity v_1 strike a stationary object of mass m_2. Application of Newton's Third Law shows that during the time Δt of the collision, the force that acts to decelerate mass m_1 is equal to the force that accelerates mass m_2. Representing these forces as F_1 and F_2, respectively, Newton's Third Law may be stated as

(3) $$F_2 = -F_1 \ .$$

If a_1 denotes the resulting deceleration of m_1 and if a_2 denotes the acceleration of m_2, using Newton's Second Law, the above may be written as

(4) $$m_2 a_2 = -m_1 a_1 \ .$$

Recalling that acceleration is defined as $a = \Delta v/\Delta t$ and the fact that the time Δt of the collision is the same for both objects, equation (4) may be rewritten as

$$m_2\Delta v_2 = -m_1\Delta v_1 \ , \text{ or}$$

(5) $$\Delta p_2 = -\Delta p_1 \ .$$

Equation (5) states simply that in a collision between two masses, the loss in momentum of the first mass is equal to the gain in momentum of the second mass.

Let the total momentum of the two masses before collision be written as

(6) $$P_i = P_1 + P_2 \ .$$

After the collision, the momentum of the two masses will be changed to

(7) $$P_1' = P_1 + \Delta P_1 \quad \text{and} \quad P_2' = P_2 + \Delta P_2 \ .$$

Hence, the total momentum of the two masses after collision may be written as

$$P_f = P_1' + P_2' \ ,$$

$$P_f = (P_1 + \Delta P_1) + (P_2 + \Delta P_2) \text{ by (7)} \ ,$$

$$P_f = (P_1 + \Delta P_1) + (P_2 - \Delta P_1) \text{ by (5)} \ ,$$

(8)

$$P_f = P_1 + P_2 \ .$$

Thus, a direct consequence of the fact that the loss in momentum of the first mass is equal to the gain in momentum of the second mass (i.e., equation (5)) is the fact that the total momentum before collision (equation (6)) is equal to the total momentum after collision (equation (8)). That is, the momentum remained constant or was conserved. This generalization is not dependent on the type of collision, whether elastic or inelastic, and forms one of the great physical laws in mechanics.

In this experiment, a ballistic pendulum (see **Figure 1**) will be used to determine if momentum and kinetic energy are conserved in an inelastic collision.

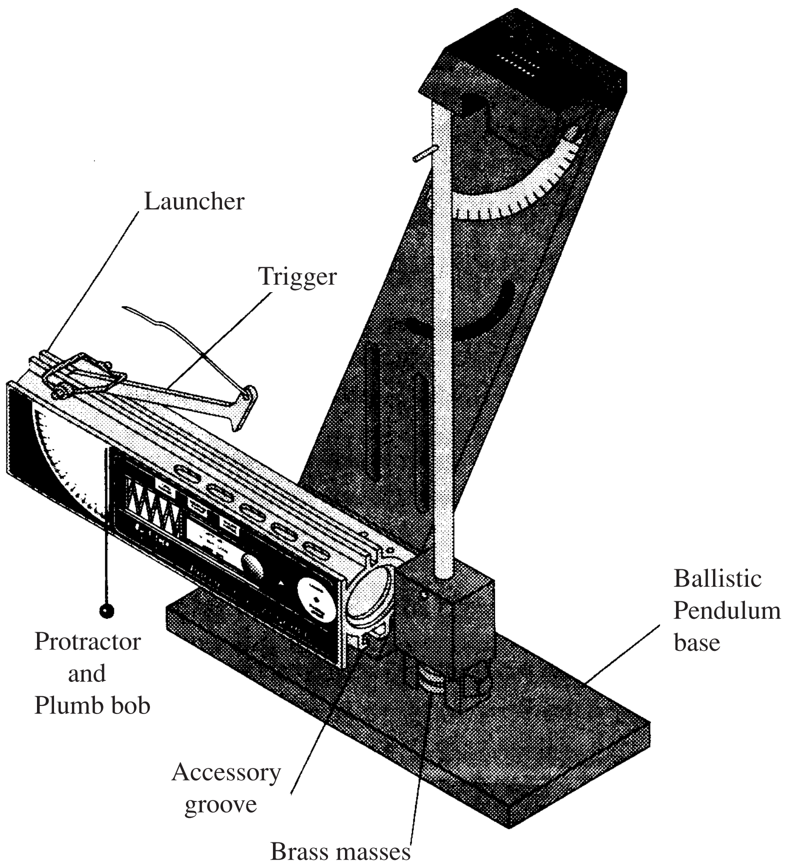


Figure 1

During the development of the theory, data taking, and calculations, the following notation will be used:

- m = mass of the ball
- M = mass of the cup
- m + M = mass of the ball and cup
- v = speed of the ball before collision
- V = speed of the ball and cup at the instant after the collision
- h = the height that the center of mass of the ball and cup rise after the collision
- H = the height of the ball off of the laboratory floor before it is fired
- R = the range of the ball if it is fired horizontally off of the table

First let's develop expressions for determining the total momentum and kinetic energy before collision. The total momentum and energy before collision may be written as

(9) $$P_i = P_{i_{ball}} + P_{i_{cup}} \quad \text{and} \quad K_i = K_{i_{ball}} + K_{i_{cup}}$$

or

(10) $$P_i = mv \quad \text{and} \quad K_i = \tfrac{1}{2} mv^2$$

since the cup is initially at rest. Since the mass m of the ball may easily be determined with a balance, the problem of determining the total momentum and kinetic energy before collision has been reduced to finding the speed of the ball before collision. The speed of the ball before collision may be determined by firing the ball horizontally off of the table.

If the ball is fired horizontally off of the table, general expressions for the x and y coordinates of its position at any time are given by

(11) $$x = x_0 + v_{0x}t + \tfrac{1}{2} a_x t^2 \ , \quad \text{and} \quad y = y_0 + v_{0y}t + \tfrac{1}{2} a_y t^2 \ .$$

The general expressions in equation (11) may be made unique for this situation (firing the ball horizontally off of the table) by inserting the following constants

(12) $$\begin{aligned} x_0 &= 0 & y_0 &= H \\ v_{0x} &= v \quad \text{and} & v_{0y} &= 0 \\ a_x &= 0 & a_y &= -g \end{aligned}$$

to obtain

(13) $$x = vt \qquad \text{and} \qquad y = H - \tfrac{1}{2} gt^2 \ .$$

When the ball lands

(14) $$\begin{aligned} x &= R\ , \text{ the range} \\ y &= 0\ , \text{ and} \\ t &= T_F\ , \text{ the time of flight.} \end{aligned}$$

When (14) is inserted into equation (13), the time of flight T_F may be eliminated between the x and y coordinates for the position of the ball to obtain an expression for the speed of the mass m before the collision.

(15) $$v = R\sqrt{\frac{g}{2H}} \ .$$

Combining equations (10) and (15), one obtains

(16) $$P_i = mR\sqrt{\frac{g}{2H}} \quad \text{and} \quad K_2 = \frac{mR^2 g}{4H}$$

Since all of the quantities on the right of equation (16) are easily obtained experimentally, we have developed a method for determining the total momentum and kinetic energy before collision.

Second, let's develop a method for determining the total momentum and kinetic energy after collision. Since the collision is inelastic (the objects stick together), the total momentum and kinetic energy after collision are

(17) $$P_f = P_{f_{ball}} + P_{f_{cup}} \quad \text{and} \quad K_f = K_{f_{ball}} + K_{f_{cup}}$$

or

(18) $$P_f = (m + M)V \quad \text{and} \quad K_f = (m + M)V^2/2$$

Since the masses m and M of the ball and cup, respectively, may easily be determined with a balance, the problem of determining the total momentum and kinetic energy after collision has been reduced to finding the speed of the combined mass after collision. The speed of the combined mass m + M may be determined by firing the ball into the cup and noting how far the center of mass rises. The kinetic energy of the combined mass the instant after collision is given by

(19) $$KE = \tfrac{1}{2}(m + M)V^2 \ .$$

If the center of mass of the combined system rises an amount h, the change in the potential energy is given by

(20) $$\Delta PE = (m + M)gh \ .$$

Since the dissipative forces in the apparatus are small enough to be ignored, all of the kinetic energy of the system after the collision goes into changing one potential energy of the system. Hence, equations (19) and (20) may be equated and solved for V to obtain

(21) $$V = \sqrt{2gh} \ .$$

Combining equations (18) and (21), one obtains

(22) $$P_f = (m + M)\sqrt{2gh} \quad \text{and} \quad K_f = (m + M)gh$$

Since all of the quantities on the right of equation (22) are easily obtained experimentally, we have developed a method for determining the total momentum and kinetic energy after collision.

If momentum is conserved, the value obtained for the momentum before collision (P_i from (16)) and the value obtained for the momentum after collision (P_f from (22)) will either be identical or will differ by a very small percent. If kinetic energy is conserved, the value obtained for the kinetic energy before collision (K_i from (16)) and the value obtained for the kinetic energy after collision (K_f from (22)) will either be identical or will differ by a very small percent.

Procedure

Part I. Momentum and Kinetic Energy Before Collision

Under no circumstances should you try to lift or move the entire ballistic pendulum apparatus by the pendulum. Contacting the pendulum in any way while trying to move or lift the entire apparatus may cause damage to the pendulum. If you must lift the entire apparatus, lift the apparatus by grasping the base and the very top of the support.

Carefully loosen the thumbscrew at the top of the pendulum and remove the pendulum from its support. Determine the mass of the ball and the combined mass of the pendulum and the ball. Also determine the length (L) between the pivot point and the Center of Mass (CM). See the Data Sheet for a diagram.

Carefully reassemble the pendulum to the support. You must make sure that the pendulum is facing in the correct direction such that the pendulum will be able to receive the ball, and you must make sure that the angle indicator will move as the pendulum swings away from the launcher. Position the entire apparatus near the edge of the table, adjust the apparatus until the launcher is level, and clamp the apparatus firmly to the bench. Swing the pendulum up until the rod of the pendulum engages and remains in a horizontal position; in this position, the launcher can be fired without the ball being caught by the pendulum. Place the ball in the launcher, and cock the launcher with the ramrod until the launcher is engaged in the medium range setting.

Launch the ball by pulling the string, and carefully watch where the ball impacts the floor. Center a piece of blank paper over the impact site and tape it to the floor. *Lay a piece of carbon paper on top of the blank piece of paper, but DO NOT tape the carbon paper down*. During subsequent launches, the carbon paper will mark the blank piece of paper with the exact location of the ball's impact.

Launch the ball five times and determine the average range. It is important that each launch be made from the medium range setting. After you have data from five launches, determine the velocity of the ball before collision and the subsequent momentum and kinetic energy before the collision.

Part II. Momentum and Kinetic Energy After Collision

Release the pendulum from its horizontal position and let it hang freely at the zero degree position. Load and launch the ball five times into the pendulum. Again, it is extremely important to cock the launcher to the medium range setting for all five launches. Before each launch, make sure that the angle indicator will slide back and record the maximum angle of elevation. Determine the average angle of elevation, and use the diagram on the Data Sheet to determine the average height (h) that the pendulum Center of Mass ascertains.

After the average height is determined, calculate the velocity just after the collision and the subsequent momentum and kinetic energy just after the collision.

Part III. Comparison

Compare the momentum and kinetic energy before and after the collision by determining the percent difference. Use these comparisons to justify whether the Momentum of the system and the Kinetic Energy of the system was conserved or not for this inelastic collision.

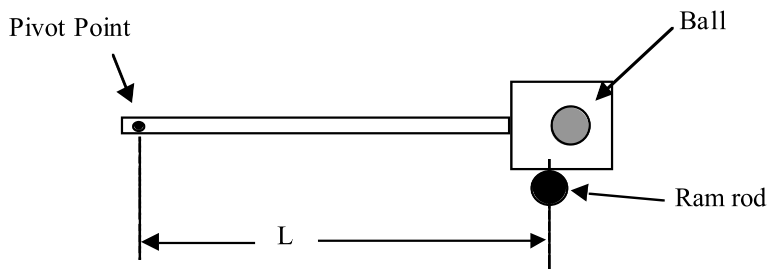
NAME ______________________________ SECTION ________________ DATE ________

DATA AND CALCULATION SUMMARY

Part I. Momentum and Kinetic Energy Before Collision

Initial Data

Mass of Ball	Mass of Pendulum and Ball	L – Distance between pivot point and CM



When the pendulum (with ball) balances on the ram rod, the point where the ram rod contacts the pendulum is the location of the Center of Mass.

Launch Data

Height of ball from Floor	H =					
Range	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
Calculation of velocity before collision						

Momentum and Kinetic Energy after Collision

Momentum Before Collision	Kinetic Energy Before Collision

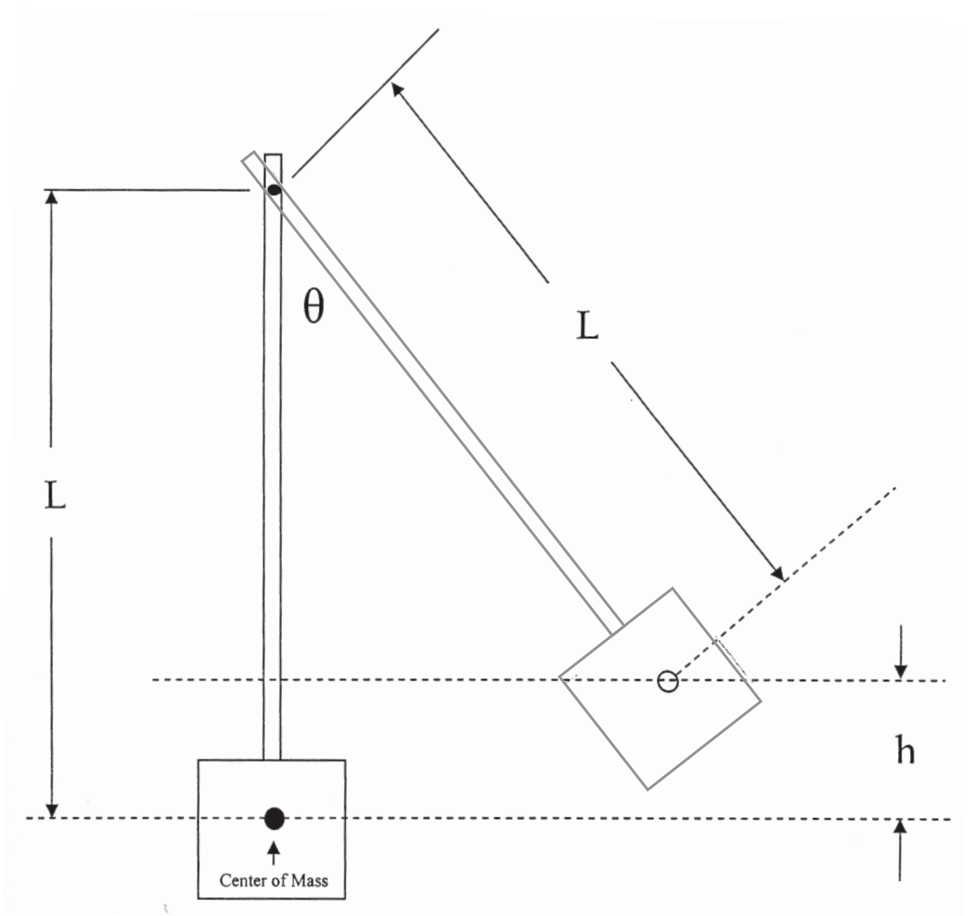
Part II. Momentum and Kinetic Energy after Collision

Collision Data

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
Angular Position						

Calculated Average h

Use the diagram below and the necessary data to calculate the average h. Label the diagram as necessary and show your work.	h =



NAME ____________________ SECTION ____________________ DATE ________

DATA AND CALCULATION SUMMARY

Calculation of the Velocity just after the Collision (Show Calculation)

Momentum and Kinetic Energy just after the Collision

Momentum After Collision	Kinetic Energy After Collision

Part III. Comparison

Percent Difference of Momentum (Show Calculation)	Percent Difference of Kinetic Energy (Show Calculation)

Questions

1. An auto weighing 4000 lbs. and traveling at a velocity of 75 mph collides head on with a smaller auto weighing 2500 lbs. and traveling 60 mph. When the autos collide, the autos lock together. Find the resultant velocity (speed and direction) of the two after impact.

2. A machine gun fires 500 bullets/minute. If each bullet has a mass of 30 grams and a muzzle velocity of 50×10^2 cm/s, find the average reaction of the gun on its support.

3. If the autos in problem 1 had collided at an intersection (at an angle of 90 degrees), find the resultant velocity (speed and direction).

Name: ______________________________ **Banner ID:** ______________________________

Lab Group ID: ______________________ **Number of Lab Partners:** _________________

Lab 6M Summary

Raw Data

H Launch Height	R Launch Range	m Ball Mass	Θ Average Pendulum Angle	L Distance between pivot point and CM of Pendulum with the ball	M Combined Mass of Pendulum and Ball

Calculated Velocities

Calculated average velocity of ball before collision.	
Calculated average velocity of pendulum and ball immediately after the collision.	

Questions

What type of collision occurred when the ball hits the pendulum? Explain.

Was Momentum Conserved? Explain.

Was Kinetic Energy Conserved? Explain