The previous plot used a grid with \( N = 200 \), i.e., 40,000 function evaluations.
One can get by with much less effort: the following plot is drawn from 1,600 function evaluations with \( N = 40 \).

Zero level curves for real (red) and imaginary (blue) parts of
\[
z = f(t) = t^7 - 3t^5 + 4t^4 - 12t^3 + t^2 - t + 17; \quad \text{with } t = x + iy
\]
Look at the two surfaces $z_{\text{real}} = \text{real} (f(x+i\cdot y)) = \text{imag} (f(x+i\cdot y)) = \text{imag}$

$$= \text{imag} \ ((x + i \cdot y)^7 - 3(x + i \cdot y)^5 + 4(x + i \cdot y)^4$$
$$-12(x + i \cdot y)^3 + (x + i \cdot y)^2 - (x + i \cdot y) + 17).$$

The real and imaginary part zero level curves $z_{\text{real}} = \text{imag} = 0$ intersect at the seven roots of the polynomial

$$f(t) = t^7 - 3t^5 + 4t^4 - 12t^3 + t^2 - t + 17.$$
The corresponding contour plot shows the disjoint level curve $z = -9$. It encircles all 7 roots of $f$:

**Question:** What is the relationship between:

- Gershgorin circles
- near zero level curve
- pseudo eigenvalues
Polynomial surface \( z = f(x + i \cdot y) = \)
\[ = -\| (x + i \cdot y)^7 - 3(x + i \cdot y)^5 + 4(x + i \cdot y)^4 \]
\[ -12(x + i \cdot y)^3 + (x + i \cdot y)^2 - (x + i \cdot y) + 17 \|. \]

The level curve \( z = -9 \) gives inclusion regions for the seven roots 
\(-2.5702, 0.35044 \pm 1.6163i, 1.8655, 1.2276, \)
and \(-0.61188 \pm 0.82553i \) of the given polynomial 
\( f(t) = t^7 - 3t^5 + 4t^4 - 12t^3 + t^2 - t + 17. \)
Computed ranks of $Z$, depending on $N$, the size of the partitions:

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$ in degrees</th>
<th>rank($Z^{N,N}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N = 40, 70, 200$</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0.00057296</td>
<td>15</td>
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<tr>
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<td>19</td>
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<tr>
<td>$N = 20$</td>
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<td>0.00057296</td>
<td>14</td>
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<tr>
<td>$N = 10$</td>
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</tr>
<tr>
<td></td>
<td>0.00057296</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>0.0011459</td>
<td>10</td>
</tr>
</tbody>
</table>
Adiabatic non-isothermal CSTR reaction equation
\[ z = y - 1 - \alpha \ast e^{(-\gamma/y)} \ast (1 + \beta - y) \]
surface and \( z = 0 \) level contour for
\( \beta = 1, \gamma = 15, 15000 \leq \alpha \leq 95000, \ 0.8 \leq y \leq 2.2 \).
With two (very small rotations) of the rectangle
\[ D = [\alpha] \times [y] = [15000, 95000] \times [0.8, 2.2] . \]
The 16 surfaces for
\[ Z = f(x, y) = |1/y + 24|^{\sin(-x/10)} + 
+ |\sin(x)/9| \cdot \log(|y|/15) \]
of part 2(c), when rotating the rectangle \( D = [1, 10] \times [1, 10] \) around \((-15, -15)\) 16 times by an angle of \( \pi/8 = 22.5^\circ \) with \( N = 100 \):

\[ 4, 6 \leq \text{rank}(Z) \leq 96, 97, 98, 100 = N \]
For a random skew-symmetric matrix $A$ of size $N = 15$ on the rectangle $D = [0 \leq x_i \leq 10] \times [0 \leq y_j \leq 10] \subset \mathbb{R}^2$:

\[
\text{rank}(A) = 14 = N - 1
\]
For a random symmetric matrix $A$ of size $N = 25$ on the rectangle $D = [0 \leq x_i \leq 10] \times [0 \leq y_j \leq 10] \subset \mathbb{R}^2$:

rank($A$) = 25 = $N$
Elevation surface and contour lines of a matrix $A_{N,N}$

$Z = z(x_i, y_j) = a_{i,j}$ for a **random matrix** $A$ of size $N = 180$ on the rectangle $D = [0 \leq x_i \leq 10] \times [0 \leq y_j \leq 10] \subset \mathbb{R}^2$:

$\text{rank}(A) = 180 = N$