Discrete and Continuous Dynamical Systems

Series S

Issue on recent advances in partial differential equations and dynamical systems: Dedicated to Georg Hetzer, on the occasion of his 75th birthday

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SPECIAL ISSUE ON RECENT ADVANCES IN PARTIAL DIFFERENTIAL EQUATIONS AND DYNAMICAL SYSTEMS: DEDICATED TO GEORG HETZER, ON THE OCCASION OF HIS 75TH BIRTHDAY

PREFACE

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This special issue is dedicated to Professor Georg Hetzer on the occasion of his 75th birthday. Georg's trajectory in mathematics started at Aachen University of Technology (RWTH Aachen) in Germany, where he received a master's degree (Staatsexamen) in mathematics and physics education in 1970 and a Ph.D. in mathematics in 1973. He served as an assistant lecturer at Aachen until 1978, when he obtained his Venia Legendi (Habilitation) and was appointed lecturer (Privatdozent). In 1984, he was promoted to adjunct professor (Außerplanmäßiger Professor). The same year, he became a research associate in a federally funded climate research program, the Aachen component of which was entitled "Mathematical Sensitivity Analysis of Global Climate Models." In 1986 Georg joined the Department of Foundations, Analysis, and Topology at Auburn University as a visiting professor. He was appointed a tenure-track full professor in 1987 and granted tenure two years later. Georg then spent the balance of his career at Auburn; he retired and was appointed professor emeritus in 2021.

Georg's early work, in part with Jochen Reinermann (his advisor) and Volker Stallbohm, on fixed-point theory, uniform-boundedness principles, Fredholm operators, coincidence degree, and global bifurcation, may be classified as nonlinear functional analysis. Subsequently, his research interests evolved in a more applied direction, with a large body of work, in part with Ed Landesman, on nonlinear PDEs, specifically elliptic boundary-value problems at resonance. Starting in 1984, while associated with the aforementioned climate research program, Georg developed an interest in the mathematical analysis of energy-balance climate models, an interest that stayed with him throughout his career and led to numerous publications, in part with Ildefonso Díaz, Lourdes Tello, and Paul Schmidt. Starting in the nineties, he worked on reaction-diffusion systems with memory, set-valued evolution equations, random and stochastic parabolic equations, almost periodic competition-diffusion systems and population dynamics, frequently collaborating with Wenxian Shen and their respective students.

Georg leaves an indelible mark as the advisor of numerous students, many of whom embarked on successful careers in academia. His doctoral students include Rüdiger Gottschalk, Manfred Trinkhaus, and Paul Schmidt (at Aachen), Bettina Schmidt, Elena Rossi, and Tung Nguyen (at Auburn). Beyond his lasting impact as a researcher and teacher, Georg played a key role in the development of mathematics at Auburn University. In an essay following this preface, his student Paul Schmidt reflects on Georg's legacy as the founding father of the Auburn PDE and Applied Mathematics group.

In addition, this special issue features fourteen research papers, concerned with recent advances in dynamical systems, partial differential equations, and their respective applications, that were contributed by collaborators and colleagues of Georg's from Europe, the United States, and China.

Specifically, Professors Tomás Caraballo, Ildefonso Díaz, Lourdes Tello, and José Ignacio Tello from Spain contributed works on nonlocal partial differential equations with long-time memory, stochastic energy-balance climate models with Legendre-weighted diffusion, diffusive logistic equations with unbounded distributed limitations, a global climate model with non-monotone multivalued coalbedo, and a Keller-Segel system with flux limitations. Professor Peter Kloeden from Germany contributed a paper on non-autonomous Laplacian lattice systems on weighted spaces. Professors Ratnasingham Shivaji, Wenxian Shen, Yanzhao Cao, Hans-Werner van Wyk, Thi-Thao-Phuong Hoang, and Aijun Zhang from the United

States contributed works on bifurcation curves for classes of elliptic systems, a parabolic-parabolic chemotaxis model with logistic source, a backward stochastic differential equation method for uncertainty quantification in deep learning, a nonautonomous chemostat model for the gut microbiome, waveform relaxation and mixed hybrid finite-element methods for transport processes, and periodic nonlocal Fisher-KPP equations with non-compact asymmetric kernels. Professors Tongtong Liang and Yejuan Wang from China contributed papers on stochastic functional Navier-Stokes equations and on stochastic delay wave equations with linear memory and nonlinear damping.

The editors believe that this collection of papers is a true reflection of Georg Hetzer's diverse research interests and vivid evidence of his lasting impact on the field.

GEORG HETZER AND APPLIED MATH AT AUBURN

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Georg and I go back a long time. We first met in the Fall of 1979, in a classroom at RWTH Aachen (Aachen University of Technology), where he was teaching a year-long course on functional analysis. He was a Privatdozent (lecturer) at the time, having obtained his Venia Legendi (habilitation) the year before; I was a third-year student in search of an advisor for my Diplomarbeit (master's thesis).

Mathematics at Aachen was organized in a very peculiar, sort of feudal, way: there were a number of Ordinarien (chaired professors), each ruling his own little fiefdom and controlling research and instruction in his field ("his" since there were no female Ordinarien). My first two years of study had been dominated by one of those overlords, and I had asked him for a thesis topic. His excellency granted me a one-on-one meeting and was willing to accommodate my request; but, of course, one of his subordinates was going to take me under her wings (yes, there were female subordinates). After meeting with her and studying some literature she recommended, I decided that this project didn't agree with me, neither in style nor in substance.

You know, the 1968 student revolution was long over, but some of its spirit survived in my generation. We were not exactly rebellious, but not subservient either. So I informed his excellency that I was going to seek greener pastures. He warned me that greener pastures could not be found, but granted me permission to withdraw, as I was obviously delusional. That's how I came to ask Georg to be my advisor. Ten years my senior, he was part of Generation 68 and, as such, not beholden to feudal power structures. Also, he had no subordinates, female or otherwise, that he could have dumped me on.

Plus, I really liked his lectures on functional analysis. They had a mathematical flavor, inspired by Bourbaki, completely different from anything I had previously encountered. It took me a while to get used to both, Georg and Bourbaki, but after I did, I never looked back. Years later, I repeatedly taught functional analysis at Auburn and never used any class materials beyond my expanded class notes based on Georg's lectures at Aachen.

I guess I was hoping he would give me a thesis topic in abstract functional analysis. Georg's early work, in part with Jochen Reinermann (his advisor) and Volker Stallbohm, on fixed-point theory, uniform-boundedness principles, Fredholm operators, coincidence degree, and global bifurcation, could in fact be classified as abstract nonlinear functional analysis. By 1979, however, his research interests had evolved in a decidedly more applied direction, leading to a large body of work, in part with Ed Landesman, on nonlinear PDEs, specifically elliptic boundary-value problems at resonance.

So, naturally, Georg introduced me to PDEs. In the following years, he supervised first my master's thesis, then my doctoral dissertation. In the Fall of 1986, while I was in the final stages of my doctoral work, Georg joined Auburn University as a visiting professor; the following year, he was hired as a full professor and granted tenure after two years on the job. After my graduation, I followed him to

Auburn, first as a postdoctoral fellow of the DFG (the German Science Foundation), then as a tenure-track assistant professor. We both spent the balance of our professional careers right there, in the loveliest village on the plains.[1]

Mathematics at Auburn was organized in a very peculiar, somewhat embarassing, way: there was a Division of Mathematics (no pun intended, I assume) with two departments; ours was called FAT (Foundations, Analysis, and Topology), the other ACA (Algebra, Combinatorics, and Analysis). FAT was dominated by T, ACA by C; the other letters were, more or less, window dressing. T and C didn't like each other much. The first time I went to see a dentist in Auburn, the good doctor, hearing that I was a math professor, curiously inquired whether the T's and the C's were still knocking each others' teeth out. Purely professional interest on his part — he was doing brisk business with both. Not only at the dentist's office, but also at conferences and other professional meetings, the Auburn division of mathematics made for a great conversation starter. ("Ah, you are at THAT place, ha-ha-ha-ha. . .")

You may have noticed that the A in FAT is the same as the second A in ACA. Before 1986, there was, in fact, not much A in FAT. There were Jack Brown, in real analysis, J. P. Holmes, in semigroups, Geraldo De Souza and Gary Sampson, in harmonic analysis. A third harmonic analyst, Henry Lin, left shortly after I came to Auburn; his position was eventually filled by Yongsheng Han. Jack Rogers, a T by education, and Steve Stuckwisch were teaching numerical analysis. FAT leadership decided that more A's, especially A's of an applied vein, were needed to be competitive or, better yet, beat the competition. I was told, probably by someone in ACA, that they would actually have preferred to focus on the essentials and rename the department the Loachapoka[2] Institute of Topology; but this is not true, I swear! Besides, it would not have gone over well with the BOT[3]. Anyway, seeking to enhance A and, thereby, FAT's viability and competitiveness, leadership turned to a good friend from Texas[4], J. P. Holmes's former advisor John Neuberger, for strategic advice.

John Neuberger knew Georg very well; I don't know where and how John and Georg met, but John and his wife Barbara visited us at Aachen. In fact, I remember sitting in Georg's tiny office and discussing my master's thesis and future plans with John. That must have been in 1983, the year I completed my Diplom (master's degree). By then, Georg had been Privatdozent (lecturer) for five years, and the following year, he was appointed Außerplanmäßiger Professor (adjunct professor); but he had not succeeded in procuring a permanent position. The way I see it, Georg was lacking "Vitamin B" (Beziehungen), which in English should be called "Vitamin C" (connections). You see, our boss, Friedhelm Erwe[5], was not like the other bosses at Aachen — much less of a feudal overlord. This is why we liked him, why the other bosses didn't like him, and why he didn't have much clout. To make

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[1] "Sweet Auburn, loveliest village of the plain" is the first line of a pastoral elegy by Oliver Goldsmith, The Deserted Village, published in 1770. It's a pretty sad poem actually.

[2] Loachapoka, with a population in excess of 150, is a hamlet five miles west of the Auburn campus, famous for its annual syrup sopping festival; the name means "turtle killing place" in the Muskogee (Creek Indian) language.

[3] Board of Trustees.

[4] The T in FAT should actually have been T^2, for Texas Topology.

[5] Prof. Dr. Friedhelm Erwe, my academic great-grandfather, passed away on July 1, 2021. He was almost 99.

a long story short, John told Georg about FAT's predicament and suggested that Georg might be just what FAT needed; the rest is history.

While visiting Auburn, Georg got along very well with everybody, especially the T's, as he was fluent in their language; he had even taught a course in algebraic topology at Aachen, in the early eighties. So he was offered a full professorship and accepted. I can't know the details, of course, but I assume there was an understanding that he would be given a chance to build up a research group in PDEs and applied math, with priority in future hiring decisions and a junior faculty position to fill the next year. Georg encouraged me to apply for that position. I did and, simultaneously, applied for a post-doctoral fellowship from the German Science Foundation, to work with Georg on energy-balance climate models. Climate modeling is a field that Georg had gotten into during his last years at Aachen, while serving as a research associate in a federally funded climate research program. This interest stayed with him throughout his career and led to numerous publications, in part with Ildefonso Díaz, Lourdes Tello, and myself. Myself because, you guessed it, I got the post-doc, and also the position, after a friendly phone interview with George Kozlowski, the FAT chair, while I was vacationing in the Austrian alps. One hell of a fight for a position it was!

George Kozlowski passed away on October 16, 2021. I fondly remember him, even though we had our run-ins over the years. He tended to be orthogonal to his deadlines, even important ones. But he sat through a whole semester of seminar talks on Mo Hirsch's theory of monotone dynamical systems, as applied to PDEs, which I delivered during my first year at Auburn. Wherever you are, George, I hope there are beautiful cohomologies to behold!

If I were to name three people who were instrumental in facilitating my academic career, I'd list John Neuberger, Georg Hetzer, and George Kozlowski. Without John, Georg would not have ended up in Auburn. Without Georg, I would have ended up neither in PDEs nor in Auburn. Without George, I might well have ended up in industry rather than academia.

The following year, in 1989, we hired A. J. Meir, without a trace of controversy, adding numerical/computational expertise to our budding research group. I picked him up at the airport, when he came to Auburn for his interview, and we took him to the "Hungry Hunter" for dinner. While singularly unimpressed by the cuisine, A. J. felt very welcome, and he was. After his talk we held a War Eagle[6] — at the Holmes's, I think, or maybe at the Kozlowski's. War Eagles were regular Friday night events, usually following a colloquium talk, where the whole tribe would meet at someone's house, to drink beer, enjoy barbeque[7] or other staples of southern cooking, bitch about ACA, the dean, and the upper administration, and reminisce about the good old days, when everybody was young and strong and the night

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[6] "War Eagle" is Auburn's battle cry and universal greeting. Supposedly, it goes back to the first time Auburn met Georgia on the gridiron in 1892. Among the Auburn fans that day was a civil war veteran with a pet eagle, whom he had found, wounded, on a battlefield during the war. Auburn was trailing badly, when the eagle suddenly broke free and soared over the football field, spurring Auburn to victory. Legend has it that, at the game's end, the poor eagle took a dive, crashed into the ground, and died; but Auburn football has been soaring ever since (except when it didn't). — There used to be a somewhat dubious establishment on Opelika Road in Auburn, called the "War Eagle Supper Club," where FAT folks liked to meet and socialize at the end of the work week. By the time I came to Auburn, that habit had fallen out of fashion; but the term "War Eagle" had entered the FAT vernacular as a moniker for beer-infused tribal powwows.

[7] J. P. Holmes had a grill the size and shape of a steam locomotive.

would typically end with the building of a human pyramid. I don't recall a human pyramid at the end of A. J.'s interview War Eagle; but I do remember a bottle of Wild Turkey.

Anyway, A. J. liked beer and barbeque and Wild Turkey and, thus, accepted our offer. At this point, we had already grown competitive with ACA, which had its own differential equations group led by Johnny Henderson and including three recent hires, Greg Harris, Darrel Hankerson, and Bertram Zinner. We cultivated good relations with them and would attend their seminars, over in the Math Annex, an old military hangar turned gym turned home of ACA in the shadow of Jordan-Hare Stadium. The Math Annex burned down some years later, in 1996, during a nationally televised football game against LSU[8], apparently due to an out-of-control charcoal grill used for tail-gating.[9] Unfortunately, the fire did not spread to Parker Hall.[10] Johnny Henderson left Auburn in 2002, to help develop a Ph.D. program in mathematics at Baylor University. Greg Harris focussed on administrative work; Darrel Hankerson moved into discrete math; Bertram Zinner found his calling in teaching statistics.

Back in FAT, we were a happy research cluster of three, Georg, A. J., and myself, until 1992, when we managed to hire Wenxian Shen, who, at the time, was finishing her Ph.D. under Shui-Nee Chow at Georgia Tech. This coup was made possible by Georg's long and carefully cultivated relationship with Jack Hale, then the director of the Center for Dynamical Systems and Nonlinear Studies at Georgia Tech. We used to meet Jack at the Dixieland Analysis Seminar, organized by Paul Waltman and Rudolf Schmid at Emory, and chat over pitchers of beer and plates of nachos after the talks.[11] Jack Hale.[12] I could add his name to the triumvirate of people who were instrumental in furthering my academic career. He accepted my first paper for publication in JDE[13] without any changes; I never saw a referee's report. I bet he noticed the acknowledgement at the bottom of the first page, where I thanked Georg for guiding my doctoral research. In the end, Georg had acquired some Vitamin C.

Jack Hale could be tough as nails, but he was always very supportive of aspiring young people. He used to go to China, scouting for talent, and I believe that is how Wenxian came to Georgia Tech. Bringing her to Auburn took a hard and protracted fight, which revealed a darker side of Georg. Some people were displeased, to put it mildly, with what they perceived as his strong-arm tactics. And even more people thought that our group had grown big enough — just big enough to beat ACA, and wasn't that the whole point? The tribal elders were reminded of the sorcerer's

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[8]Louisiana State University.

[9]What a terrible loss! LSU won 19-15, after Auburn attempted a two-point conversion to tie the game and LSU intercepted a pass, running it all the way back for two points.

[10]Parker Hall is the main math building at Auburn, and has been ever since the dawn of time. Adjacent Allison Hall used to house the physics department and, after demolition of the Math Annex, our friends from ACA. There are a number of important constants that govern the physical universe, such as Newton's gravitational constant, the speed of light, or the Hubble constant. Lesser known, but equally important, is the Parker-Allison constant, which is 5 years. Math will get a new building 5 years from now. Always.

[11]Rudolf claimed that beer and nachos were paid for by the dean's office. Which made us think that Emory was a much better place than Auburn, in terms of research support.

[12]Jack K. Hale passed away on December 9, 2009, aged 81. He hailed from Kentucky, drove a vintage Ford Mustang, and was proud of both.

[13]Journal of Differential Equations.

apprentice: "Die ich rief, die Geister, werd' ich nun nicht los..."[14] ("The spirits that I summoned I cannot rid myself of now...") In view of her record, the hiring of Wenxian Shen was one of the best things that ever happened to mathematics at Auburn, and in hindsight, even former detractors must admire Georg's foresight and thank their respective gods for his persistence! Well, let's not get carried away...

At the time of Wenxian's hiring, we were still FAT. The four of us had our own little computer network and email server, administered and maintained by A. J. The domain name was "cam," for computational and applied math; our workstations were called "is," "isnot," "was," and "maybe." Get it? Give you a hint: ajm.is.fat, pgs.isnot.fat, gh.was.fat, ws.maybe.fat. We were having lots of fun making fun of ourselves. And we were working pretty hard. In 1994, around the time we came up for T & P[15], A. J. and I procured NSF[16] funding for our work on magnetohydrodynamics. In view of negative historical precedents in FAT, we were somewhat worried about T & P; but, in the end, it was no issue at all, neither within the department nor higher up. The same year, also Wenxian landed her first NSF grant, and she went on to have almost continuous NSF funding for the next two decades; her T & P was, of course, a no-brainer.

Not only for our group, but for all of FAT, 1994 was a banner year in terms of NSF funding. There were three more research awards that year, to Gary Gruenhage (in set theoretic topology), to Krystyna Kuperberg (in dynamical systems), and to Wlodek Kuperberg (in geometry). Before 1994, nobody in FAT but Gary Gruenhage had ever had NSF research funding. So, it is fair to say that FAT was finally on a roll. Yet, instead of cheering us on, the dean and upper administration attempted to starve us to death. Go figure. After Wenxian, we did not hire anybody, in applied math or any other area, for more than a decade. Parker Hall was slowly turning into a home for the aged. But big changes were in the offing... First, we witnessed a steady trickle of faculty from ACA to FAT, which ultimately left ACA unviable. FAT and parts of ACA then turned into the Department of Mathematics, the remnants of ACA into the Department of Discrete and Statistical Sciences, a.k.a. DSS. (If you don't know what discrete sciences are, you are not alone...) In the course of this restructuring, our applied math group was eventually joined by Johnny Henderson (if only temporarily) and Greg Harris. Also Frank Uhlig joined the Department of Mathematics, reinforcing the numerical analysis detail. Curiously, Frank had come from Aachen, like Georg and myself, but well before us; yet he had made his home in ACA. Other additions included analysts Narendra Govil, Ted Kilgore, and Richard Zalik, as well as, eventually, the entire probability group (Olav Kallenberg, Jerzy Szulga, Ming Liao, Bill Hudson, and Jerry Veeh).

Alas, also this new division of math wasn't meant to last: Dean Schneller[17] had had enough of quarreling mathematicians. More importantly, he and the upper administration wanted to create a permanent home for statistics within COSAM[18]. Statistics at Auburn was organized in a very peculiar, let's say, unconventional, way: numerous departments across campus had their own experts teaching statistics; the few true statisticians in residence were housed in DSS, but operated under the umbrella of a campus-wide entity, called the Academy of Statistics (no kidding),

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[14]Johann Wolfgang von Goethe, Der Zauberlehrling.

[15]Tenure and promotion.

[16]National Science Foundation.

[17]Stewart Schneller, Dean of COSAM, 1994–2010.

[18]College of Sciences and Mathematics.

under the auspices of the provost's office. Given this less than enticing environment, recruiting and retaining quality statistics faculty was all but impossible. Seeking to solve all the problems once and for all, Dean Schneller formed the Interdepartmental Committee for the Consolidation of Auburn Mathematics, a.k.a. the Merger Committee, and charged it with drafting a constitution for a united department. I served on that committee for more than three years. In the end, there emerged the Department of Mathematics and Statistics, as we know it today. The year was 2004.

During the merger negotiations, the dean had promised an end to the hiring freeze, stipulating that we prioritize applied and interdisciplinary research. Our first new hire was Anotida Madzvamuse, who fit the bill nicely. Although he interacted very well with our group and collaborated with A. J., Georg, and Wenxian, Anotida unfortunately never found the broad administrative support he had been promised to grow his research at the interface between mathematics and biology. Disillusioned, he left us after only three years, to go back to England and pursue a career at the University of Sussex. Their gain, our loss.

Growing applied and interdisciplinary research at Auburn was, of course, what Georg, A. J., and I had been hired to do — and we obliged. As mentioned earlier, Georg had been affiliated with a multidisciplinary climate research program, back in Germany; he continued his work on energy-balance climate models at Auburn, collaborating with A. J. and myself and, in later years, with Ildefonso Díaz and his group in Madrid. At the same time, A. J. and I were doing MHD[19] and collaborated with engineers at Auburn and Tuscaloosa on liquid-metal flows and electromagnetic stirring. I spent part of a sabbatical at the Forschungszentrum Rossendorf in Dresden, interacting with a group of applied physicists, who were interested in our computational approach to MHD flow problems. Meanwhile, A. J. was collaborating with geophysicists at Auburn on porous media, poroelasticity, and thermochronology. While working in a somewhat less applied and interdisciplinary vein, Wenxian was quickly attaining international stature as an authority on the dynamics of almost periodic systems, lattice dynamical systems, and nonautonomous and random differential equations, frequently collaborating with Georg on applications to competition-diffusion systems and population dynamics.

As you can imagine, our work was extremely popular in the department — whenever it came to generating another progress report, vision statement, or long-term hiring and program development plan and selling the same to the dean. Less so after the sale. I still have every departmental vision statement we ever contributed to and had a lot of fun reading them again, after all these years, while writing this essay. We had been preaching the same old gospel ever since the good old days of FAT; alas, we were prophets in the desert. Lacking sustained administrative support, none of those visions ever materialized; Anotida's grievously truncated trajectory at Auburn makes for the most telling evidence.

But then we had a stroke of luck... In 2007, there was broad departmental consensus to fill two junior faculty positions in support of our nascent and very promising undergraduate program in actuarial mathematics, initiated by Jack Brown and Tom Pate. Candidates were expected to have significant actuarial credentials, but research interests in another field, compatible with existing departmental programs. We hired Dmitry Glotov and Maggie Han. By sheer serendipity (really?), both had research interests in applied math and PDEs, providing our group of four (all full

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[19]Magnetohydrodymamics.

professors at the time) with a much needed infusion of fresh blood. Both Dmitry and Maggie did very well, on the actuarial side and way beyond. Finally, the train was moving again.

Following Anotida's departure and the retirement of Jack Rogers, there was an obvious need to hire in numerical analysis. Through A. J., we had acquired Vitamin C with his former advisor, Max Gunzburger, a prolific producer of academic offspring. One of his offspring, Yanzhao Cao, was languishing at a minor university in Florida and gladly accepted our offer to join us as an associate professor.[20] In subsequent hiring cycles, following the retirements of Frank Uhlig and Steve Stuckwisch, Yanzhao was instrumental in bringing Junshan Lin to Auburn, and we recruited another numerical analyst connected to Max Gunzburger in Hans-Werner van Wyk. At the same time, and very unfortunately for us, A. J., long tired of Auburn, retired from Auburn, to escape our eternal bickering and find a more conducive environment at SMU[21] (not sure he did). Three years later, we landed another coup by hiring Thi-Thao-Phuong Hoang. Competition was tough, and I played a small part in tipping the scales. Which may well be the best thing I ever did for the department. Keep going, Phuong!

The next year, following a brief and utterly unsuccessful stint as department chair, yours truly retired, a bit prematurely and not exactly according to plan. So it goes. The moment was structured that way.[22] Georg stayed on to witness the hiring of Selim Sukhtaiev — for all I can tell yet another strong addition to the applied math group. I met one of Selim's advisors, Yuri Latushkin, many years ago at Georgia Tech, while he was visiting the Center for Dynamical Systems and Nonlinear Studies. Looks like we can't escape the shadow of Jack Hale... (Selim and Yuri, I wish that, by the time this essay appears in print, there will be peace in your country. Slava Ukraïni!)

Almost three years removed from Auburn, I don't really know any longer what's going on. Never met Selim, but we've emailed. He's organizing the applied math seminar now. And he told me that we just landed another coup, hiring Yimin Zhong, currently at Duke, and Yuming (Paul) Zhang, currently at UCSD[23], both of whom will join us in the fall. Us?

Georg retired effective August 2021. I don't know whether you realized it, Wenxian, but you are in charge now! You've been the queen all along, but the three musketeers who fought for you rode off into the sunset. You are safe though — Richelieu[24] and Milady[25] are gone too. Your realm is in great shape, your people is thriving, and the future is bright. Make sure the young ones know and remember how it all started. And, you young ones, mi raccomando[26]: don't let us down! Carry the torch forward and make sure Georg's legacy lives on. Tous pour un, un pour tous. All for one, and one for all.

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[20]If I recall correctly, Yanzhao had another offer from someplace in upstate New York; but he really dreaded the winters...

[21]Southern Methodist University.

[22]Kurt Vonnegut, Slaughterhouse Five.

[23]University of California San Diego.

[24]Armand Jean du Plessis, Duke of Richelieu, a.k.a. Cardinal Richelieu, a.k.a. the Red Eminence. Chief minister to Louis XIII of France. Known for his cunning and political gamesmanship.

[25]Milady Laurence de Winter, a.k.a. the Countess. Fictional character in the 1844 novel "The Three Musketeers" by Alexandre Dumas. Richelieu's agent and femme fatale.

[26]Italian, and untranslatable. Means something like "I beg you," but with a subtly threatening tinge ("you'd better...").