Introduction to Vector Calculus

Phillip Zenor
Department of Mathematics
Auburn University

Edward E. Slaminka
Department of Mathematics
Auburn University

Donald Thaxton
Department of Physics
Auburn University

November 21, 2008

SECTION 5.4

1. 2kg/m^2 3. 4

5. $\frac{28\pi^2}{15}$

7. $2\pi^2\int_0^2\int_0^x(x^2+y^2)(x+y)\,dy\,dx=\frac{80\pi^2}{3}$

9. $\frac{1}{2}(2\pi)^2\int_0^2\int_0^x((x+1)^2+(y+1)^2)$
$(x+y)\,dy\,dx=80\pi^2$

11. $\int_0^2\int_0^x(x^2+x)\,dy\,dx=\frac{20}{3}$

13. $\left(\frac{35}{48},\frac{35}{54}\right)$ 15. $\frac{2\pi^2}{63}$ 17. $\frac{\pi^2}{180}$

19. $\frac{1}{30}$ 21. $\int_0^1\int_0^1((1-t)+(1-t)\sqrt{2})\,ds\,dt=\sqrt{2}$

23. $3k\pi^2\sqrt{2}$ 25. $4\pi^2\sqrt{2}$

27. $\frac{1}{2}(2\pi f)^2\int_0^R\int_0^{2\pi}(s\cos t)^2\rho s\;dt\,ds=\frac{1}{2}\pi^3R^4\rho f^2$

29. $\int_0^1\int_{\pi/2}^{3\pi/2}-r^2\cos(\theta)\;d\theta\,dr=\frac{2}{3}$

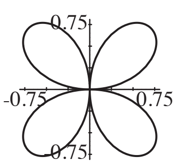
31. $\int_0^{2\pi}\int_0^1 2s\sqrt{1-s^2\cos^2 t}\;ds\,dt$

SECTION 5.5

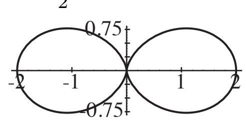
1. $\frac{7}{30}$ 3. $\frac{\sqrt{3}e}{2}$

5. $8\sqrt{2}\pi^2\rho$ 7. $\frac{28\pi^2\rho\sqrt{2}}{3}$

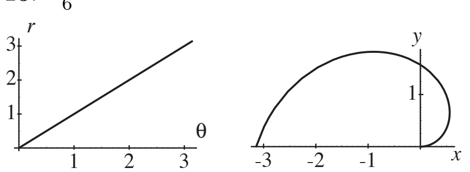
9. $\frac{\pi}{2}$



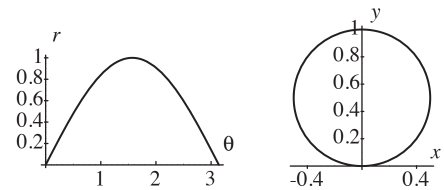
11. $\frac{3\pi}{2}$



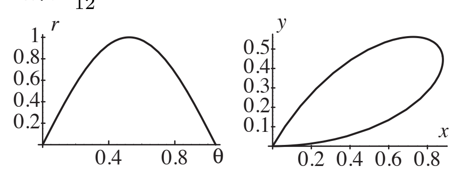
13. $\frac{\pi^3}{6}$



15. $\frac{\pi}{4}$



17. $\frac{\pi}{12}$



19. $\frac{\pi}{8}$ 21. $\frac{3\pi}{16}$ 23. $\sqrt{3}$

25. $\int_0^4\int_0^{(1-x)}\sqrt{1+4y^2}\,dy\,dx.$ 27. $\dfrac{\pi(5\sqrt{5}-1)}{6}$

29. $\int_0^{\sqrt{3}}\int_0^{2\pi}\Big[(r\cos(\theta)+1)+2(r\sin(\theta)-2)^2$
$+(r\cos(\theta)+1)^2\Big]\Big[1+4(r\cos(\theta)+1)^2$
$+4(r\sin(\theta)-2)^2\Big]^{1/2}r\,d\theta\,dr.$

31. $$\int_0^{2\pi}\int_0^1\frac{r}{\sqrt{1-r^2\cos^2\theta}}\,dr\,d\theta$$

SECTION 5.6

1. $\vec{h}(u,v,w)=$
$$\begin{pmatrix}-v+w\\u+2v+w\\u+v+w\end{pmatrix},\qquad\begin{cases}0\le u\le 1\\0\le v\le 1\\0\le w\le 1\end{cases}$$
$J(u,v,w)=1$

3. $\vec{h}(u,v,w)=$
$$\begin{pmatrix}-1+u+2w\\2+u-v\\u+v+4w\end{pmatrix},\qquad\begin{cases}0\le u\le 1\\0\le v\le 1\\0\le w\le 1\end{cases}$$
$J(u,v,w)=6$

5. $\vec{h}(t,s,\theta)=$
$$\begin{pmatrix}s\\(s^2-ts^2+st)\cos\theta\\(s^2-ts^2+st)\sin\theta\end{pmatrix},\qquad\begin{cases}0\le t\le 1\\0\le s\le 1\\0\le\theta\le 2\pi\end{cases}$$
$J(s,t,\theta)=(s^3-s^2)(st-s-t)$

7. $\vec{h}(r,\phi,\theta) = \begin{pmatrix} r\cos\theta\sin\phi \\ r\sin\theta\sin\phi \\ r\cos\phi \end{pmatrix}, \quad \begin{cases} 0 \le r \le 3 \\ 0 \le \phi \le \pi \\ 0 \le \theta \le 2\pi \end{cases}$
$J(r,\phi,\theta) = r^2\sin\phi$

9. $\vec{h}(s,t,\theta) = \begin{pmatrix} s\cos\theta \\ s^2 t \\ s\sin\theta \end{pmatrix} \quad \begin{cases} 0 \le t \le 1 \\ 0 \le s \le 1 \\ 0 \le \theta \le 2\pi \end{cases}$
$J(s,t,\theta) = s^3$

11. $\vec{h}(s,t,\theta) = \begin{pmatrix} s\cos\theta \\ t \\ s\sin\theta \end{pmatrix} \quad \begin{cases} 2 \le t \le 52 \\ 1 \le s \le 3 \\ 0 \le \theta \le 2\pi \end{cases}$
$J(s,t,\theta) = s$

13. $\vec{h}(r,\phi,\theta) = \begin{pmatrix} \cos(\theta)(3 + r\cos\phi) \\ 4 + r\sin\phi \\ \sin(\theta)(3 + r\cos\phi) \end{pmatrix} \quad \begin{cases} 0 \le r \le 2 \\ 0 \le \phi \le 2\pi \\ 0 \le \theta \le 2\pi \end{cases}$
$J(r,\phi,\theta) = r(3 + r\cos\phi)$

15. $\vec{h}(r,t,\theta) = \begin{pmatrix} 3 + (3 - r)\cos\theta \\ rt \\ (r - 3)\sin\theta \end{pmatrix} \quad \begin{cases} 0 \le r \le 1 \\ 0 \le t \le 1 \\ 0 \le \theta \le 2\pi \end{cases}$
$J(r,t,\theta) = |r(r-3)|$

17. $h(r,t,\theta) = \begin{pmatrix} r \\ 6 + (-6 + r\,t)\,\cos\theta \\ (-6 + r\,t)\,\sin\theta \end{pmatrix} \quad \begin{cases} 0 \le r \le 1 \\ 0 \le t \le 1 \\ 0 \le \theta \le 2\pi \end{cases}$
$J(r,t,\theta) = |r(-6 + rt)|$

19. $h(r,t,\theta) = \begin{pmatrix} r \\ -2 + (2 + t)\,\cos\theta \\ (2 + t)\,\sin\theta \end{pmatrix} \quad \begin{cases} 1 \le r \le 3 \\ 2 \le t \le 52 \\ 0 \le \theta \le 2\pi \end{cases}$
$J(r,t,\theta) = |t + 2|$

21. $\vec{h}(t,s,\theta) = \begin{pmatrix} -1 + (4 + r\cos\phi)\cos\theta \\ 4 + r\cos\phi \\ (4 + r\sin\phi)\sin\theta \end{pmatrix} \quad \begin{cases} 0 \le r \le 3 \\ 0 \le \phi \le w\pi \\ 0 \le \theta \le 2\pi \end{cases}$
$J(r,\phi,\theta) = r(r + r\cos\phi)$

23. $\vec{h}(t,s,u) = \begin{pmatrix} t \\ st^2 \\ 2u \end{pmatrix} \quad \begin{cases} 0 \le t \le 1 \\ 0 \le s \le 1 \\ 0 \le u \le 1 \end{cases}$

25. $\vec{h}(t,s,u) = \begin{pmatrix} t \\ t^2\left(\frac{u}{2} + (1 - u)v\right) \\ \frac{1}{2}t^2 u\sqrt{3} \end{pmatrix} \quad \begin{cases} 0 \le t \le 1 \\ 0 \le s \le 1 \\ 0 \le u \le 1 \end{cases}$

27. $\vec{h}(t,s,\theta) = \begin{pmatrix} t \\ \frac{t^2}{2} + \frac{t^2 s}{2}\cos\theta \\ \frac{t^2 s}{2}\sin\theta \end{pmatrix} \quad \begin{cases} 0 \le t \le 1 \\ 0 \le s \le 1 \\ 0 \le \theta \le \pi \end{cases}$

SECTION 5.7

1. 0   3. $12 - \frac{16}{\pi^2}$ 5. $(1 - e^2)\ln^2(2)$

7. $\frac{7}{2}$ 9. $\frac{40\rho\pi^2 f^2}{3}$ 11. $(0,0,0)$m

13. $\frac{\pi}{3}$ 15. 5400π 19. $\frac{2}{3}$

21. $\frac{41}{30}$ 23. $\frac{34}{105}$

SECTION 5.8

1. $\frac{1}{6}$ 3. $\frac{209}{420}$

5. The rectangle $0 \le y \le 2,\ 0 \le z \le 1$.

7. The region in the $xy-$plane bounded by the graphs of $y = -x,\ y = 2 + x^2,\ x = 0$, and $x = 2$.

9. The region in the $yz-$plane bounded between the graphs of $y = -z,\ y = 2 + z^2,\ z = 0$, and $z = 2$.

11. The integration limits do not define a solid.

13. The integration limits do not define a solid.

15. The integration limits do not define a solid.

17. 16   19. 2π 21. $\frac{31}{6}$

23. $\frac{56}{5}$ 25. 16   27. $\frac{9}{2}$

29. $\frac{8}{15}$ 31. $\frac{\sqrt{2}}{3}$ 33. π

35. $\frac{608}{21}$ 37. $\frac{19}{4}$ 39. $\frac{4544}{105}$