0.1 What is new in this course

Instead of computation, we emphasize more on understanding math. We will think about math objects and relations in axiomatic way.

Tips:

1. Understand definitions, theorems/corollaries, and examples first. Then understand the proofs.
2. Notations (pp487) and Index (pp513) are helpful to find the definitions of one word.

Roughly speaking:

Def 0.1.1 (Algebraic Structure). A set on which we define some operations (eg. $+$, $\cdot$, inverse). These operations follow certain rules.

Ex 0.1.2. $(\mathbb{R}, +)$, $(\mathbb{R}^+, \cdot)$, $(\mathbb{Z}, +)$, $(\mathbb{R}, +, \cdot)$, $(\mathbb{R}^2, +)$, vector spaces.

0.2 Sets and Relations

Def 0.2.1. Sets, elements, empty set, $a \in S$, $a \notin S$, subset, proper subset. Cartesian product, cardinality $|S|$.

Ex 0.2.2 (Ex 0.5, p3, Cartesian product).

Def 0.2.3. A relation between sets $A$ and $B$ is a subset $R$ of $A \times B$.

$$(a, b) \in R \iff a \in A \text{ has relation with } b \in B \iff aRb.$$ 

Def 0.2.4. A function $\phi$ mapping $X$ into $Y$ is a relation between $X$ and $Y$, such that each $x \in X$ appears as the first member of exactly one ordered pair $(x, y)$ in $\phi$. We write $\phi : X \to Y$ and express $(x, y) \in \phi$ by $\phi(x) = y$. Set $X$ is called the domain. Set $Y$ is called the codomain. The range of $\phi$ is $\phi(X) = \{ \phi(x) \mid x \in X \}$. Clearly $\phi(X) \subseteq Y$.

Def 0.2.5. A function $\phi : X \to Y$ is one-to-one (or injective) if $\phi(x_1) = \phi(x_2)$ only when $x_1 = x_2$. The function $\phi$ is onto $Y$ (or surjective) if the range of $\phi$ is $Y$ (i.e. $\phi(X) = Y$). If $\phi$ is one-to-one and onto, we call it a bijection.

Ex 0.2.6. Consider the following functions $f_1 : \mathbb{R} \to \mathbb{R}$. (by graphs)
1. $f_1(x) := e^x$ is one-to-one, but not onto $\mathbb{R}$.

2. $f_2(x) := x\sin x$ is onto $\mathbb{R}$, but not one-to-one.

3. $f_3(x) := 2x - 1$ is one-to-one and onto. It is a bijection.

**Def 0.2.7.** A **partition** is to divide a set into several subsets. Each two subsets have no interception.

**Def 0.2.8.** An **equivalent relation** $\mathcal{R}$ on a set $S$ is one satisfying the three properties for $x, y, z \in S$:

1. (Reflexive) $x \mathcal{R} x$.

2. (Symmetric) If $x \mathcal{R} y$ then $y \mathcal{R} x$.

3. (Transitive) If $x \mathcal{R} y$ and $y \mathcal{R} z$ then $x \mathcal{R} z$.

Equivalent relation $\iff$ Partition

**Ex 0.2.9.** Ex 0.20, p7. (Congruence Modulo $n$).

### 0.2.1 Homework (optional)

Section 0, p10, 29, 30, 32