A DYNAMIC MONTHLY DEMAND MODEL
OF U.S.-PRODUCED SOFTWOOD LUMBER
WITH A FUTURES MARKET LINKAGE

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ABSTRACT

In this paper we estimate a dynamic demand model of U.S. produced softwood lumber using a cointegrated vector autoregression model. We find that demand for U.S.-produced lumber responds to prices of softwood lumber, housing starts, and lumber prices in the futures market, and that various trade measures against Canadian softwood lumber imports have boosted this demand. These results suggest that U.S. lumber producers and consumers could use price information from futures markets to manage price risks and adjust their production/consumption activities and that U.S. producers’ political actions have paid huge dividends.

JEL codes: C32, L66, G13, Q18

Keywords: softwood lumber demand, cointegrated vector autoregression, hedging, softwood lumber war, lumber futures

INTRODUCTION

As softwood lumber is the largest single category of forest products output in the United States, its demand and supply are of special interest to both private entrepreneurs and public policy-makers. For example, to the extent that softwood lumber in U.S. markets is primarily sourced in the U.S. and Canada, it is not surprising to see that U.S. producers have lobbied for

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† The opinions expressed are those of the authors and not those of the Commodity Futures Trading Commission or its Commissioners.
restrictions on Canadian lumber imports and that the two countries have engaged in a long-lasting softwood lumber trade war (Zhang 2007). The various trade-restrictive measures on Canadian lumber imports historically and currently, as well as the market structure for softwood lumber, characterized by inelastic demand and supply, collectively render lumber prices volatile in the U.S. (Zhang and Sun 2001). Thus, understanding the dynamics of demand for U.S. produced lumber is helpful for many U.S. lumber producers to make their production decisions and U.S. consumers to adjust their consumption.

Herein, the estimated U.S. softwood lumber demand model differs from previous estimated models (e.g., Uri and Boyd 1990; Adams et. al. 1992) in three ways: (a) our demand is estimated with monthly data, while most previous work used annual data; (b) we focus on U.S.-sourced lumber, while previous work emphasized annual U.S. demand for softwood lumber from all sources (U.S. production and imports); and more importantly (c) we test for and incorporate an empirical link between current demand for U.S.-produced lumber and the lumber futures market, whereby producers and consumers may use prices of lumber futures to adjust their production and consumption activities. In addition, we demonstrate the cointegrated VAR model’s policy-analytic usefulness in empirically assessing the positive impacts that various trade restrictions measures have had on demand for U.S.-produced softwood lumber. Since we examine the success with which some of these trade measures accomplished their purpose and augmented supply and demand of U.S.-sourced softwood lumber, we focus on quantities of U.S. produced lumber. The next section presents the demand model for U.S. produced lumber, followed by estimation methods, data and results. The final section concludes.

**A DYNAMIC DEMAND MODEL FOR U.S.-PRODUCED LUMBER**

As Uri and Boyd (1990) stated, the demand for softwood lumber at the regional level in a given period is often expressed as a Cobb-Douglas function:

\[
Q_L = \mu P_L^\alpha P_S^\beta H^\gamma M^\delta
\]

or

\[
\ln Q_L = \ln \mu + \alpha \ln P_L + \beta \ln P_S + \gamma \ln H + \delta \ln M + \varepsilon
\]

where \(Q_L\) is quantity demanded for softwood lumber, \(P_L\) is lumber price, \(P_S\) is price of a substitute good, \(H\) is housing starts, \(M\) is maintenance, remodeling, and repairing activities; \(\mu\) (or \(\ln \mu\)), \(\alpha\), \(\beta\), \(\gamma\), and \(\delta\) are parameters to be estimated, and \(\varepsilon\) is an error term. Often the quantity of softwood lumber demanded in previous periods is also included. At the national level, other demand models such as the end-use approach (Adams et al. 1992) and product diffusion approach (Spelter 1985) have been used. Again, all previous demand studies have used annual data and irrespective of where the lumber is produced.

Our interest in this paper is monthly demand for U.S. produced lumber. Demand of lumber substitutes is highly inelastic in such short-term (monthly) horizons so that the term, \(\gamma \ln P_S\), is close to zero (\(\gamma \ln P_S \rightarrow 0\)). Further, the maintenance, remodeling, and repairing
activities are highly correlated with housing starts. Thus, U.S. producers may consider price expectations, expressed in the lumber futures prices, as an indicator of demand in the near future. Thus, we have

\[ Q_L = \mu P_L^\alpha P_F^\omega H^\gamma \]  \hspace{1cm} (3)

where \( P_F \) is the price of lumber futures and \( \omega \) is a parameter to be estimated. Here, \( P_L \) and \( P_F \) may be seen as prices of close substitute lumber products: the currently valued lumber priced at \( P_L \) that would have a negative exponent \( (\alpha < 0) \) and its time-differentiated substitute delivered at a later date and priced at \( P_F \) that would have a positive coefficient \( (\omega > 0) \).

Equation 3 is the basic model of this paper. Should \( \alpha \) and \( \omega \) be statistically identical, but with opposite signs (i.e., \( \alpha = -\omega \)), we would have

\[ Q_L = \mu \left( \frac{P_L}{P_F} \right)^\alpha H^\gamma \]  \hspace{1cm} (4)

or

\[ \ln Q_L = \ln \mu + \alpha \ln P_L - \alpha \ln P_F + \gamma \ln H + \epsilon \]  \hspace{1cm} (5)

Obviously, demand for U.S. produced lumber is affected by the various trade restriction measures placed on Canadian lumber imports. Hence we used various dummy variables to account for the effects of important and potentially market-influencing events. This model was estimated with time series data using a cointegration approach (cointegrated VAR model).

**ESTIMATION METHODS AND DATA**

**The Cointegration Approach and Data**

As is well known, economic time series often fail to meet conditions of weak stationarity (also known as stationarity and ergodicity) required of valid inference. In some cases, applying regression to time-ordered data generate biased estimators (Granger and Newbold 1986, p. 1-5). On the other hand, while often individually non-stationary, such series can form vectors with stationary linear combinations, whereby the series move in tandem and in a stationary manner as a group known as an error-corrected cointegrated system (Johansen and Juselius 1990).

Based on Equation 5, we found monthly data for the following endogenous variables (denoted by parenthetical labels) with which to conduct this study:

- U.S. softwood lumber production \( (Q_L) \) in millions of board feet. These data were obtained from Western Wood Products Association and Southern Forest Products Association (G. Andrew 2012; V. Barabino, 2012. Pers. Comm.).
• U.S. housing starts (H) in thousands of units, not seasonally adjusted (U.S. Census Bureau 2012).
• U.S. wholesale price of softwood lumber (P_L): This is the U.S. producer price index or PPI for softwood lumber in the lumber and wood products group of PPIs, Series no. WPU0811. This variable represents current U.S. softwood lumber price.
• Price of Softwood Lumber Futures (P_F): This is the average monthly settlement price of the CME Group’s Random Length Lumber Futures contract that trades in volumes of 110,000 board feet of random length (8 to 20 feet) softwood 2-by-4s.\(^1\) \(P_F\) is the average of softwood lumber futures 45 days forward from the current pricing point, \(P_L\), above.\(^2\)

Modeled in natural logarithms, our data are shown to be non-stationary or integrated of order 1. An estimation period of January, 1992 through May, 2012 (1992:01 – 2012:05) was chosen because previous U.S. softwood lumber production data are not available at the monthly level.

Following Juselius and Toro (2005) and Juselius (2006, chs. 1-4), we examined the logged levels and differences to assess the data’s non-stationarity properties. Such examinations led to formulation of specification implications of these properties that utilize inherent stores of information to avoid compromised inference, and in some cases, biased estimates (Granger and Newbold 1986). Incorporating statistically supported specification implications in turn results in a statistically adequate underlying VAR model (and algebraically equivalent unrestricted VEC) with which the cointegrated properties of the four endogenous variables can be exploited.

**The Underlying Statistical Model: The Levels VAR and Unrestricted VEC Equivalent\(^3\)**

Sims (1980) and Bessler (1984) note that a VAR model posits each endogenous variable as a function of \(k\) lags of itself and of each of the system’s remaining endogenous variables. The above lumber-related variables render the following 4-equation model in lagged levels:

\[
X(t) = a(1,1)*Q_L(t-1) + \ldots + a(1,k)*Q_L(t-k) + \\
a(2,1)*H(t-1) + \ldots + a(2,k)*H(t-k) + \\
a(3,1)*P_L(t-1) + \ldots + a(3,k)*P_L(t-k) + \\
a(4,1)*P_F(t-1) + \ldots + a(4,k)*P_F(t-k) + \\
\text{a(c)*CONSTANT + a(T)*TREND + a(s)*SEASONALS + \gamma(t)}
\]

(6)

where \(X(t) = Q_L(t), H(t), P_L(t),\) and \(P_F(t)\). The asterisk denotes the multiplication operator; \(t\)

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\(^1\) The chosen roll methodology on the Bloomberg terminal prices the front month contract with a roll into the next nearest contract on the first business day of the front month contract’s expiration month.

\(^2\) The average estimate that \(P_F\) prices a position at an average horizon of 45 days forward of \(P_L\) arises from a number of factors. The 45-day average estimate uses the assumptions that (i) a monthly average price presents approximately half a month (15 days), (ii) the contract lists every other month, and (iii) average settlement occurs about 15 days into the delivery month. Hence, the following summation arises: 15+15 or 15+30+15.

\(^3\) The 45-day average arises from taking an average of the latter two possibilities.

\(^3\) This section draws heavily on Johansen and Juselius (1990) and Juselius (2006).
refers to current time period; and $\gamma(t)$ is a vector of white noise residuals. The $a$-coefficients are ordinary least squares regression estimates with the first parenthetical digit denoting the four endogenous variables as ordered in $X(t)$'s definition, and the second reflecting the lagged value. The lag structure, $k=3$, was suggested from the application of Tiao and Box's (1978) lag search procedure. The $a(c)$ denotes the intercept generated on a vector of unity values, while $a(T)$ is the coefficient generated on a time trend or TREND. Equation 6 also includes a vector of 11 centered seasonal variables and a number of other binary variables discussed below.

It is well known that Equation 6, known as a levels VAR, with a lag order-$k$ can be equivalently written more compactly as an unrestricted vector error correction (unrestricted VEC) model (Juselius 2006, p. 59-63; Johansen and Juselius 1990):

$$
\Delta x(t) = \Gamma(1)\Delta x(t-1) + \ldots + \Gamma(k-1)\Delta x(t-k+1) + \Pi^*x(t-1) + \Phi D(t) + \varepsilon(t)
$$

(7)

The endogenous variable number, $p$, is 4. The $\varepsilon(t)$ are white noise residuals, the delta is the difference operator, while the $x(t)$ and $x(t-1)$ are $p$ by 1 vectors of the endogenous variables in current and lagged levels. The $\Gamma(1), \ldots, \Gamma(k-1)$ terms are $p$ by $p$ matrices of short run regression coefficients, and $\Pi$ is a $p$ by $p$ long run error correction term to account for endogenous levels. The $\Phi D(t)$ is a set of deterministic variables, including an array of binary (dummy) variables that will be added to address stationarity issues and policy and market events. The error correction (EC) term is decomposed as follows:

$$
\Pi = a^*\beta
$$

(8)

The $\alpha$ is a $p$ by $r$ matrix of adjustment coefficients ($r$ is the number of cointegrating relationships or the reduced rank of $\Pi$ discussed below). The $\beta$ is a $p$ by $r$ vector of cointegrating parameters.

The error correction or EC term retains the levels-based and other long run information: linear combinations of non-differenced and individually I(1) levels variables (under cointegration); permanent shift binaries to capture more enduring effects of policy/market events (presented below); and a linear trend. The term $[\Gamma(1)\Delta x(t-1) \ldots \Gamma(k-1)\Delta x(t-k+1), \Phi D(t)]$ collectively comprises the model's short run/deterministic component (hereafter denoted short run component) that includes the permanent shift binaries in differentiated form, observation-specific outlier binaries (introduced below), and seasonal binaries.

Having followed Zhang (2007), Nagubadi et al. (2009) and Majumdar, et al. (2011), along with market knowledge and expertise, we initially restricted the following non-differenced permanent shift binary variables to the levels-based error-correction space to account the long run effects of seven important and potentially market-influencing events:

- **NAFTA**: This binary is defined for the January, 1994 implementation of the North American Free Trade Agreement (NAFTA) and takes a value of unity for the 1994:01 – 2012:05 period and zero otherwise.
- **URUGUAY**: This binary is defined for the January, 1995 implementation of the Uruguay Round Trade Agreement and takes a value of unity for the 1995:01 – 2012:05 period and zero otherwise.
Estimation Results

We followed Juselius’ (2006, ch. 6) method of identifying and including extraordinarily influential effects of month-specific events through specification of “outlier” binaries. When a potentially includable outlier was identified with a “large” standardized residual, an appropriately specified variable was included in differenced form as part of Equation 7’s short run component, and retained if a battery of diagnostics (discussed below) moved favorably to suggest enhanced specification.4

Table 1’s battery of diagnostic values for the levels VAR (and its unrestricted algebraic VEC equivalent) before and after efforts focusing on enhanced specification suggest clear benefits to such efforts.

The trace correlation, a goodness of fit indicator, increased 84% to 0.644. While serial correlation was initially a likely issue, the finally estimated levels VAR after specification efforts generated evidence that serial correlation was no longer an issue. While initial evidence strongly rejected the null of no heteroscedasticity before specification efforts, the

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4 An observation-specific event was judged as potentially “extraordinary” if its standardized residual exceeded 3.0 in absolute value. Such a rule for outliers was designed based on the effective sample size of 242 observations using the Bonferroni criterion: \( \text{INVNORMAL}(1-0.025) \) where \( T=242 \) and INVNORMAL is a function for the normal distribution that returns the variable for the cumulative density function of a standard normal distribution (Estima 2007). The Bonferroni variate had an absolute value of 3.7. Having realized that there were some month-specific events with potentially extraordinary effects with absolute standardized residual values of about 3.0, we opted to follow recent research and chose a more conservative Bonferroni absolute value criterion of 3.0 rather than 3.7 (Babula and Rothenberg 2012). Observations with absolute standardized residual values of 3.0 or more were thereby considered as potential outliers, and we specified an appropriately defined variable for relevant observations for the sequential estimate procedure. Ten binaries were ultimately included. Due to space limitation considerations, we do not report the binaries as they are part of the estimated model’s short run component that is not a focus of this study on long run cointegration relationships. The binaries are available from the authors on request.
finally restricted model after specification efforts suggested that heteroscedasticity was likely not an issue.

Doornik-Hansen (D-H) values test the null that the estimated model’s residuals behave normally. The D-H values for the estimated system improved notably such that the system of estimated residuals ultimately achieved strongly normal behavior. The univariate D-H values suggest that ultimately, evidence at the five percent significance level was insufficient to reject the null of normally behaving residual estimates for all four equations.

Finally, Table 1 suggests that the finally estimated and statistically adequate model displayed skewness and kurtosis indicators that fell within literature-established ranges.

<table>
<thead>
<tr>
<th>Test and/or equation</th>
<th>Null hypothesis and/or test explanation</th>
<th>Prior efforts at specification adequacy</th>
<th>After efforts at specification adequacy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trace correlation</td>
<td>system-wide goodness of fit: large proportion desirable</td>
<td>0.350</td>
<td>0.644</td>
</tr>
<tr>
<td>LM Test for serial correlation (lag 2)</td>
<td>Ho: no serial correlation by lag-2. Reject for p-values of 0.05 or less</td>
<td>29.6 (p=0.02)</td>
<td>7.3 (p=0.97)</td>
</tr>
<tr>
<td>ARCH, lag 2</td>
<td>Ho: No heteroscedasticity. Reject for p values of 0.05 or less</td>
<td>254.1 (p=0.006)</td>
<td>222.8 (p=0.13)</td>
</tr>
<tr>
<td>Doornik-Hansen test, system-wide normality</td>
<td>Ho: modeled system behaves normally. Reject for p-values below 0.05.</td>
<td>15.8 (p=0.05)</td>
<td>6.7 (p=0.57)</td>
</tr>
<tr>
<td>Doornik-Hansen test for normal residuals</td>
<td>Ho: equation residuals are normal. Reject for p-values at or below 0.05</td>
<td>5.12 (p=0.08)</td>
<td>0.70 (p=0.71)</td>
</tr>
<tr>
<td>(univariate)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∆Q_l</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.12</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>(p=0.08)</td>
<td>(p=0.71)</td>
<td></td>
</tr>
<tr>
<td>∆P_l</td>
<td></td>
<td>5.5</td>
<td>4.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(p=0.062)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>∆P_f</td>
<td></td>
<td>2.02</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(p=0.36)</td>
<td>(p=0.60)</td>
</tr>
<tr>
<td>∆H</td>
<td></td>
<td>3.01</td>
<td>3.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(p=0.02)</td>
<td>(p=0.17)</td>
</tr>
<tr>
<td>Skewness(kurtosis) univariate values</td>
<td>Skewness: ideal is zero; &quot;small&quot; absolute value acceptable kurtosis: ideal is 3.0; acceptable range is 3.0-5.0.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∆Q_l</td>
<td></td>
<td>-0.35 (3.14)</td>
<td>-0.13 (2.9)</td>
</tr>
<tr>
<td>∆P_l</td>
<td></td>
<td>0.34 (3.49)</td>
<td>0.33 (3.41)</td>
</tr>
<tr>
<td>∆P_f</td>
<td></td>
<td>0.02 (3.33)</td>
<td>0.14 (3.06)</td>
</tr>
<tr>
<td>∆H</td>
<td></td>
<td>0.01 (3.44)</td>
<td>-0.036 (3.48)</td>
</tr>
</tbody>
</table>

**Cointegration: Testing For and Imposing an Appropriate Reduced Rank**

The endogenous variables are shown below to be non-stationary. Juselius (2006, p. 80) notes that cointegrated variables are driven by common trends and stationary linear combinations called cointegrating vectors or CVs. The Π-matrix in Equation 8 is a p by p
matrix equal to the product of the two p by r matrices: β of error correction estimates that under cointegration combine into r < p stationary CVs of the four individually non-stationary endogenous variables, and α of adjustment coefficients (beta, alpha estimates, respectively). Under cointegration, the rank of β'x(t) is reduced despite the non-stationarity of x(t)'s four series.

The EC space's reduced rank has traditionally been selected based on the widely-applied trace tests (Johansen and Juselius 1990). However, Juselius and Toro (2005), Juselius and Franchi (2007), and Juselius (2006, ch. 8) strongly recommend against a sole reliance on the trace test results in determining the reduced rank r < p, and in turn the number of cointegrating vectors that error-correct the system. More specifically, they suggest that determination of reduced rank (r) should consider other relevant sources of evidence. Thus, our determination of r is a three-tiered process that considers three sources of relevant evidence: the traditionally consulted nested trace tests, patterns of α-estimate statistical significance in relevant CVs, and patterns of characteristic roots in companion matrices (Juselius 2006, ch. 8).

All three sources of evidence suggest that r could be as low as 1 and as high as 3, with most evidence suggesting that r is likely 1.

Table 2 provides nested trace test results. A strict reading of these results suggests that evidence at the five percent significance level is sufficient to reject Table 2's first three hypotheses, and is insufficient to reject the fourth, suggesting that r ≤ 3. Given the nested nature of these four trace tests, they suggest that r = 3. However, evidence only marginally rejected the third hypothesis that r ≤ 2. Such indicates that the appropriate number of error-correcting CVs may be smaller than 3, perhaps 2 or less.

The patterns of adjustment coefficient or α-estimate significance further suggest that r is not only not less than 3, but is likely 2 or less. A CV that actively participates in, and that should be considered part of, the EC mechanism should display a high number of statistically significant alphas. When many of a CV's α-estimates are statistically significant, then including that CV in the EC space is justified since the CV is contributing to the model's explanatory power. When many of the α-estimates in a CV are not statistically significant, Juselius (2006, p. 141-143) notes that including that particular CV in the EC mechanism would likely not improve the explanatory power of the model; may invalidate inference; and likely should be excluded from the cointegration space.

The patterns of pseudo-t values on the three CVs from the rank-unrestricted VEC model suggest that CV1 contributes most to the model's explanatory power; CV2 contributes the second-highest levels of explanatory power; and that the third CV contributes little and should perhaps be excluded from the EC process. This is because of the four α-estimates generated on the four endogenous variables in each CV: three were significant in CV1; two were significant in CV2; and only 1 was significant in CV3. This evidence concerning α-estimate significance suggests that the EC mechanism should include CV1 and possibly CV2, and should not include CV3, suggesting that r is 1 or 2 rather than 3.

Due to space considerations, the alpha estimates, pseudo t-values and other results from the rank-unrestricted VEC model are not reported, and are available from the authors on request. An α-estimate is deemed statistically non-zero at the five percent significance level if its absolute pseudo-t value is 2.6 or more (Juselius 2006, p. 142). More specifically for this paper's model, the following were the statistically significant pseudo-t values generated by the CVs: -3.1 on Q₁, -2.7 on P₁, and 3.6 on P₁ in CV1; 6.0 on Q₂ and 2.8 on P₁ in CV2; and -4.4 on H in CV3.
Table 2. Nested Trace Tests and Test Statistics

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Trace Value</th>
<th>95% Fractile</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank or $r \leq 0$</td>
<td>134.84</td>
<td>78.06</td>
<td>Reject null that $r \leq 0$</td>
</tr>
<tr>
<td>Rank or $r \leq 1$</td>
<td>81.21</td>
<td>57.17</td>
<td>Reject null that $r \leq 1$</td>
</tr>
<tr>
<td>Rank or $r \leq 2$</td>
<td>42.23</td>
<td>40.13</td>
<td>Reject null that $r \leq 2$, although marginally</td>
</tr>
<tr>
<td>Rank or $r \leq 3$</td>
<td>14.61</td>
<td>26.36</td>
<td>Fail to reject that $r \leq 3$</td>
</tr>
</tbody>
</table>

Notes: As recommended by Juselius (2006), CATS2-generated fractiles are increased by 8*1.8 or 14.4 to account for the eight previously discussed deterministic variables that were restricted to lie within the cointegration space. As recommended by Juselius (2006, ch. 8) and programmed by Dennis (2006), trace values are corrected with Bartlett's small sample adjustment.

The third source of rank-relevant evidence is patterns of the characteristic roots under alternative reduced ranks that suggest $r$ is likely 1 and not 2 or 3. Generally, if the chosen $r$ is appropriate, then the companion matrix under $r$ should generate $(p - r)$ unit roots, and the $(p - r + 1)$st root should be substantially below unity. Should the $(p - r + 1)$st root be near-unity, then $r$ should likely be reduced (Juselius 2006, ch. 8). The following summary results clearly suggest that not only is the appropriate reduced rank less than 2 or 3, it is likely even smaller, that is $r=1$:

- Under $r = 3$, there was $(p - r) = (4-3)$ or 1 unit root with the second root having been 0.96 that is nearly unity. This suggests that $r=3$ should be reduced.
- Under $r = 2$, there were $(p - r) = (4-2)$ or two unity roots with the third being 0.83, a value deemed near enough to unity to suggest that $r=2$ may also be reduced.
- Under $r = 1$, there were $(p - r) = (4-1)$ or three unit roots with the fourth of 0.68 far enough below unity to suggest that $r=1$.

Based on the three sources of above-cited references, we conclude that the EC space's reduced rank is more likely 1 than 2 or 3 and we restricted the model for $r=1$ with a single CV error-correcting the system.

Hypothesis Tests on the Three Unrestricted Cointegrating Relations

One begins with the unrestricted CV (not reported here) that emerged from Johansen and Juselius' (1990) reduced rank estimation of Equation 7 after having imposed a rank of $r=1$ on the EC space. A sequence of hypothesis tests were then conducted on the EC space; the statistically supported hypotheses were imposed; and the restriction-ridden model was re-estimated with the Johansen and Juselius' (1990) reduced rank estimator to generate the finally restricted cointegrating relation that error-corrects the system and that is presented and analyzed below. Hypothesis tests on the betas take the form:

$$\beta = H^*\varphi$$  \hspace{1cm} (9)

6 Due to space considerations, the authors have summarized and have not reported the companion matrices and related results under 1, 2, and 3. These results and matrices are available from the authors on request.
Above, $\beta$ is a $p \times 1$ by $r$ vector of coefficients included in the cointegration space, and $H$ is a $p \times 1$ by $s$ design matrix, with $s$ being the number of unrestricted or free beta coefficients. The $\varphi$ is an $s \times r$ matrix of unrestricted beta coefficients. The hypothesis test value or statistic is:

$$2\ln(Q) = T^*\sum_i \left( \frac{1-\lambda_i^2}{1-\lambda_i} \right)$$ for $i = 1 (=r)$ \hspace{1cm} (10)

Asterisked (non-asterisked) eigenvalues ($\lambda_i$, $i = 1$) are generated with (without) the tested restrictions imposed.

The first group consists of four system-based and rank-dependent stationarity (unit root) tests on the four endogenous variables. Juselius (2006), Juselius and Toro (2005), and Juselius and Franchi (2007) recommend this approach over univariate unit root tests (e.g., Dickey-Fuller and Phillips-Perron tests) for such multivariate models as ours. The four unit root tests were conducted in CATS2 (Dennis 2006). Evidence suggested that all four endogenous variables are non-stationary or I(1) in logged levels.

The second group of tested hypotheses contains those that emerged and/or were suggested by (a) values of the estimated CV’s parameter estimates, (b) market/industry knowledge and expertise, and (c) economic and econometric theory. It is well known from Sims (1980) that the unrestricted levels VAR of Equation 6 that underlies Equation 7 is a reduced form one, where estimated relations reflect a mix of demand- and supply-side elements without clear structural interpretations. Further, these reduced form relations encompass an intertwined mix of influences of long run and short run components. The advantage of dichotomizing Equation 6 into Equation 7’s long run EC component and its short run component is to permit researchers to focus on the long-run component’s equilibrium or cointegrating relationships in Equation 8 and to work economic/econometric theory and market knowledge into Equation 8’s estimation through imposition of statistically supported restrictions obtained from hypothesis tests. In so doing, long run theoretical relationships are illuminated and separated from short run influences, and economic rationalization of Equation 8’s relationships may become clearer than in prior work that did not so-dichotomize the model. As well, such dichotomization of long run and short run effects, as well as due diligence in imposing statistically supported hypothesis test restrictions, may render a finally restricted CV whose parameter values vary from those in prior literature whose methods did not have the benefit of their models’ dichotomization into long run and short run components.

The following five restrictions arose from (a) – (c). They were tested and strongly accepted by the data using Equations 9 and 10 and were imposed on the EC space that was then re-estimated using Johansen and Juselius’ (1990) reduced rank estimator: 

\footnote{The p1 equals 12: it is the sum of $p=4$ endogenous variables plus the eight deterministic variables (previously presented) that were restricted to lie in the cointegration space.}

\footnote{More specifically, Equation 9 is re-written as $\beta^* = [b, \varphi]$. Let $p_1$ be the new dimension of 12 to reflect the four endogenous variables and the eight deterministic variables restricted to lie in the EC space. The $\beta^*$ is a $p_1 \times 1$ by $r$ or $11 \times 3$ matrix with one of the variable’s levels restricted to a unit vector and $b$ is a $p_1 \times 1$ by $12$ vector with a unity value corresponding to the variable the stationarity of which is being tested. The $\varphi$ is a $p_1 \times r$ by $1$ matrix that vanishes under $r=1$ since $(r-1)$ is zero. Given the rank of 1, the test values and parenthetical p-values for the four stationarity tests are as follows with the null of stationarity rejected for p values below 0.05: 26.3 (0.000008) for $Q_t$, 34.96 (0.00000012) for $P_{ct}$, 28.2 (0.000003) for $P_r$ and 28.4 (0.000003) for $H$.}

\footnote{The Chi-square test value (5 degrees of freedom) using the Bartlett small sample correction programmed in CATS2 by Dennis (2006) was 8.97 and had a p-value of 0.11. Thus, evidence at the five percent significance level was insufficient to reject the five restrictions.
• \( \beta(P_L) = -\beta(P_F) \). The reasoning and importance of this test is highlighted below.
• Zero restrictions on the \( \beta \)-estimates on the binaries defined for the Uruguay Round agreement, the two recessions, and time trend. The issue of multicollinearity of these non-trend binaries with other temporally concurrent permanent shift binaries ultimately included in the finally restricted CV [Equation 11] is addressed below as a reason why the data accepted these restrictions.

What resulted is the finally-restricted cointegrating relationship provided in Equation 11 as a monthly U.S. demand for softwood lumber.

\[
Q_L = -1.25*(P_L - P_F) + 0.30*H - 0.09*NAFTA + 0.10*SLA96
\]

\[
(-7.10) \quad (4.3) \quad (-1.34) \quad (1.85)
\]

\[
+ 0.16*ADCVD +0.19*SLA06
\]

\[
(3.00) \quad (2.88)
\]

The parenthetical values below the estimated coefficients are pseudo t-values. The absolute critical value to test that the CV \( \beta \)-estimate is statistically zero is 2.6 at the 5% significance level (Juselius 2006, p. 142).\(^{10}\)

**INTERPRETATION AND USEFULNESS OF U.S.-PRODUCED SOFTWOOD LUMBER DEMAND MODEL**

As expected of a demand relation, the quantity of U.S. softwood lumber (\( Q_L \)) is negatively related to its price, \( P_L \). As we used a cointegrated VAR approach, our elasticity of demand is a long-run elasticity. It is thus not surprising that our elasticity estimate (-1.25) is much higher than those presented in the literature (e.g., Uri and Boyd 1990; Adams et al. 1992).

Perhaps more interestingly, such demand is positively and equally dependent on futures price that prices a closely-substitutable (and time-differentiated) futures lumber position an average of 45 days forward. Insofar as data were modeled in natural logarithms, Equation 11’s price difference term implies that U.S. softwood lumber demand depends on the relative softwood lumber/futures price, such that the emergent demand takes the form of Equation 4. This implication is reinforced by the term’s notable statistical strength: The term’s pseudo-t value (-7.1) far exceeds the ±2.6 critical values at the 5% significance level (Juselius 2006, p. 142).

At first glance, Equation 11’s \( P_L/P_F \) term may suggest that concurrently equal movements of the two prices could or would be mutually offsetting with no effect on softwood lumber demand. However, such a precise offset is unlikely, since the modeled softwood lumber and futures prices do not define (and are not intended to define) the CME Group’s Random Length Lumber contract’s underlying basis. Rather, \( P_L \) is a nationally-surveyed PPI intended to capture national softwood lumber price trends, and is not the contract-specific cash price

\(^{10}\) Juselius (2006, p. 142) notes that these pseudo-t values are not Student t-values and as a result, the critical values for the hypothesis concerning Equation (11)’s \( \beta \)-estimates are not the same as Student-t critical values.
used to settle the CME Group’s contract that in turn generates $P_F$. And while the two prices are expected to qualitatively move in tandem, there is no hard expectation that a related event or policy shock should generate equal percent changes in the two prices.

Nonetheless, softwood lumber priced concurrently at its own price and priced forward at $P_F$ are highly substitutable products that are differentiated by time. The above lumber/futures price relation results are therefore consistent with U.S. softwood lumber demand that is negatively related to its own current price ($P_L$) and positively related to the price of its close substitute priced forward at $P_F$. Hence, Equation 11 is that log linear form of the previously discussed Cobb-Douglas demand in Equation 3, and more specifically Equation 4 where demand is a function of the $P_L/P_F$ ratio.

Risk-managing activity, including hedging, is likely under Equation 11’s $P_L/P_F$ price term. As softwood lumber price rises relative to futures price, demand for softwood lumber at the current pricing point, $P_L$, becomes relatively more expensive than at the futures pricing point, $P_F$, at an average time-stamp of 45 days forward. As $P_L/P_F$ consequently rises, there is a willingness of some agents to postpone demand and allocate some of their total demand towards the relatively cheaper futures pricing point some 45 days forward through taking positions with the CME Group’s softwood lumber contract.

Likewise, as softwood price declines relative to futures price, demand for lumber at the current pricing point, $P_L$, becomes relatively cheaper than at the $P_F$ point some 45 days ahead. As $P_L/P_F$ declines, there is a $P_L$-induced increase in current demand that may be offset by a decline in demand at the futures price 45 days ahead, as agents hedge through position-taking on the CME Group contract. Finally, it is important to note that policies or events that induced effects on $P_F$ have similarly reasoned effects on softwood lumber demand through changes in the relative softwood lumber/futures price.

Observed data associated with notably pronounced movements in lumber price, futures price, relative $P_L/P_F$ price, and CME Group lumber contract trading volumes seem consistent with the above reasoning and analysis of Equation 11’s lumber/futures price term. One well-known instance occurred during the 10 months ending March, 1993 when softwood lumber rose 43% and futures price soared even more, such that the softwood lumber price ratio fell by about 20%.

As demonstrated in Figure 1, the resulting 20% drop in relative lumber/futures price rendered demand for U.S. softwood lumber cheaper at the current $P_L$ pricing point than at $P_F$, the pricing point some 45-days forward. This relative price ratio decline was concurrently met with a notable 38.3% escalation in the trading volume of CME Group’s random length lumber contract as reflected by the monthly averages in the contract’s daily trades, particularly after November, 1992.

Although Figure 1 provides daily trades (both long and short positions combined) and does not provide levels of open interest, this rise in trading activity in response to sharp movements in the relative lumber/futures price ratio likely included hedging activity along with some speculative trading.

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11 Reasons for the pronounced lumber price increases included enhanced lumber demand from a then-recovering economy’s increased residential construction, a reduction in timber supplies from Pacific Northwest forests due to environmental concerns, and from allegations of heightened levels of trader speculation (Gorte 1993; Bianco 2012).
Estimated Effects of Specific Policies/Events

Since this study’s cointegrated VAR model was estimated in natural logarithms (logs), interpreting coefficient estimates generated by binary (or dummy) variables follow Halvorsen and Palmquist’s (1980) well-known method (Babula and Rothenberg 2012). The Halvorsen and Palmquist (HP) values calculated for the β-estimates generated by binary variables indicate, on average, the percentage by which the dependent variable (Q_t in Equation 11) is above (for positive β-estimates) or below (for negative β-estimates) during the binary’s period of definition than during sub-periods of the sample when the binary’s defining event was not in force. The HP values provide an important avenue of the cointegrated VAR model by which policy-analytic results may be obtained.


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12 As noted in Halvorsen and Palmquist (1980), for log/log estimations such as ours, one takes “e,” the base of the natural logarithm, raises it to the power of the binary’s β-estimate; subtracts 1.0; and then multiplies the result by 100 to render he noted HP value for that estimated coefficient.

13 The periods of binary variable definition ending with May, 2012 are asterisked. May, 2012 values were the most recent available at the point of model estimation, and this date serves as the end of the estimation period and is not the end of the subsample for which the binaries are defined.
and in some cases, entire sub-periods of binary definition are included in those of other binaries.

As such, these initially included permanent shift binaries likely generated highly collinear coefficients, insofar as all seven coefficients were picking-up concurrent influences. The most obvious example involved the sub-periods defined for NAFTA (1994:01 onward) and URUGUAY (1995:01 onward): \( \beta(\text{NAFTA}) \) initially captured all influences captured by \( \beta(\text{URUGUAY}) \).

Such collinearity among binaries is likely responsible for the acceptance of zero restrictions on the beta estimates for URUGUAY and the two recessions during implementation of the hypothesis tests, and perhaps for the seemingly low significance levels suggested by low absolute pseudo-\( t \) values on Equation 11’s NAFTA and SLA96 coefficients that may well, in reality, be statistically non-zero.\(^{14}\) Of the four remaining permanent shift binaries that the hypothesis test results suggested should remain, the ADCVD and SLA06 variables generated positive and significant coefficients. Yet NAFTA and SLA96 generated pseudo \( t \) values that suggested insignificance. We decided to retain NAFTA to compare with results of other commodity trade studies (e.g., Babula and Rothenberg 2012) where it is shown to be significant and to retain SLA96 to compare with results of other studies on softwood lumber (e.g. Zhang 2006). And as Zhang (2006) notes, including the final year of SLA96 which were expected to expire might have caused the insignificance.

Equation 11’s ultimately included four permanent shift binaries generated \( \beta \)-estimates that, through HP values, suggested the following effects on U.S. demand for domestically produced lumber:

- Implementation of NAFTA resulted in U.S. demand levels for softwood lumber that were, on average, not significantly different from the treaty’s 1994 implementation. This is consistent with our expectation because softwood lumber was excluded from the U.S.-Canada Free Trade Agreement which became NAFTA (Zhang 2007).
- Similar to Zhang (2006), the first SLA in 1996 resulted in demand for U.S. softwood lumber that was, on average, 10% higher than when SLA96 was not in force.
- The U.S. antidumping and countervailing duty orders that were imposed on certain imports of Canadian-sourced softwood lumber appeared effective in augmenting U.S. demand for its own softwood lumber. The HP value on \( \beta(\text{ADCVD}) \) suggests that such demand was, on average, 17.4% higher than during the sample periods when the orders were not in force. This finding is consistent with Nagubadi and Zhang (forthcoming).
- The second softwood lumber agreement established in 2006 appeared effective insofar as HP value on \( \beta(\text{SLA06}) \) suggests that U.S. demand for its own lumber was about 21% higher than sub-samples prior to the agreement’s implementation. We must relegated to future research how much of a net effect this is, insofar as RECESS_0709’s defined sub-period falls entirely within that of the SLA06. This finding is consistent with Nagubadi and Zhang (forthcoming).

\(^{14}\) More specifically, the entire URUGUAY sub-period is enveloped into that of NAFTA. The RECESS_0709 sub-period is included completely in the sub-period of SLA06, while RECESS_2001 overlaps partially with the sub-periods of ADCVD. Further, for the four coefficients ultimately included in Equation 11, the sub-period of ADCVD (2001:08 – 2006:09) overlays with the defined sub-period of NAFTA (1994:01 – 2012:05).
CONCLUSION

In this paper we use the cointegration approach to establish a demand model for U.S. produced softwood lumber that displays noticeable statistical strength. Our model is consistent with economic theory and robust under various diagnostic tests. Our results show that the demand for U.S.-produced lumber responds to current lumber prices, futures lumber prices, and housing starts. Further, various trade restriction measures on Canadian lumber imports have been successful and have had positive impacts on demand for U.S.-produced softwood lumber.

The results suggest that demand for U.S.-produced softwood lumber is related to a ratio of current to futures prices as noted within the Cobb-Douglas form in Equations 3 and 4, or alternatively, to both current and futures prices but in opposite ways. Softwood lumber priced currently and forwardly can be taken as closely substitutable lumber products differentiated by time. U.S. producers could use this relationship to manage price risks and to adjust their short-run production plans. Similarly, lumber buyers could hedge their consumption activities. Finally, the forest economist profession, which has not paid much attention to futures markets in the literature, may glean substantial amounts of useful information and insights on market relationships by studying lumber prices in the futures market -- for example, relationships among the lumber/futures price ratio and contract trading volumes from hedging and from speculative trades.

REFERENCES


