

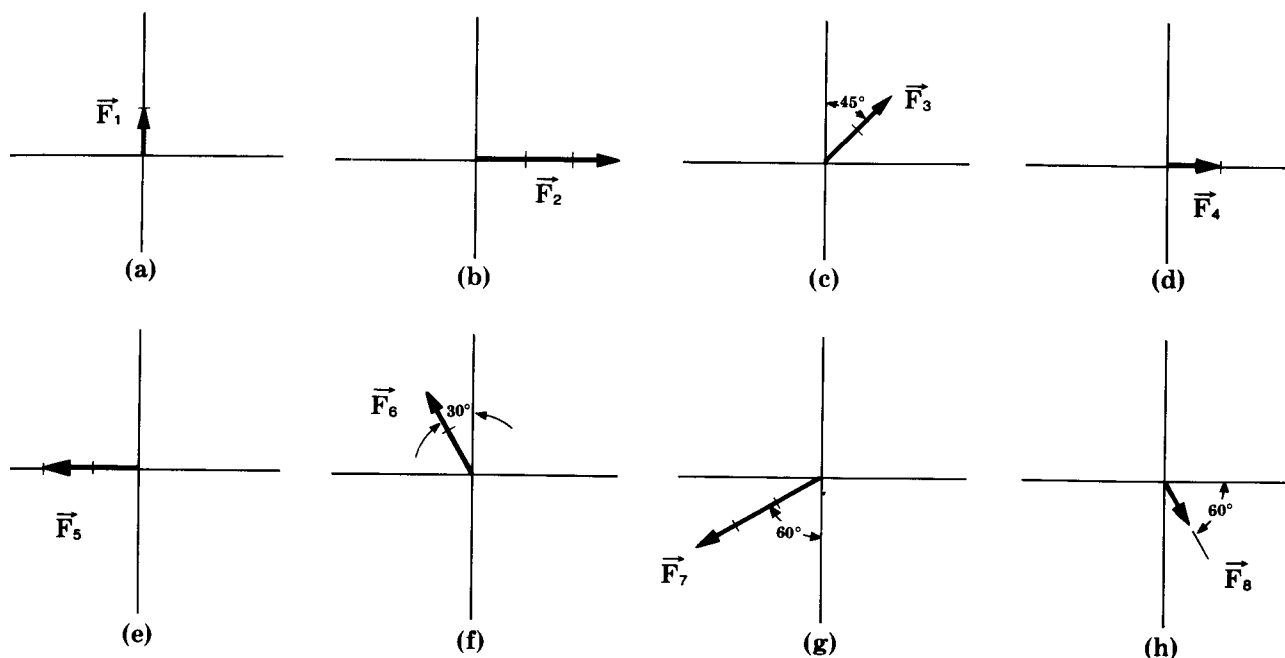
Vector Addition

Object: To study graphical and analytical methods of adding several vectors and to perform an experimental check on graphical and analytical solutions.

Apparatus: Force table with four attached pulleys, ring with four strings, four weight hangers, weights, ruler, protractor, and drawing compass.

Physical quantities may be classed as either **scalars** or **vectors**. A **scalar** is a physical quantity that has magnitude only and is represented by a number and a unit. For example, mass, time, and speed are scalars. We say that an object has a mass of 0.10 kg, the time elapsed was 10.0 seconds, or the speed limit is 55 miles/hour. Note that only a magnitude and unit have been given to specify these scalar physical quantities. A **vector** is a physical quantity that has magnitude and direction and is represented by an arrow. The length of the arrow (using some convenient scale) represents the magnitude and the orientation of the arrow represents the direction of the vector. For example, displacement, force, and velocity are vectors. We say a displacement of 10 miles North, a force of 10 pounds to the right, and a velocity of 30 meters/sec directed 45° North of East. Note that not only a magnitude but also a direction has been given to specify these vector physical quantities.

We shall conduct our study of vector addition with the vector quantity force. This choice is one of convenience since forces may be easily produced in the laboratory in order to experimentally check our graphical and analytical



- (a) A vector representing a force of 0.10 kg.wts. directed North
- (b) A vector representing a force of 0.30 kg.wts. directed East
- (c) A vector representing a force of 0.20 kg.wts. directed 45° East of North
- (d) A vector representing a force of 0.10 kg.wts. directed East
- (e) A vector representing a force of 0.20 kg.wts. directed West
- (f) A vector representing a force of 0.20 kg.wts. directed 30° West of North
- (g) A vector representing a force of 0.30 kg.wts. directed 60° West of South
- (h) A vector representing a force of 0.10 kg.wts. directed 60° South of East

Figure 1

solutions. The force that a 0.10 kg mass hanging on a string can exert on that string (assuming no acceleration) is (0.10 kg) (9.80 m/sec²), however again for convenience let's agree to call this a 0.10 kg.wt. force. That is the force exerted by a hanging mass of 0.10 kg, 0.20 kg, and 0.25 kg is 0.10 kg.wt., 0.20 kg.wt., and 0.25 kg.wt., respectively. If we use this method, we will not have to multiply all of our masses by 9.80 m/sec², yet the units name indicates that we know that the mass must be multiplied by this factor in order to obtain the force expressed in proper units.

Shown in **Figure 1** are several vectors representing forces. The scale used is 1 cm represents a magnitude of 0.10 kg.wts.

At this point it is important to realize that a vector may be slid around as long as we don't change its magnitude (i.e., stretch or compress it) or change its direction (i.e., reorient it).

When two or more vectors are added together their sum is called the **resultant vector**. The resultant vector can replace all of the other vectors with the same result. Shown in **Figure 2** is the sum or resultant of several of the vectors shown in **Figure 1**.

Notice that the vectors have been slid around (but not stretched, compressed, or reoriented) so that the arrows are placed tail to head and the resultant is drawn from the tail of the first vector to the head of the last vector. It should also be noted that the sum or resultant of the vectors is independent of the order of addition of the vectors.

The problem of vector addition becomes a little more complicated if the vectors to be added are not parallel or anti-parallel. One graphical method of adding such vectors is the **parallelogram method**. Shown in **Figure 3** is the graphical parallelogram method of adding vectors.

Notice that the two vectors are drawn with the same origin, the parallelogram is completed, and the resultant or sum of the two vectors is the diagonal of the parallelogram.

In order to add three or more vectors by the parallelogram method just repeat this procedure using the diagonal from the first parallelogram and the third vector, then the new diagonal and the fourth vector, etc. This process is shown in **Figure 4**.

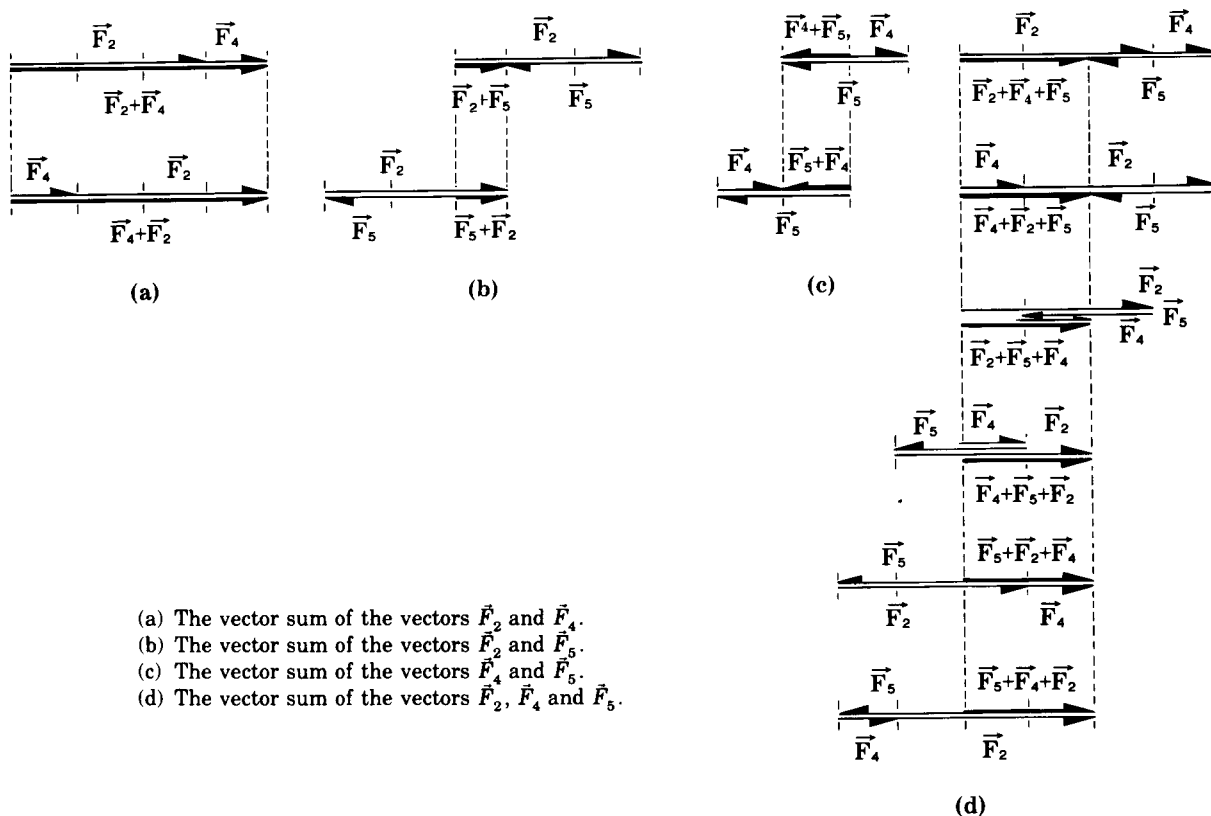
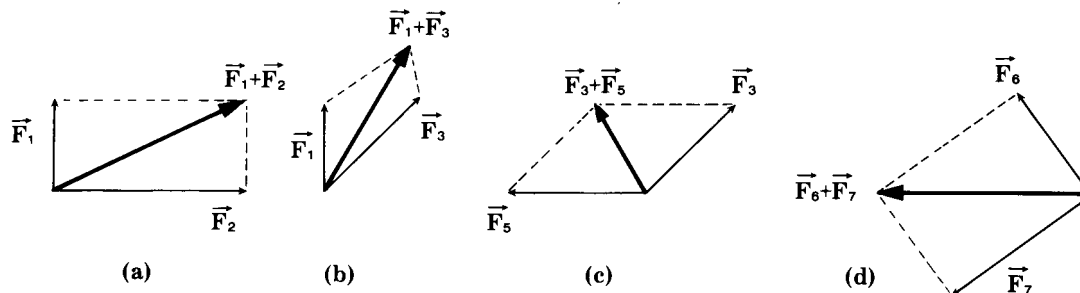


Figure 2

Once again it should be noted that the resultant or vector sum of the three vectors and \vec{F}_1 , \vec{F}_2 , and \vec{F}_3 , is independent of the order in which they are added.

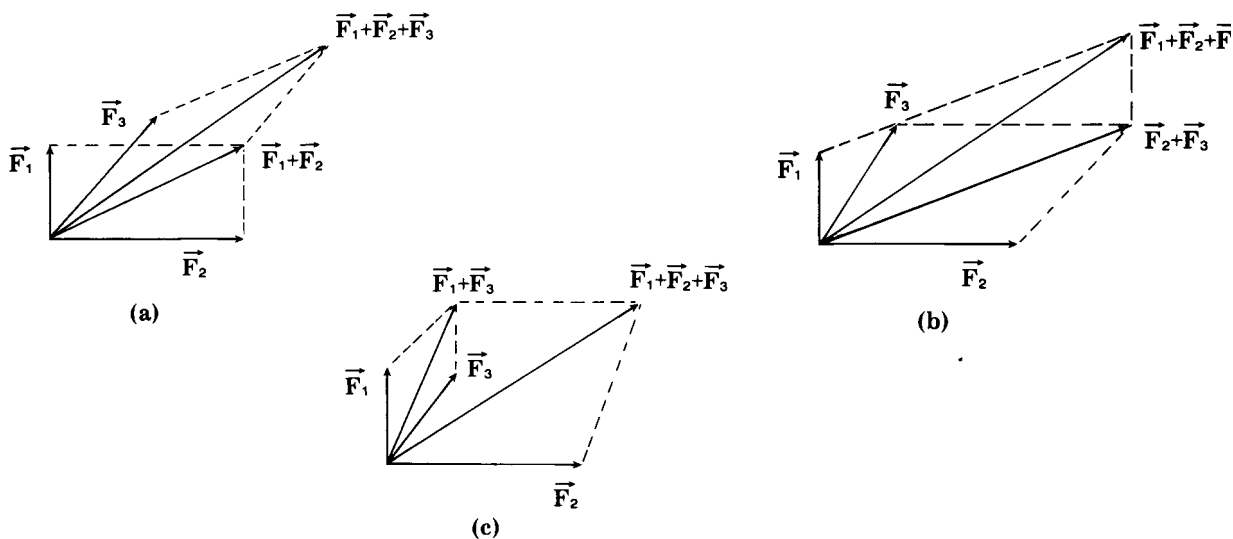
Alternately, if four or more vectors are to be added by the parallelogram method, they can be added two at a time using parallelograms and then these resultants can be added two at a time using parallelograms.

By now you have probably concluded that the parallelogram method is a somewhat tedious means of adding two or more vectors. If as shown in **Figure 5** instead of adding the vectors by the parallelogram method we slide the second vector over and attach its tail to the head of the first vector and then draw the resultant vector from the tail of the first vector to the head of the last vector we obtain the same resultant.



- (a) The vector sum of the vector \vec{F}_1 and \vec{F}_2 by the graphical parallelogram method.
- (b) The vector sum of the vector \vec{F}_1 and \vec{F}_3 by the graphical parallelogram method.
- (c) The vector sum of the vector \vec{F}_3 and \vec{F}_5 by the graphical parallelogram method.
- (d) The vector sum of the vector \vec{F}_6 and \vec{F}_7 by the graphical parallelogram method.

Figure 3

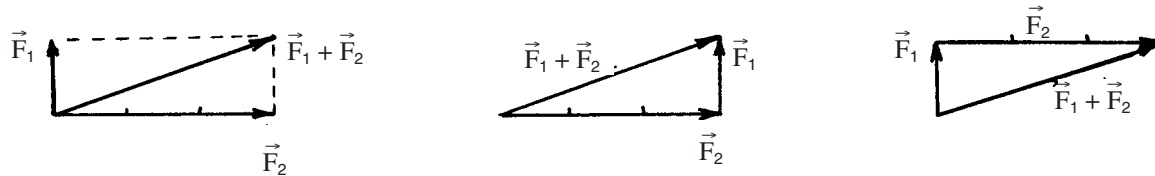


- (a) \vec{F}_1 and \vec{F}_2 are added by the parallelogram method and their resultant diagonal is then added to the third vector \vec{F}_3 by the parallelogram method.
- (b) \vec{F}_2 and \vec{F}_3 are added by the parallelogram method and their resultant diagonal is then added to the third vector \vec{F}_1 by the parallelogram method.
- (c) \vec{F}_1 and \vec{F}_3 are added by the parallelogram method and their resultant diagonal is then added to the third vector \vec{F}_2 by the parallelogram method.

Figure 4

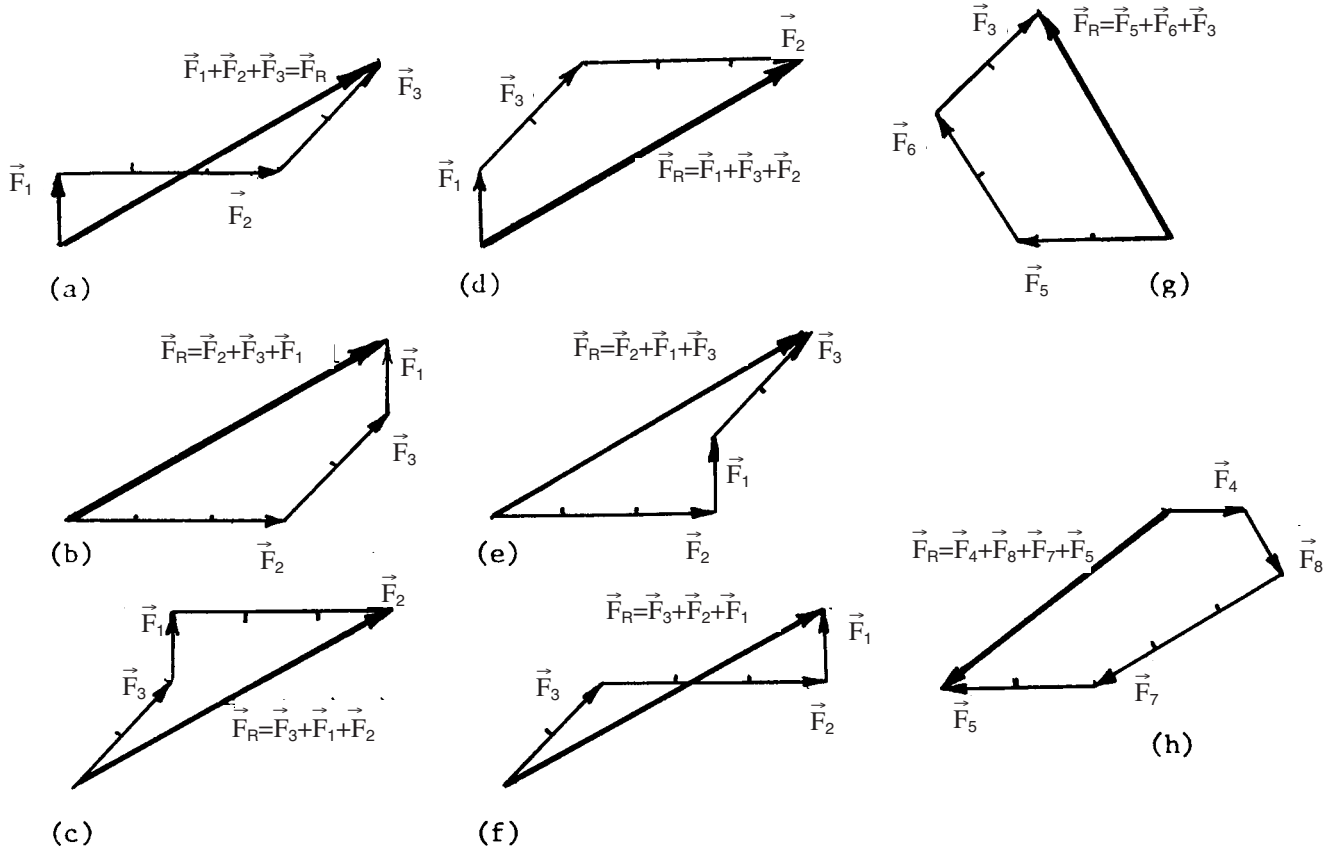
What we have done is form a closed, many-sided figure, hence this method of adding vectors is called the **closed polygon method**. Shown in **Figure 6** is the sum of several vectors by the closed polygon method. Again, notice that the order of addition does not effect the resultant or vector sum.

Another method of graphically adding vectors is the **component method**. What one does here is graphically resolve all of the vectors into their x and y components, graphically add these components, and then use the sum of the components as the x and y components of the resultant vector. This method is shown in **Figure 7**.



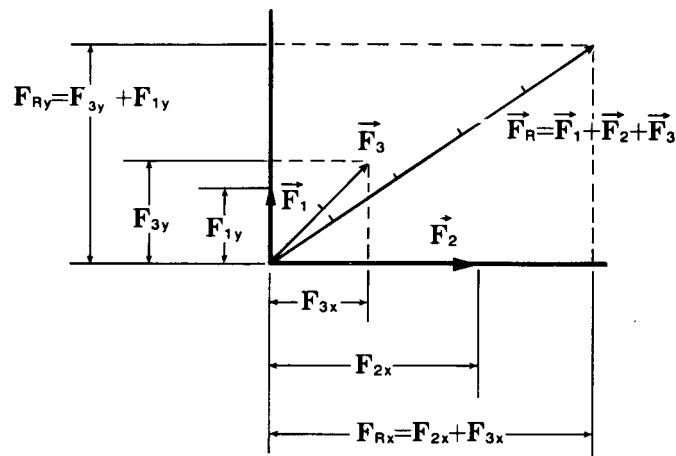
(a) The vector sum of the vectors \vec{F}_1 and \vec{F}_2 by the parallelogram method
 (b) $\vec{F}_2 + \vec{F}_1$ by the graphical closed polygon method
 (c) $\vec{F}_1 + \vec{F}_2$ by the graphical closed polygon method

Figure 5



(a) through (f) Show the vector sum of the three vectors \vec{F}_1 , \vec{F}_2 , and \vec{F}_3 by the graphical closed polygon method.
 (g) Shows the vector sum $\vec{F}_5 + \vec{F}_6 + \vec{F}_3$ by the graphical closed polygon method.
 (h) Shows the vector sum $\vec{F}_4 + \vec{F}_8 + \vec{F}_7 + \vec{F}_5$ by the graphical closed polygon method.

Figure 6



$$\begin{aligned}
 F_1 &= 1 \text{ cm} & \theta_1 &= 90^\circ \\
 F_2 &= 3 \text{ cm} & \theta_2 &= 0^\circ \\
 F_3 &= 2 \text{ cm} & \theta_3 &= 45^\circ \\
 F_R &= 5 \text{ cm} & \theta_R &= 29^\circ
 \end{aligned}$$

Figure 7

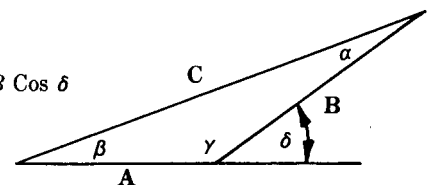
Vectors may also be added analytically by the **component method**. The analytical method for adding vectors by the component method follows readily by formulating the graphical method into mathematical expressions. This is shown below for **Figure 7**.

$$\begin{aligned}
 F_{1x} &= F_1 \cos \theta_1 = (0.10 \text{ kg wt}) \cos 90^\circ = 0 \text{ kg wt} \\
 F_{2x} &= F_2 \cos \theta_2 = (0.30 \text{ kg wt}) \cos 0^\circ = 0.30 \text{ kg wt} \\
 F_{3x} &= F_3 \cos \theta_3 = (0.20 \text{ kg wt}) \cos 45^\circ = 0.14 \text{ kg wt} \\
 F_{Rx} &= F_{1x} + F_{2x} + F_{3x} = 0.43 \text{ kg wt} \\
 F_{1y} &= F_1 \sin \theta_1 = (0.10 \text{ kg wt}) (\sin 90^\circ) = 0.10 \text{ kg wt} \\
 F_{2y} &= F_2 \sin \theta_2 = (0.30 \text{ kg wt}) (\sin 0^\circ) = 0 \text{ kg wt} \\
 F_{3y} &= F_3 \sin \theta_3 = (0.20 \text{ kg wt}) (\sin 45^\circ) = 0.14 \text{ kg wt} \\
 F_{Ry} &= F_{1y} + F_{2y} + F_{3y} = 0.24 \text{ kg wt} \\
 F_R &= \sqrt{F_{Rx}^2 + F_{Ry}^2} = 0.50 \text{ kg wt} \\
 \theta_R &= \tan^{-1} \left(\frac{F_y}{F_x} \right) = 29^\circ
 \end{aligned}$$

Finally, vectors may be added analytically by what we shall refer to as the **trigonometry method**, because it employs the law of Sines and Cosines. The law of Sines and Cosines is reviewed below.

Law of Sines:
$$\frac{A}{\sin \alpha} = \frac{B}{\sin \beta} = \frac{C}{\sin \gamma}$$

Law of Cosines:
$$\begin{aligned}
 C^2 &= A^2 + B^2 - 2AB \cos \gamma = A^2 + B^2 + 2AB \cos \delta \\
 B^2 &= A^2 + C^2 - 2BC \cos \beta \\
 A^2 &= B^2 + C^2 - 2BC \cos \alpha
 \end{aligned}$$



The vectors \vec{F}_4 and \vec{F}_3 are added in **Figure 8** by the analytical trigonometry method.

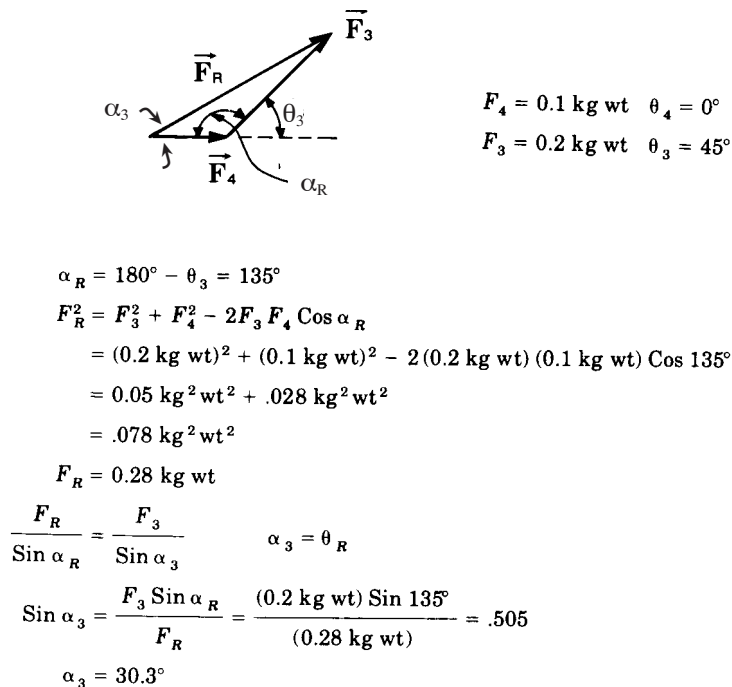


Figure 8

According to the work done in **Figure 8**, the resultant or vector sum of the vectors \vec{F}_4 and \vec{F}_3 is the vector \vec{F}_R of magnitude 0.28 kg. wt. and orientation 30.3° with respect to the 0° reference. If one wishes to add more than two vectors by the analytical trigonometry method, it must be done as a repeated process. That is, add two of the vectors by this method and then add their resultant and a third vector by this method, then add this new resultant and a fourth vector, etc.

In order to experimentally check our graphical and analytical solutions we will make use of the following information. A vector called the **equilibrant** is equal in magnitude and opposite in direction to the resultant vector. The **equilibrant** is the vector that can hold the system in equilibrium. Shown in **Figure 9** are several vectors, their resultant and the equilibrant.

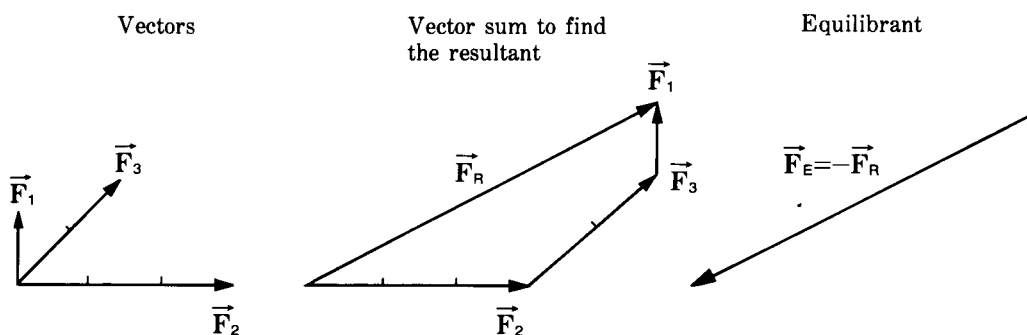


Figure 9

What we will do experimentally is set up the vectors and experimentally determine the equilibrant. Knowing the experimental equilibrant, we can determine an experimental resultant, and then we will compare this experimental resultant to the graphical and analytical solutions.

Procedure

Part I. The sum of two vectors.

The vector forces will be assigned by the laboratory instructor. Determine the resultant force graphically by the parallelogram method, closed polygon method, and the component method. Next, determine the resultant analytically by the component and trigonometry method. Finally, determine the equilibrant and resultant vectors experimentally by means of the force table. The experimental results are to be compared to the graphical and analytical solutions.

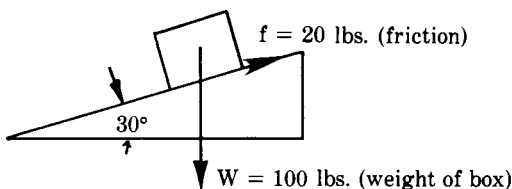
To use the force table, place the ring with the three strings over the pin in the center of the force table. Mount the pulleys on the force table so that forces may be applied in the desired directions by attaching the appropriate masses to the string. Once the given forces are set up, search for the equilibrant, keeping in mind that you can vary both the magnitude and direction. When you think that you have established equilibrium, displace the center ring slightly from the center and see if it returns to the center. If the ring does not return to the center, you need to make a few fine adjustments. To get an idea of the sensitivity of your force table, you should add masses to the equilibrant until you destroy equilibrium and reorient the equilibrant until you destroy equilibrium.

Part II. The sum of three vectors.

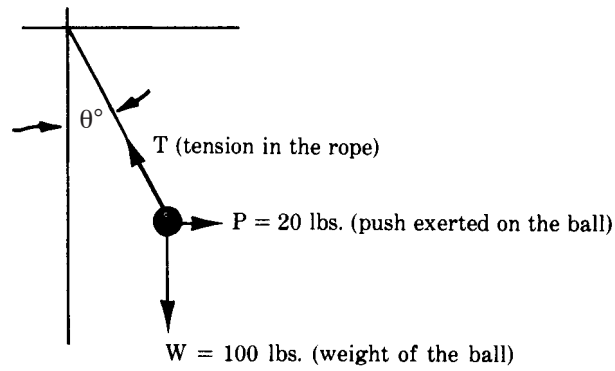
Three vector forces will be assigned by the laboratory instructor. Determine the resultant force graphically by the parallelogram method, closed polygon method, and the component method. Next, determine the resultant analytically by the component and trigonometry method. Finally, determine the equilibrant and resultant vectors experimentally by means of the force table. The experimental results are to be compared to the graphical and analytical solutions.

Questions and Problems

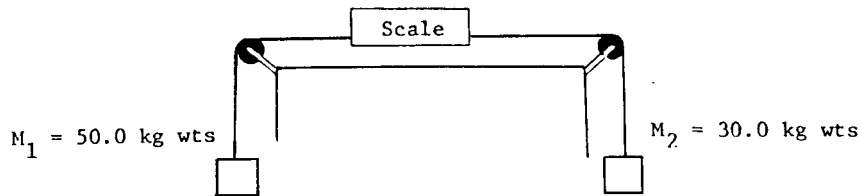
1. If the weight hangers were exactly the same, could their weight be neglected? Explain.
2. Suppose you reach an equilibrium situation and then discover that your force table was not level. What effect will leveling the table have on your equilibrium situation?
3. Could the weight of the ring have been neglected if it had weighed considerably more? Explain.
4. Suppose that you reach an equilibrium situation but the ring is not centered about the center of the table. How does this affect your result?
5. By the graphical and analytical component method, determine the net force tending to move the box down the incline.



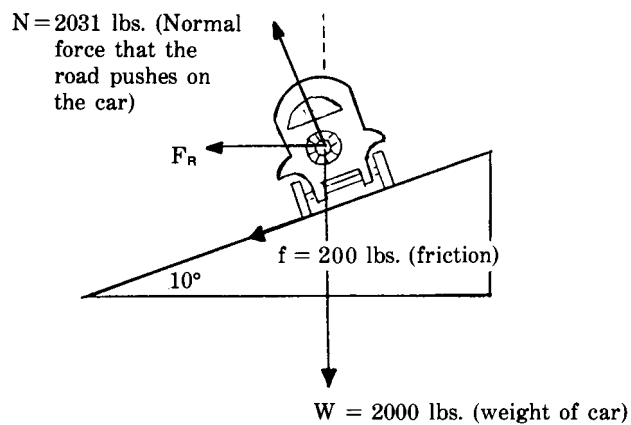
6. By the graphical closed polygon and analytical trigonometry method, determine the tension in the rope and the angle of the rope.



7. By the graphical closed polygon method, determine the scale reading. Is this system static?



8. By the graphical and analytical component method, determine the central force on the car tending to make it round the curve.



DATA AND CALCULATION SUMMARY

Part I. Addition of Two Vectors			VECTOR	MAGNITUDE in kg.wts.	ORIENTATION in degrees	SENSITIVITY	
			DATA	\vec{F}_1	$F_1 =$	$\theta_1 =$	$\Delta F =$
				\vec{F}_2	$F_2 =$	$\theta_2 =$	$\Delta\theta =$
				\vec{F}_E	$F_E =$	$\theta_E =$	
	Graphic Methods	Experimental	\vec{F}_R	$F_R =$	$\theta_R =$	Percent Difference with $F_{R (exp)}$	
		Parallelogram	\vec{F}_R	$F_R =$	$\theta_R =$		
		Closed Polygon	\vec{F}_R	$F_R =$	$\theta_R =$		
		Component	\vec{F}_R	$F_R =$	$\theta_R =$		
		Analytical Methods	Component	\vec{F}_R	$F_R =$	$\theta_R =$	
			Trigonometry	\vec{F}_R	$F_R =$	$\theta_R =$	

Part II. Addition of Three Vectors			VECTOR	MAGNITUDE in kg.wts.	ORIENTATION in degrees	SENSITIVITY	
			DATA	\vec{F}_1	$F_1 =$	$\theta_1 =$	$\Delta F =$
				\vec{F}_2	$F_2 =$	$\theta_2 =$	$\Delta\theta =$
				\vec{F}_3	$F_3 =$	$\theta_3 =$	
	\vec{F}_E	$F_E =$		$\theta_E =$	Percent Difference with $F_{R (exp)}$		
	Graphic Methods	Experimental	\vec{F}_R	$F_R =$	$\theta_R =$		
		Parallelogram	\vec{F}_R	$F_R =$	$\theta_R =$		
		Closed Polygon	\vec{F}_R	$F_R =$	$\theta_R =$		
		Component	\vec{F}_R	$F_R =$	$\theta_R =$		
		Analytical Methods	Component	\vec{F}_R	$F_R =$	$\theta_R =$	
Trigonometry			\vec{F}_R	$F_R =$	$\theta_R =$		

