1.7 Generating Sets and Cayley Digraphs

1.7.1 Generating Sets

Ex 1.81 (Ex 7.1, p68). The Klein 4-group \( V = \{ e, a, b, c \} \) is generated by \( \{ a, b \} \). It is also generated by \( \{ b, c \}, \{ c, a \}, \{ a, b, c \} \).

Ex 1.82. The generating sets of \( \mathbb{Z}_6 \).

Def 1.83. Let \( \{ S_i \mid i \in I \} \) be a collection of sets. Here \( I \) may be any set of indices. Then \( \bigcap_{i \in I} S_i \) is the set of all elements that are in all the sets \( S_i \); that is,

\[
\bigcap_{i \in I} S_i = \{ x \mid x \in S_i \text{ for all } i \in I \}.
\]

Ex 1.84. \( S_1 \cap S_2 \cap \cdots \cap S_n \).

Thm 1.85. If \( G \) is a group, and \( H_i \) is a subgroup of \( G \) for all \( i \in I \). Then \( \bigcap_{i \in I} H_i \) is a subgroup of \( G \).

Def 1.86. Let \( G \) be a group and let \( a_i \in G \) for \( i \in I \). The smallest subgroup of \( G \) containing \( \{ a_i \mid i \in I \} \) is the subgroup generated by \( \{ a_i \mid i \in I \} \).

If this subgroup is \( G \), then \( \{ a_i \mid i \in I \} \) generates \( G \), and \( \{ a_i \mid i \in I \} \) is a set of generators of \( G \). If \( G \) is generated by a set of finite elements then \( G \) is finitely generated.

Thm 1.87. If \( G \) is a group and \( a_i \in G \) for \( i \in I \), then the subgroup \( H \) of \( G \) generated by \( \{ a_i \mid i \in I \} \) has as elements precisely those of \( G \) that are finite products of integral powers of the \( a_i \), where powers of a fixed \( a_i \) may occur several times in the product.

1.7.2 Cayley Digraphs (omit)

Similar to group table, a Cayley digraph of a group \( G \) uses digraph to represent the multiplication relationship via a set of generators of \( G \).

1.7.3 Homework, I-7, p72-73

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