Matlab project 2

1. Create a $5 \times 5$ square matrix of random elements using the matlab function `rand(5)`, call it $A$.

2. Find all the eigenvalues of $A$ using the matlab function `eig(A)` to produce a column vector containing all the eigenvalues of $A$. Notice that the complex eigenvalues, if there are any, should appear in conjugate pairs.

3. The matlab function `eig(A)` can also be used to get both eigenvalues and eigenvectors. To do so, enter command

   $[V \ E] = \text{eig}(A)$

   to produce a diagonal matrix $E$ of eigenvalues and a full matrix $V$ of corresponding eigenvectors of unit length as columns. Perform this operation in your project.

4. Verify the columns of $V$ are indeed eigenvectors associated to eigenvalues in the diagonal matrix $E$ by picking any column of $V$, say the first one $v_1$, and checking whether matrix $A$ multiplied by this vector on right is equal to the first eigenvalue $\lambda_1$ multiplied by this vector, that is, checking whether the difference $Av_1 - \lambda_1 v_1$ is equal to 0.

   Note: In matlab, the element of a matrix $A$ at place say $(1, 2)$ is coded as $A(1, 2)$, and the first column of the matrix is coded as $A(:, 1)$. Do not forget to use * to perform multiplication in matlab. Due to roundup error, 0 may be expressed as a very small number and such a number may contains an exponential factor such as e-010 meaning that the decimal point should be moved 10 places to left.

5. Now form a symmetric matrix $B$ by setting $B = AA^T$. Note that in matlab, the transpose is entered using the prime $'$. 

6. Using the command $[U \ D] = \text{eig}(B)$ to produce a diagonal matrix $D$ of eigenvalues of $B$ and a full matrix $U$ of associated eigenvectors as columns. Notice that all the eigenvalues should be real.

7. Verify $B = UDU^{-1}$ by displaying both sides of this equality and comparing. In matlab, to display a variable, such as a matrix, just type its name and press enter.

8. Verify $U$ is an orthogonal matrix by checking $U^T U = I$.

Submit a record of your matlab session which should include both matlab commands and output, and contain no error.