INTERNATIONAL REVIEW OF ATOMIC AND MOLECULAR PHYSICS (IRAMP)
Volume 1, No. 2, July-December 2010, pp. 169-179, © *International Science Press, ISSN: 2229-3159*

RESEARCH LETTER

STARK BROADENING OF HYDROGENIC SPECTRAL LINES BY LANGMUIR TURBULENCE IN MAGNETIC FUSION PLASMAS: DIAGNOSTIC POSSIBILITIES

EUGENE OKS

Department of Physics, 206 Allison Lab, Auburn University, Auburn, AL 36849 USA.

Abstract: We derive the dynamical Stark width and shift of hydrogenic spectral lines caused by Langmuir turbulence in magnetic fusion plasmas. We show that this additional broadening mechanism can dominate over the Stark broadening by the plasma microfield, theories of the latter broadening being also discussed. We also derive conditions necessary for Langmuir-wave-caused dips/depressions to occur in the profiles of the components of the Zeeman triplet. Based on this analysis, we propose methods for the spectroscopic diagnostics of Langmuir turbulence in magnetic fusion plasmas.

PACS: 52.55.-s, 52.35.Ra, 32.60.+i, 32.70.Jz.

1. INTRODUCTION

The magnetic fusion research community is interested to find out whether Langmuir turbulence develops in magnetic fusion and, if it does, to determine its para-meters. It is desirable to have spectroscopic diagnostics for this purpose, because it is "non-intrusive": it does not perturb parameters to be measured.

A number of spectroscopic methods for diagnosing Langmuir turbulence/oscillations in different kinds of plasmas has been developed and practically implemented by Oks and his collaborators, as presented in book [1]. All these methods related to situations where the radiator (e.g., a hydrogen or deuterium atom) is subjected to a quasistatic electric field — in addition to the oscillatory electric field of Langmuir turbulence and to the broad-band dynamic microfield due to plasma electrons. The quasistatic electric field was usually represented by the ion microfield (in the case where the latter was mostly quasistatic) and/or by a low-frequency electrostatic turbulence (e.g., by ionic sound).

In this situation, there occur the following two major effects of Langmuir turbulence on profiles of hydrogenic spectral lines. The first effect is an appearance of dips/depressions at distances from the unperturbed line position (in the frequency scale) that are proportional to the plasma electron frequency ω_p, the proportionality coefficients being rational numbers (expressed via the corresponding quantum numbers). Langmuir-wave-caused dips (hereafter, *L-dips*) in profiles of hydrogenic spectral lines were discovered experimentally in 1977 [2] and explained theoretically in papers [2 – 6]. This effect was observed and used for diagnostics in a large number of experiments conducted by various experi-mental groups at different plasma sources (see, e.g., book [1]). The latest experimental results (obtained in a laser-produced plasma) can be found in [7].

The second effect is an additional dynamical broa-dening [8, 9] (presented also in book [1]). In distinction to the first one, it was not widely used for diagnostics.

In all of the above experiments, magnetic fields did not play any substantial role. Therefore, for magnetic fusion plasmas, characterized by a strong magnetic field of several (up to 10) tesla, possible effects of Langmuir turbulence on hydrogenic lines should be analyzed afresh. Below we present such analysis and propose — on its basis — a method for the spectroscopic diagnostic of Langmuir turbulence in magnetic fusion plasmas. The primary focus will be the additional dynamical broa-dening — for reasons explained below.

2. THEORETICAL ANALYSIS AND DIAGNOSTIC POSSIBILITIES

2.1 Line Broadening by Langmuir Turbulence

In 1975 Oks and Sholin [8] derived analytically additional contributions to the width and shift of hydrogenic spectral lines due to Langmuir turbulence for the case where the separation ω_F between sublevels of the principal quantum number n is caused by a quasistatic electric field F (hereafter, the "electric" case, for brevity). In this case the separation between the Stark sublevels in the frequency scale is

$$\omega_F = 3n\hbar F/(2Z_r m_e e), \quad \ldots (1)$$

where Z_r is the nuclear charge of the radiator. The stochastic electric field of Langmuir turbulence was represented in [8] in the form

$$\mathbf{E}_p(t) = \sum_{j=1}^{J} \mathbf{E}_j(t)\cos\left[\omega_j t + \varphi_j(t)\right] \quad \ldots (2)$$

where the phase $\varphi_j(t)$ and the amplitude $\mathbf{E}_j(t)$ change their values with the every change of the state of a Poisson process characterized by the average change frequency γ_p. Between the changes, the quantities $\varphi_j(t)$ and the components $\mathbf{E}_j^{\sigma}(t)$ are constant taking random values characterized by a certain distribution. In particular, the phase φ_j has a uniform distribution in the interval (0, 2π) with the density $1/(2\pi)$. The stochastic function $\mathbf{E}_j(t)$ in (2) is the realization of a kangaroo-type uniform Markovian stationary stochastic process. A convenient characteristic is the root-mean-square average $E_0 = (<|\mathbf{E}_j(t)|^2>)^{1/2}$, which is called for brevity the average amplitude.

The main frequencies ω_j are all approximately equal to the plasma electron frequency

$$\omega_p = \left(4\pi e^2 N_e/m_e\right)^{1/2} \cong 5.641\times10^4 \left[N_e\left(\mathrm{cm}^{-3}\right)\right]^{1/2} \quad \ldots (3)$$

Here N_e is the electron density.

The frequency $\gamma_p < \omega_p$ is the largest of the characteristic frequencies of various nonlinear processes in the plasma — the processes such as, e.g. the generation of the Langmuir waves, the induced scattering of the Langmuir waves on the charged particles, the nonlinear decay into ionic sound and so on. The frequency γ_p is assumed to control the width of the power spectrum of Langmuir turbulence.

The additional contributions to the width and shift of hydrogenic spectral lines due to Langmuir turbulence, derived analytically by Oks and Sholin [8], depend on the separation between the Stark sublevels ω_F caused by a quasistatic electric field. However, for the conditions typical for magnetic fusion plasmas — in particular, in the tokamak divertor region — the ion microfield is not quasistatic for the most intense hydrogenic spectral lines. Therefore, at the absence of a low-frequency electrostatic turbulence, the separation between sublevels of the principal quantum number n is caused by a relatively strong magnetic field B (so that in this case these are Zeeman sublevels rather than the Stark sublevels):

$$\omega_B = eB/(2m_e c) \quad \ldots (4)$$

Thus, in this "magnetic" case, the Langmuir-wave-caused contributions to the diagonal elements $\Gamma_{\alpha\beta} = -Re\,\Phi_{\alpha\beta}$ and $D_{\alpha\beta} = -Re\,\Phi_{\alpha\beta}$ of the impact broadening operator Φ can be obtained from the corresponding Oks-Sholin's results by substituting ω_F by ω_B. Here α and β label sublevels of the upper (a) and lower (b) levels involved in the radiative transition. For brevity we call $\Gamma_{\alpha\beta}$ and $D_{\alpha\beta}$ the width and the shift, respectively. In this way, we obtain the following expressions for $\Gamma_{\alpha\beta}$ and $D_{\alpha\beta}$ (the corresponding expressions for nondiagonal elements of the operator Φ will be published elsewhere):

$$\Gamma_{\alpha\beta} = \Gamma_\alpha + \Gamma_\beta - d_{\alpha\alpha}d_{\beta\beta}E_0^2\gamma_p/[3\hbar^2(\gamma_p^2+\omega_p^2)],$$
$$D_{\alpha\beta} = D_\alpha + D_\beta, \quad \ldots (5)$$

where

$$\Gamma_\alpha = [E_0^2\gamma_p/(12\hbar^2)]\ \{2\mathbf{d}_{\alpha\alpha}{}^2/(\gamma_p^2+\omega_p^2) + (|\mathbf{d}_{\alpha,\alpha-1}|^2 + |\mathbf{d}_{\alpha,\alpha+1}|^2)\,[1/(\gamma_p^2+(\omega_B-\omega_p)^2) + 1/(\gamma_p^2+(\omega_B+\omega_p)^2)]\}, \quad \ldots (6)$$

$$D_\alpha = [E_0^2\gamma_p/(12\hbar^2)](|\mathbf{d}_{\alpha,\alpha-1}|^2 - |\mathbf{d}_{\alpha,\alpha+1}|^2)\,[(\omega_B-\omega_p)/(\gamma_p^2+(\omega_B-\omega_p)^2) + (\omega_B+\omega_p)/(\gamma_p^2+(\omega_B+\omega_p)^2)]. \quad \ldots (7)$$

Here the matrix elements of the dipole moment operator are

$$\mathbf{d}_{\alpha\alpha}^2 = [3ea_0 n_\alpha q_\alpha/(2Z_r)]^2,\ |\mathbf{d}_{\alpha,\alpha-1}|^2 - |\mathbf{d}_{\alpha,\alpha+1}|^2$$
$$= \mathbf{d}_{\alpha\alpha}^2/q_\alpha^2,\ |\mathbf{d}_{\alpha,\alpha-1}|^2 + |\mathbf{d}_{\alpha,\alpha+1}|^2)$$
$$= \mathbf{d}_{\alpha\alpha}^2\,(n^2-q^2-m^2-1)_\alpha/(2q_\alpha^2), \quad \ldots (8)$$

where a_0 is the Bohr radius; $q = n_1 - n_2$; n_1, n_2 and m are the parabolic quantum numbers. In Eqs (6) and (7), in the subscripts we used the notation $\alpha + 1$ and $\alpha - 1$ for the Zeeman sublevels of the energies $+\hbar\omega_B$ and $-\hbar\omega_B$, respectively (compared to the energy of the sublevel α). Formulas for Γ_β and D_β entering Eq. (5) can be obtained from Eqs (6) and (7) by substituting the superscript α by β.

Let us analyze the above results for the width—because it is practically more important than the shift. The expressions for the width demonstrate the following two characteristic features.

For relatively large magnetic fields, such that

$$\omega_B >> \omega_p, \qquad \text{... (9)}$$

the term containing the diagonal matrix element $\mathbf{d}^2_{\alpha\alpha}$ predominates, so that the other term can be neglected. The dominating term is the *adiabatic* contribution: it does not couple (by virtual transitions) different Zeeman sublevels—in distinction to the neglected term. Under the same condition (9), the nondiagonal matrix elements of the impact broadening operator become much smaller than the diagonal elements, so that the quantity $\Gamma_{\alpha\beta}$ from Eq. (5) becomes a "true width".

Figure 1 shows the ratio $R = \omega_B/\omega_p$ versus the magnetic field B for three different electron densities N_e. It is seen that even for $N_e = 10^{13}$ cm^{-3}, which is usually considered as the lowest electron density relevant to magnetic fusion plasmas, the fulfillment of the condition (9) requires magnetic fields greater than 10 tesla*/.

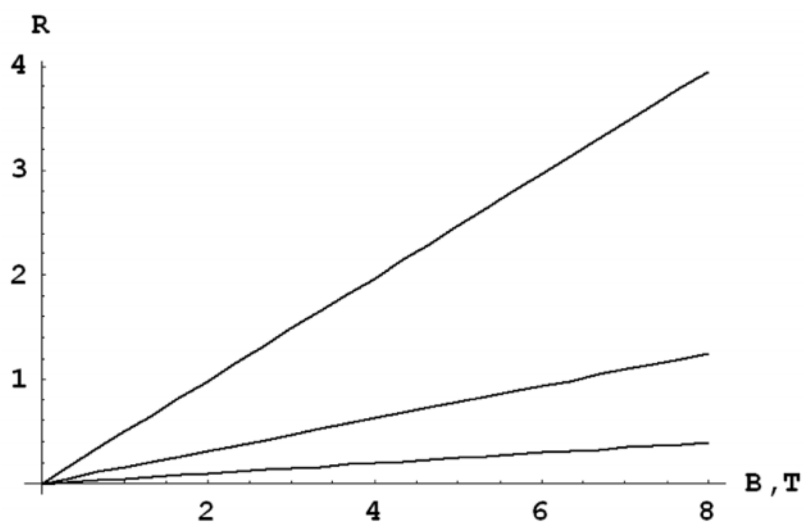

Fig. 1: The ratio of the frequencies $R = \omega_B/\omega_p$ versus the magnetic field B in tesla for three different electron densities N_e: 10^{13} cm^{-3} (the upper line), 10^{14} cm^{-3} (the middle line), 10^{15} cm^{-3} (the lower line)

*/ In principle, there might exist also another adiabatic effect of the stochastic electric field of Langmuir turbulence if $\gamma_p << \omega_p$: the formation of satellites separated by $\pm k\omega_p$ (in the frequency scale) from each component of the Zeeman triplet ($k = 1, 2, 3, \ldots$). For the case, where Langmuir turbulence develops anisotro-pically in such a way, that its electric field is linearly-polarized, the satellite intensities were calculated analytically by Lifshitz [10] (see also book [1]). However, the satellite intensities are relatively small. Even for the most intense satellite ($k = 1$), the ratio of its intensity I_s to the intensity of the corresponding component of the Zeeman triplet I_0 is

$$I_s/I_0 \sim (n^2T_e/U_{Hi})\,[E_0^2/(8\pi N_eT_e)].$$

Here U_{Hi} = 13.6 eV is the ionization potential hydro-gen/deuterium atoms, T_e is the electron temperature; the quantity $E_0^2/(8\pi N_eT_e)$, which is called the degree of turbulence, is the ratio of the energy density of the Langmuir turbulence to the thermal energy density of the plasma. The latter ratio is always much smaller than unity: usually it is in the range $10^{-2} - 10^{-4}$. Given that for spectroscopic experiments related to tokamak divertors, where the most intense hydrogenic lines are used (L_α, L_β, H_α, H_β) one has $n^2T_e/U_{Hi} \sim 1$, it is seen that indeed $I_s/I_0 \sim (10^{-2} - 10^{-4}) << 1$. Thus, these satellites do not seem to be useful for diagnostics of magnetic fusion plasmas unless highly-excited hydrogenic lines ($n >> 1$) are employed.

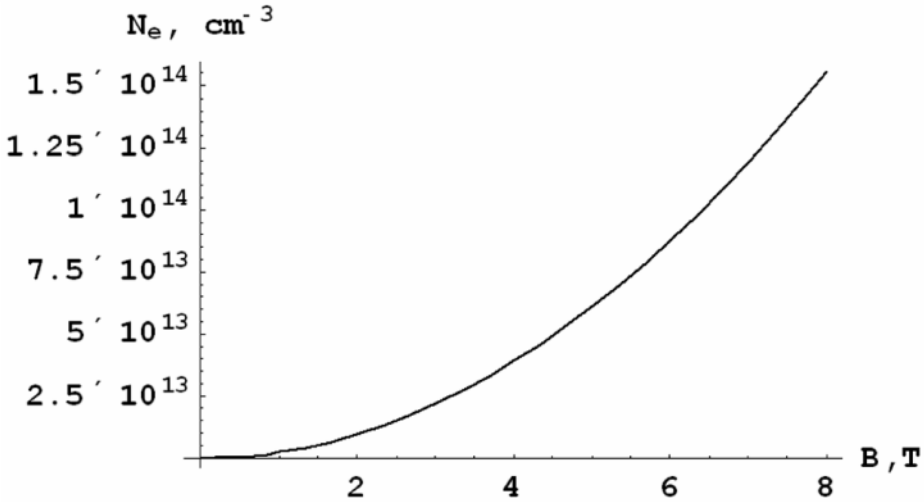

Fig. 2: The line (the geometric set of points) in the plane (B, N_e) corresponding to the resonance: $\omega_B = \omega_p$. Here B is the magnetic field in tesla, N_e is the electron density in cm^{-3}

The most interesting is another scenario, where

$$\omega_B = \omega_p. \qquad \text{... (10)}$$

This resonance can occur exactly or approximately for a number of pairs (B, N_e) typical for the conditions of tokamak divertors. Indeed, from Fig. 2, which shows the line (the geometric set of points) in the plane (B, N_e) corresponding to the resonance (10), it follows that the resonance takes place, e.g. for $B = 2\,T$ and $N_e = 10^{13}\,cm^{-3}$, or for $B = 5\,T$ and $N_e = 6 \times 10^{13}\,cm^{-3}$, or for $B = 8\,T$ and $N_e = 1.6 \times 10^{14}\,cm^{-3}$.

In the conditions close to the resonance (10), the Langmuir-wave-caused Stark width dramatically increases. Neglecting the non-resonance terms in Eqs (5), (6), it can be represented in the form:

$$\Gamma_{\alpha\beta} = (|\mathbf{d}_{\alpha,\alpha-1}|^2 + |\mathbf{d}_{\alpha,\alpha+1}|^2 + |\mathbf{d}_{\beta,\beta-1}|^2 + |\mathbf{d}_{\beta,\beta+1}|^2)\,E_0^2/(12\hbar^2\gamma_p) \qquad \text{... (11)}$$

We note that all terms in Eq. (11) correspond to the *nonadiabatic* contribution: they couple by virtual transitions different Zeeman sublevels.

Now let us compare $\Gamma_{\alpha\beta}$ from Eq. (11) to the width due to the competing Stark broadening mechanism. For the conditions typical for tokamak divertors, the latter is the dynamical Stark broadening by ions. In 1994 Derevianko and Oks [11] analyzed the dynamical Stark broadening of hydrogenic lines by ions in magnetized plasmas using an advanced analytical semiclassical theory: the Generalized Theory (GT). The GT originated from papers by Ispolatov and Oks [12] and by Oks, Derevianko, and Ispolatov [13]; it was significantly enhanced later on, as summarized in [14, 15].

The GT is based primarily on a *generalization* of the formalism of Dresses Atomic States (DAS) in plasmas. DAS is the formalism initially designed to describe the interaction of a *monochromatic* (or quasi-monochromatic) field – e.g., laser or maser radiation – with gases. Later it was applied for the interaction of a laser or maser radiation with plasmas [1]. The employment of DAS led to the enhancement of the accuracy of analytical calculations and to more robust codes.

The *generalization* of DAS in the GT is based on using atomic states dressed by a *broad-band* field of plasma electrons of ions [12-15]. Therefore generalized DAS is a more complicated concept tha usual DAS, where the dressing was due to a monochromatic field.

The GT allows analytically in the exact way (in all orders) for the component of the dynamic plasma microfield parallel to the additional static (electric or magnetic) field. Thus the GT made a significant advance compared to the simplest semiclassical theory of the dynamical Stark broadening (see, e.g. [16])—sometimes called the Standard Theory (ST) or the conventional theory—since the ST allowed for the same component of the dynamic plasma microfield only in the 2nd order of the Dyson perturbation expansion. In distinction to the ST, the GT is not divergent at small impact parameters. In paper by Touma et al. [17] it was shown analytically that for the overwhelming majority of hydrogenic lines the GT does not violate the unitarity of the *S*-matrix at any impact parameter and therefore does not have to separate collisions into "weak" and "strong" — in distinction to the ST. The latter has to separate collisions into "weak" and "strong" for all hydrogenic lines (to avoid the divergence) and defines the boundary between the "weak" and "strong" collisions only by the order of magnitude. Only for few hydrogenic lines (such as, e.g. for L_α, and to a lesser extent for L_β and H_α) the GT might violate the unitarity at small impact parameters (as discussed by Oks, Derevianko, and Ispolatov [13]) and could use the separation into "weak" and "strong" collisions for enhancing the accuracy.

In the GT, the dynamical Stark width due to ions consists of *adiabatic* contribution (proportional to the sum of the diagonal matrix elements $\mathbf{d}_{\alpha\alpha}{}^2 + \mathbf{d}_{\beta\beta}{}^2$) and *nonadiabatic* contribution (proportional to the sum of the nondiagonal matrix elements $|\mathbf{d}_{\alpha,\alpha-1}|^2 + |\mathbf{d}_{\alpha,\alpha+1}|^2 + |\mathbf{d}_{\beta,\beta-1}|^2 + |\mathbf{d}_{\beta,\beta+1}|^2$) – similar to Eqs. (5) – (7) for the Langmuir-wave-caused contributions to the Stark width.

The main result of the GT for magnetic fusion plasmas is the following. At values of the magnetic field B typical for magnetic fusion plasmas, practically the entire dynamical Stark width due to ions is due only to the *adiabatic* contribution. This is because, as the magnetic field B increases, causing the increase of the separation ω_B between the Zeeman sublevels of hydrogenic atoms, the nonadiabatic contribution to the dynamical Stark broadening by ions decreases – specifically, it decreases dramatically at magnetic fields typical for magnetic fusion plasmas. Some further details on the main result of the GT for magnetic fusion plasmas are presented in Appendix */.

The ratio of $\Gamma_{\alpha\beta}$ from Eq. (11) to the corresponding contribution $\gamma_{\alpha\beta}$ due to the dynamical broadening by ions calculated by the *GT* can be represented as the product of four dimensionless factors as follows:

$$\Gamma_{\alpha\beta}/\gamma_{\alpha\beta} \sim (m_e/M)^{1/2}\,[T_e r_D/e^2]\,(\omega_p/\gamma_p)\,[E_0^2/(8\pi N_e T_e)] \qquad \text{... (12)}$$

Here M is the reduced mass of the pair "radiator — perturbing ion" and r_D is the Debye radius. We note that the right side of Eq. (12) can be simplified to a more explicit scaling: $\Gamma_{\alpha\beta}/\gamma_{\alpha\beta}$ is proportional to $E_0^2 T_e^{1/2}/(N_e \gamma_p M^{1/2})$, if $T_i = T_e$. However, the representation of $\Gamma_{\alpha\beta}/\gamma_{\alpha\beta}$ as the product of the four dimensionless factors in (12) provides a better physical understanding and is more convenient for estimates. A practical formula for the product of the first two factors in the right side of Eq. (12).

$$(m_e/M)^{1/2}\,[T_e r_D/e^2] = 1.204 \times 10^8\,[T_e\,(\text{eV})]^{3/2}\,[N_e\,(\text{cm}^{-3})]^{-1/2}\,(M_p/M)^{1/2}, \qquad \text{... (13)}$$

where M_p is the proton mass.

Let us estimate the ratio $\Gamma_{\alpha\beta}/\gamma_{\alpha\beta}$ for a hydrogen plasma (so that $M = M_p/2$) of the electron density $N_e = 6 \times 10^{13}$ cm^{-3} and of the temperature $T_e = 5$ eV. From Eq. (13) we get: $(m_e/M)^{1/2}\,[T_e r_D/e^2] = 246 >> 1$, so that $\Gamma_{\alpha\beta}/\gamma_{\alpha\beta} \sim 2 \times 10^2 (\omega_p/\gamma_p)\,[E_0^2/(8\pi N_e T_e)]$. The ratio ω_p/γ_p is a large quantity — typically in the range of $(10^2 - 10^4)$, while the degree of turbulence $E_0^2/(8\pi N_e T_e)$ is a small quantity — typically in the range of $(10^{-4} - 10^{-2})$. So, we obtain the following range: $\Gamma_{\alpha\beta}/\gamma_{\alpha\beta} \sim (2 - 2 \times 10^4)$.

This example shows that for magnetic fusion plasmas, the contribution to the dynamical Stark width due to the Langmuir turbulence can dominate over the competing dynamical Stark broadening by ions, so that the half-width-at-half-maximum of a hydrogenic line will be

$$\delta\lambda_{1/2} = [\lambda_0^2/(2\pi c)]\,\Gamma_{\alpha\beta}, \qquad \text{... (14)}$$

*/ In 2009 Rosato et al. [18] revisited the subject studied by Derevianko and Oks [11] in 1994: the dynamical Stark broadening of hydrogen/deuterium lines by ions in magnetized plasmas. Paper [18] presented some analytical results and some simulations. The analytical results in [18] were based on the ST. In Appendix to the present paper it is shown that the analytical results from [18] yield a very dramatic inaccuracy – up to two orders of magnitude (!).

Rosato et al. [18] knew that the dynamical Stark broadening of hydrogen/deuterium lines by ions in magnetized plasmas had been already described by an advanced theory, such as the GT, by Derevianko and Oks [11]: in [18] there is a reference to paper [11]. Nevertheless, they decided to recycle the obsolete theory, such as the ST, in application to the same phenomena. Judging by this and by a later comment by Rosato [19], it seems that they are confused/misguided concerning this issue, which is why the Appendix at the end of the present paper should by useful to them.

where λ_0 is the unperturbed wavelength and $\Gamma_{\alpha\beta}$ is given by Eq. (11). Thus, it can be used for diagnostics of Langmuir turbulence. Specifically, from the experimentally measured Stark width of hydrogenic spectral lines in the conditions close to the resonance, it is possible to determine the quantity E_0^2/γ_p — as it is seen from Eq. (11).

2.2 L-dips in Line Profiles

Let us now briefly discuss L-dips. They were discovered experimentally and explained theoretically for dense plasmas, where one of the electric fields experienced by hydrogenic radiators is quasistatic — due to the ion microfield and/or a low-frequency electrostatic turbulence (see [1–7]). In this situation, the central point of the *L*-dip phenomenon was a resonant coupling between a quasistatic electric field **F** and an oscillatory electric field of the Langmuir wave. In the profile of the Stark component of the Lyman line originating from the sublevel *q*, the resonance could manifest, generally, as two dips (L^+-dip and L^--dip) located at the following distances $\Delta\lambda_\pm^{\,dip}$ from the unperturbed wavelength λ_0 of this Lyman line:

$$\Delta\lambda_\pm^{\,dip} = -[\lambda_0^2/(2\pi c)]\,\{q\omega_p + [2\omega_p^3/(27n^3 Z_r Z_p \omega_{at})]^{1/2}$$

$$[n^2(n^2 - 6q^2 - 1) + 12n^2q^2 \pm 6n^2 q]\} \qquad \text{... (15)}$$

Here Z_p is the charge of the perturbing ions, $\omega_{at} = m_e e^4/\hbar^3 \cong 4.14 \times 10^{16}\ s^{-1}$ is the atomic unit of frequency, *n* is the principal quantum numbers. The first, primary term in braces reflects the dipole interaction with the ion microfield. The second term in braces takes into account — via the quadrupole interaction — a spatial nonuniformity of the ion microfield. This second term is, generally speaking, a correction to the first term — except for the case of the central Stark component ($q = 0$), for which the first term vanishes. We note that in the profile of the central Stark component there could be only one *L*-dip (hereafter, "central *L*-dip") since the term $\pm\, 6n^2q$ vanishes. A formula for the *L*-dip positions in profiles of hydrogenic lines from other spectral series (Balmer, etc.) can be found in [1, 6].

It is important to emphasize the following. For a given electron density N_e, the value of the plasma electron frequency ω_p is fixed — in accordance to Eq. (3). The resonance occurs when the separation between the Stark sublevels of the principal quantum number *n* caused by the field *F*

$$\omega_F = 3n\hbar F/(2Z_r m_e e) \qquad \text{... (16)}$$

is equal to ω_p:

$$\omega_F = \omega_p. \qquad \text{... (17)}$$

The quasistatic electric field in plasmas has a broad distribution over the ensemble of radiators — regardless of whether this field represents the ion microfield or the low-frequency electrostatic turbulence. Therefore, if the ion microfield is mostly quasistatic or a low-frequency electrostatic turbulence has been developed in the plasma, then there would always be a fraction of radiators, for which the resonance condition (4) is satisfied.

However, for the conditions typical for magnetic fusion plasmas — in particular, in the tokamak divertor region — the ion microfield is not quasistatic. Therefore, at the absence of a low-frequency electrostatic turbulence, the separation $\omega_B = eB/(2m_e c)$ between sublevels of the principal quantum number *n* is caused by a relatively strong magnetic field *B* (so that in this case these are Zeeman sublevels rather than the Stark sublevels). Then the resonance condition is given by Eq. (10) instead of Eq. (17).

In this situation, the following two conditions are necessary for observing *L*-dips. First, the magnetic field to have a noticeable nonuniformity ΔB across the region, from which a particular hydrogenic line is emitted:

$$\Delta B/B > [\lambda_0/(2\pi c)]\; n^2\hbar E_0/(m_e e Z_r), \qquad \text{... (18)}$$

Second, the Langmuir electric field should not be too strong:

$$n^2\hbar E_0/(m_e e Z_r) < \gamma_p. \qquad \text{... (19)}$$

Under conditions (18), (19), it could be possible to observe an *L*-dip in the profile of each component of the Zeeman triplet. The halfwidth of the *L*-dip $\delta\lambda_{1/2}$ would be controlled only by one parameter of Langmuir turbulence — by the averaged amplitude E_0

$$\delta\lambda_{1/2} \cong (3/2)^{1/2}\,\lambda_0^2 n^2\hbar E_0/(8\pi m_e e c Z_r), \qquad \text{... (20)}$$

so that the other parameter, namely γ_p, would not enter the formula (20).

Therefore, the following diagnostic method can be proposed. If *L*-dips are observed in the profiles of the components of the Zeeman triplet, one can first deduce the averaged amplitude E_0 of the Langmuir electric field from the experimental halfwidth of the *L*-dip using Eq. (20). Then from the experimental halfwidth of the components of the Zeeman triplet, one can deduce the quantity E_0^2/γ_p via Eq. (14) and thus (since E_0 would be already determined) the characteristic frequency γ_p of the nonlinear process controlling the width of the power spectrum of Langmuir turbulence.

3. CONCLUSIONS

We derived the dynamical Stark width and shift of hydrogenic spectral lines caused by Langmuir turbulence in magnetic fusion plasmas. We showed that this additional broadening mechanism can dominate over the Stark broadening by the plasma microfield. We also derived conditions necessary for Langmuir-wave-caused dips/depressions to occur in the profiles of the components of the Zeeman triplet. Based on this analysis, we proposed methods for the spectroscopic diagnostics of Langmuir turbulence in magnetic fusion plasmas.

We obtained the results (5) – (7) for the case, where an additional static field experienced by radiators is magnetic, from the corresponding Oks-Sholin's results [8] derived for the case, where an additional static (quasistatic) field experienced by radiators was electric. We note that the latter results were derived under the assumption that Langmuir turbulence develops isotropically. Later Oks and Sholin [9] extended the analysis to the situation where the Langmuir turbulence is developed anisotropically. This led to the difference in the Langmuir-wave-caused Stark width (and shift) observed in two perpendicular linear polarizations. Details on the corresponding results for the magnetic case will be presented elsewhere. Here we only emphasize that the polarization analysis is an effective tool to find out whether Langmuir turbulence developed anisotro-pically and to determine experimentally its parameters, including also the degree of anisotropy.

ACKNOWLEDGEMENTS

The author would like to thank Roland Stamm and Joël Rosato for discussions.

APPENDIX A
BRIEF COMPARISON OF THEORIES OF STARK BROADENING OF HYDROGENIC LINES IN NON-TURBULENT MAGNETIC FUSION PLASMAS

A.1 The Primary, Adiabatic Contribution to the Linewidth

At magnetic fields typical for magnetic fusion plasmas, the nonadiabatic contribution to the dynamical Stark broadening by ions decreases dramatically [11, 14] – as discussed also in Sect. A.2 below. So, the main result of the GT for magnetic fusion plasmas is that practically the entire dynamical Stark width due to ions is due only to the adiabatic contribution.

In accordance to [11-15], the adiabatic contribution to the dynamical Stark width due to ions γ_{ad} has the following form, which is the *exact*, *nonperturbative* analytical result:

$$\gamma_{ad} = 18(\hbar m_e)^2 (X_{\alpha\beta}/Z_r)^2 Z_i^2 N_i(2\pi M / T_i)^{1/2} I(R_i),$$
$$X_{\alpha\beta} = |nq_\alpha - n_\beta q_\beta|. \quad \text{… (A.1)}$$

Here Z_i, N_i, and T_i are the charge, the density, and the temperature of the plasma ions, respectively; M is the reduced mass of the pair "radiator – perturbing ion"; the function $I(R_i)$ is defined as follows:

$$I(R_i) = \{R_i^2[3 - \cos(1/R_i)] + (R_i - 2R_i^3)\sin(1/R_i) - ci(1/R_i)\}/6, \quad \text{… (A.2)}$$

In Eq. (A.2), $ci(1/R_i)$ is the cosine integral function, the quantity R_i being

$$R_i = r_D/r_{Wa}, \quad \text{… (A.3)}$$

where

$$r_D = [T_e/(4\pi e^2 N_e)]^{1/2} = 743.40\,[T_e(eV)/N_e(\text{cm}^{-3})]^{1/2}, \text{cm} \quad \text{… (A.4)}$$

is the Debye radius and

$$r_{Wa} = 3X_{\alpha\beta}\hbar / (Z_r m_e v_i) = 3.5486\times10^{-6}\,(X_{\alpha\beta}/Z_r)(M/M_p)^{1/2}/\,[T_i(eV)]^{1/2}, \text{cm} \quad \text{… (A.5)}$$

is the adiabatic Weisskopf radius (M_p is the proton mass). The quantity r_{Wa} naturally arises in the GT with the exact coefficient given in (A.5) – in distinction to the Weisskopf radius of the ST defined only by the order of magnitude. The practical part of formula (A.5) was obtained using the fact that the average over ion velocities is performed with the effective statistical weight factor $W_M(v_i)/v_i$, where $W_M(v_i)$ is the Maxwell distribution, and that $W_M(v_i)/v_i$ has the maximum at $v_i = (T_i/M)^{1/2}$.

For comparing the adiabatic Stark widths of the GT and of the ST, it is convenient to introduce the adiabatic broadening cross-section $\sigma_a(v_i)$ related to the adiabatic width γ_a as follows:

$$\gamma_a = N_i \int_0^\infty dv_i W(v_i) v_i \sigma_a(v_i), \qquad \text{… (A.6)}$$

where $W(v_i)$ is the velocity distribution. In the GT, the adiabatic broadening cross-section $\sigma_{a\mathrm{GT}}(v_i)$ is

$$\sigma_{a\mathrm{GT}}(v_i) = 2\pi\, [r_{Wa}(v_i)]^2\, I[R_i(v_i)], \qquad \text{… (A.7)}$$

where $I[R_i(v_i)]$ is given by (A.2) and $r_{Wa}(v_i)$ is given by the first equality in (A.5). This is the *exact* analytical result equivalent to the summation of *all orders* of the Dyson perturbation expansion.

In paper by Rosato et al [18], based on the *second order* of the Dyson perturbation expansion of the ST, the adiabatic broadening cross-section $\sigma_{a\mathrm{Ros}}(v_i)$ is

$$\sigma_{a\mathrm{Ros}}(v_i) = \pi[r_{\mathrm{str}}(v_i)]^2 + 2\pi[r_{W,\mathrm{Ros}}(v_i)]^2 \ln[r_D/r_{\mathrm{str}}(v_i)]$$
$$\approx 2\pi[r_{W,\mathrm{Ros}}(v_i)]^2\{1/2 + \ln[r_D/r_{W,\mathrm{Ros}}(v_i)], \qquad \text{… (A.8)}$$

where $r_{\mathrm{str}}(v_i)$ is a so-called "strong collision radius" (i.e., the boundary between weak and strong collisions); $r_{\mathrm{str}}(v_i) \approx r_{W,\mathrm{Ros}}(v_i)$ for the adiabatic contribution. The Weisskopf radius in the ST is defined only by the order of magnitude (which is one of the major sources of the inaccuracy of the ST): it is $\sim n_\alpha^2 \hbar / (Z_r m_e v_i)$. Rosato et al [18] arbitrarily chose the following numerical coefficient in the Weisskopf radius of the ST:

$$r_{W,\mathrm{Ros}}(v_i) = (2/3)^{1/2}\, n_\alpha^2 \hbar / (m_e v_i). \qquad \text{… (A.9)}$$

We note that they set $Z_r = 1$ because their work was limited to hydrogen /deuterium spectral lines. Therefore, in the comparison below we also set $Z_r = 1$.

We denote the ratio of the adiabatic broadening cross-sections as follows:

$$\kappa = \sigma_{a\mathrm{Ros}} / \sigma_{a\mathrm{GT}}. \qquad \text{… (A.10)}$$

Below we provide examples of the values of the ratio κ for several hydrogen lines in a hydrogen plasma (so that $M = M_p/2$) at the conditions typical for tokamak divertors. The components of a particular line are identified by the parabolic quantum numbers:

$(n_1 n_2 m)_\alpha - (n_1 n_2 m)_\beta$; we also indicate the polarization of the component (π or σ). The ratio κ is calculated at $T = 4$ eV and $N_e = (1 - 3)\times 10^{13}$ cm^{-3}.

For the Paschen-alpha line, for the component (102) – (101), which is one of the two most intense σ-components: $\kappa = 80$.

For the Balmer-gamma line, for the intense π-component (220) – (010): $\kappa = 50$.

For the Balmer-alpha line, for the component (101) – (100), which is one of the two most intense σ-components: $\kappa = 30$.

The above shows that *Rosato et al [18] overestimated the primary, adiabatic contribution to the dynamical Stark broadening by ions by up to two orders of magnitude* (!).

A.2 The Secondary, Nonadiabatic Contribution to the Linewidth

The nonadiabatic contribution to the dynamical Stark width due to ions γ_{na}, calculated for magnetic fusion plasmas using the GT, has a more complicated form than Eq. (A.1), as can be seen from Eqs. (5) – (8) of [11] or Eqs. (4.4.5) – (4.4.6) of book [14]. It is controlled by the integral of a so-called width function $A\_(\chi, Y, Z)$ over scaled (dimensionless) impact parameters Z:

$$a\_(\chi, Y, Z_D) = \int_0^{Z_D} dZ A\_(\chi, Y, Z)/Z. \qquad \text{… (A.11)}$$

The scaled impact parameter Z is defined as

$$Z(\rho) = \rho/\rho_B = 2m_e c v \rho / (eB). \qquad \text{… (A.12)}$$

The upper limit of the integration in (A.11) is $Z_D = Z(r_D)$, where r_D is the Debye radius. Typically $r_D > r_{Wa}$, which is assumed in (A.11).

Compared to the ST, there are two new parameters that enter the width function. The first one χ stands for

$$\chi = (n_\alpha q_\alpha - n_\beta q_\beta)/n_\alpha. \qquad \text{… (A.13)}$$

The second new parameter Y is physically the most important: it is a coupling parameter defined as

$$Y = 3 n_v Z_i \hbar eB / (2 m_e^2 c v_i^2)$$

$= 0.31885\, n_v Z_i (M/M_p) B(T)\, /T_i(eV), \quad v = \alpha \text{ or } \beta,$ … (A.14)

where Z_i is the charge of the plasma ions.

For example, for the D_α or L_β line ($n_v = 3$) from a deuterium plasma ($M/M_p = 1$, $Z_i = 1$), Eq. (A.14) yields: $Y = 0.95655\, B(T) / T_i(eV)$. For the typical parameters of tokamak divertors, the ratio $B(T) /T_i(eV)$ is greater or of the order of unity, so that Y is also greater or of the order of unity. At these values of the coupling parameter, first, the ST becomes quite inaccurate, and second (but most importantly), there occurs a dramatic decrease of the nonadiabatic contribution to the dynamical Stark width due to ions. Thus, the total contribution to the dynamical Stark width due to ions can be well represented by the adiabatic contribution given by Eq. (A.1).

The finding, that the nonadiabatic contribution significantly decreases with the increase of the magnetic field, was quite clear already in 1994: from the results of Derevianko and Oks [11] (where Eqs (A.11)-(A.13) were first presented) complemented by the results of Ispolatov and Oks [12] (where it was shown that the function a_(χ, Y, Z_D), controlling the nonadiabatic contribution, significantly decreases with the increase of the coupling parameter Y). Therefore, the claim by Rosato et al that they were the first to "discover" this effect in their paper [18] published in 2009 is without merit.

Finally, let us discuss the relation between the unitarity of the *S*-matrix and the nonadibatic contribution calculated by the GT or by the ST. In both theories, the nonadibatic contribution is calculated via $\{1 - S_{na}\}_{ang}$, which is the angular average of the nonadiabatic part of the *S*-matrix. It is calculated up to the second order of the Dyson perturbation expansion, but using the different basis: the basis of the dressed atomic states in the GT as opposed to the usual atomic basis in the ST. At small impact parameters, the ST would violate the unitarity of the *S*-matrix. To avoid the violation, the ST has to separate collisions into "weak" and "strong", the boundary between them being defined from the condition

$$\left|\{1 - S_{na}\}_{ang}\right| = C, \qquad 0 \le C \le 2. \quad \text{… (A.15)}$$

The uncertainty in the choice of the constant C in (A.15) is yet another major source of inaccuracy of the ST.

Touma et al [17] showed analytically that for the overwhelming majority of hydrogenic spectral lines, the nonadiabatic contribution calculated by the GT does not violate the unitarity of the *S*-matrix – in distinction to the ST. Therefore, for the overwhelming majority of hydrogenic spectral lines the lower limit of the integration in (A.11) can remain to be zero.

As an illustration of this important distinction between the GT and ST, we present Fig. A.1. For the most intense π-component (400) – (100) of the Balmer-gamma line, Fig. A.1 shows the dependence of the integrand $A\_/Z$ in (A.11) versus the scaled impact parameter Z: by the GT (solid curve) and by the ST (dashed curve). The solid curve is calculated by the GT for the coupling parameter $Y = 0.85$, which corresponds, e.g., to $B = 4$ *T* and $T = 1.5$ eV, or $B = 6$ *T* and $T = 2.25$ eV, or $B = 8$ *T* and $T = 3$ eV. Two possible unitarity restrictions are presented by straight lines. The solid straight line corresponds to the choice $C = 1$ in (A.15), the dashed straight line corresponds to the choice $C = 2$ in (A.15).

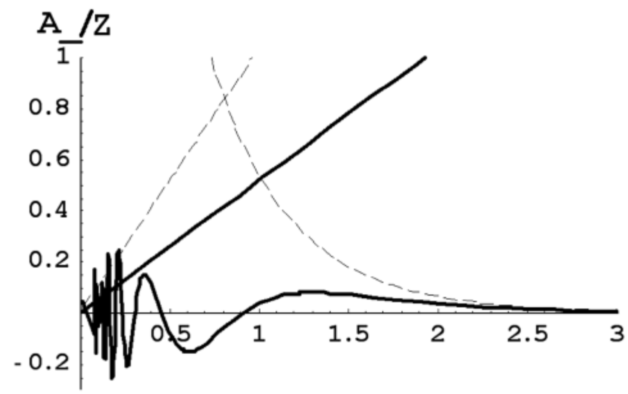

Fig. A.1: Dependence of the integrand $A\_/Z$ in (A.11) versus the scaled impact parameter Z: by the GT (solid curve) and by the ST (dashed curve). The solid curve is calculated by the GT for the coupling parameter $Y = 0.85$, which corresponds, e.g., to $B = 4$ T and $T = 1.5$ eV, or $B = 6$ T and $T = 2.25$ eV, or $B = 8$ T and $T = 3$ eV. Two possible unitarity restrictions are presented by straight lines. The solid straight line corresponds to the choice $C = 1$ in (A.15), the dashed straight line corresponds to the choice $C = 2$ in (A.15). The entire illustration is for the most intense π-component (400) – (100) of the Balmer-gamma line.

Figure A.1 clearly demonstrates the following:

1. The ST violates the unitarity of the *S*-matrix and has to separate collisions into weak and strong at the value of Z somewhere between 0.8 and 1.
2. The GT does not need to engage the unitarity cutoff: the integrand $A\_/Z$ strongly oscillates at small Z and thus practically "kills" the contribution from the small impact parameters to the integral.

3. Even after engaging the unitarity cutoff, the ST significantly overestimates the nonadiabatic contribution – by several times (in addition to dramatically overestimating the adiabatic contribution by up to two orders of magnitude).

We note that for the Lyman-alpha line, *as an exception*, the GT might need to engage the unitarity restriction and therefore separate collisions into weak and strong. Figure A.2 presents the plot for the same conditions as in Fig. A.1, but for the σ-components of the Lyman-alpha line: (001) – (000), (00-1) – (000).

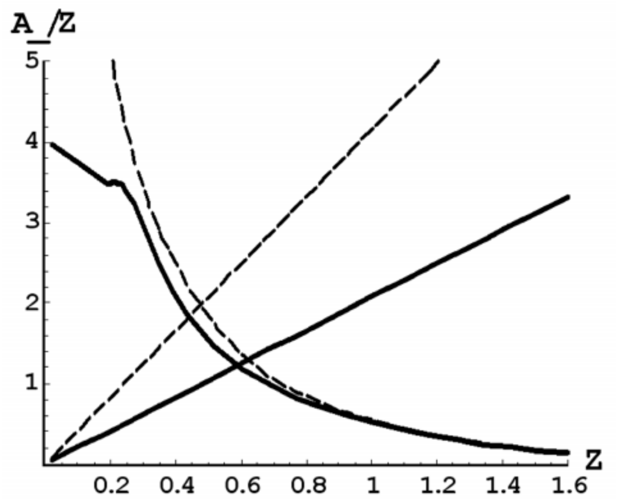

Fig. A.2: The same as in Fig. A.1, but for the σ-components of the Lyman-alpha line: (001) – (000), (00-1) – (000).

Figure A.2 shows that for the chosen plasma conditions, both the ST and the GT need engaging the unitarity cutoff. It shows also that, after engaging the unitarity cutoff for both theories, the ST still overestimates the nonadiabatic contribution to the broadening, though only slightly.

It should be emphasized that the Lyman-alpha line has zero or little practical importance for diagnostics of magnetic fusion plasmas because additional broadening mechanisms (opacity broadening and/or Doppler broadening) would usually dominate over the Stark broadening of this line. Therefore, the fact that for the Lyman-alpha line, *as an exception*, the GT might need engaging the unitarity cutoff (just like the ST) for describing the secondary, nonadiabatic contribution to the broadening, has zero or little practical importance.

The *summary* of the above results on the Stark broadening of hydrogenic lines in non-turbulent magnetic fusion plasmas is the following:

1. The analytical results by Rosato et al [18], which are based on the ST, are extremely inaccurate: they *dramatically overestimate the primary, adiabatic contribution* to the broadening – *by up to two orders of magnitude* (!). Therefore, the analytical results by Rosato et al [18] are practically useless for diagnostics of magnetic fusion plasmas. At the same time, the corresponding results by the GT are *exact* (within the semiclassical approach used by both theories). They are equivalent to the summation of *all orders* of the Dyson perturbation expansion, while the ST results are limited to the second order of the Dyson perturbation expansion.

2. The *nonadiabatic contribution* to the broadening is of a *secondary importance* because it is much smaller than the adiabatic contribution for the conditions of magnetic fusion plasmas. But even with respect to this secondary contribution, Rosato et al [18] significantly overestimate it – by several times – for the overwhelming majority of hydrogen/deuterium lines. In distinction, the GT describes the nonadiabatic contribution much more accurately because it does not need engaging the unitarity cutoff for the overwhelming majority of hydrogenic lines.

3. The finding, that the nonadiabatic contribution significantly decreases with the increase of the magnetic field, was quite clear already in 1994: from the results of Derevianko and Oks [11] complemented by the results of Ispolatov and Oks [12]. Therefore, the claim by Rosato et al that they were the first to "discover" this effect in their paper [18] published in 2009 is without merit.

4. For the Lyman-alpha line, *as an exception*, the GT might need to engage the unitarity cutoff while describing the secondary, nonadiabatic contribution to the broadening of hydrogenic lines – as first noted in Oks-Derevianko-Ispolatov's paper [13] in 1995. However, the Lyman-alpha line has zero or little practical importance for diagnostics of magnetic fusion plasmas because additional broadening mechanisms (opacity broadening and/or Doppler broadening) would usually dominate over the Stark broadening of this line. Therefore, the fact that for the Lyman-alpha line, *as an exception*, the GT might need engaging the unitarity cutoff (just like the ST) for describing the secondary, nonadiabatic contribution to the broadening, has zero or little practical importance.

5. In view of the above, the attempts by Rosato and his coworkers [19] to claim that the ST is just as accurate or even better than the GT for describing the Stark broadening of hydrogenic lines in non-turbulent magnetic fusion plasmas are futile.

REFERENCES

[1] E. Oks, "Plasma Spectroscopy: The Influence of Microwave and Laser Fields", *Springer Series on Atoms and Plasmas*, **9**, Springer: New York, (1995).

[2] A.I. Zhuzhunashvili and E. Oks, *Sov. Phys. JETP*, **46**, (1977), 2142.

[3] E. Oks and V.A. Rantsev-Kartinov, *Sov. Phys. JETP*, **52**, (1980), 50.

[4] V.P. Gavrilenko and E. Oks, *Sov. Phys. JETP*, **53**, (1981), 1122.

[5] V.P. Gavrilenko and E. Oks, *Sov. Phys. Plasma Phys.*, **13**, (1987), 22.

[6] E. Oks, St. Böddeker, and H.-J. Kunze, *Phys. Rev. A*, **44**, (1991), 8338.

[7] O. Renner, E. Dalimier, E. Oks, F. Krasniqi, E. Dufour, R. Schott, and E. Förster, *J. Quant. Spectr. Rad. Transfer*, **99**, (2006), 439.

[8] E. Oks and G.V. Sholin, *Sov. Phys. JETP*, **41**, (1975), 482.

[9] E. Oks and G.V. Sholin, *Opt. Spectrosc.*, **42**, (1977), 434.

[10] E.V. Lifshitz, *Sov. Phys. JETP*, **26**, (1968), 570.

[11] A. Derevianko and E. Oks, *Phys. Rev. Lett.*, **73**, (1994), 2079.

[12] Ya. Ispolatov and E. Oks, *J. Quant. Spectr. Rad. Transfer*, **51**, (1994), 129.

[13] E. Oks, A. Derevianko, and Ya. Ispolatov, *J. Quant. Spectr. Rad. Transfer*, **54**, (1995), 307.

[14] E. Oks, "Stark Broadening of Hydrogen and Hydrogenlike Spectral Lines in Plasmas: The Physical Insight", *Alpha Science International*, Oxford, U.K., 2006.

[15] E. Oks, "Spectral Line Shapes", **18**, *AIP Conf. Proc.*, 874, Auburn, Alabama, USA, 2006, pp. 19.

[16] H.R. Griem, *Spectral Line Broadening by Plasmas*, Academic, New York, 1974.

[17] J. Touma, E. Oks, S. Alexiou, and A. Derevianko, *J. Quant. Spectr. Rad. Transfer*, **65**, (2000), 543.

[18] J. Rosato, Y. Marandet, H. Capes, S. Ferri, C. Mosse, L. Godbert-Mouret, M. Koubiti, and R. Stamm, *Phys. Rev.*, **E79**, (2009), 046408.

[19] J. Rosato, *Private Communication*.