Approximate average bit error probability for DQPSK over fading channels

Y. Sun, Á. Baricz, M. Zhao, X. Xu and S. Zhou

The bit error probability (BEP) of DQPSK with Gray coding over an AWGN channel can be computed simply, although it is hard to integrate and derive the average BEP for fading channels. Presented are novel approximations of the average BEP of DQPSK with Gray coding over fading channels. Numerical results show that the novel formulations are quite accurate.

Introduction: The bit error probability (BEP) of differential quaternary phase shift keying (DQPSK) with Gray coding over an additive white Gaussian noise (AWGN) channel can be computed simply with mathematical packages such as Matlab. However, it is hard to be further integrated with the fading statistics and derive analytical results for the average BEP for fading channels. To address this problem, accurate bounds and approximations for the BEP of an AWGN channel are proposed in [1–3]. While the bounds and approximations involving only erfc and exponential functions can facilitate analytical integration of the BEP, they are usually looser than the ones which involve the modified Bessel function of the first kind and zero-order, i.e. $I_0(x)$. In this Letter, the I_0-type bounds, which are hard to integrate further, are approximated by an expression involving only erfc and exponential functions. This result is then averaged over fading statistics and an approximate average BEP for fading channels is derived.

Approximate BEP for AWGN channel: The BEP of DQPSK with Gray coding over an AWGN channel is [4]:

$$P_b(\gamma) = Q(a\sqrt{\gamma}, b\sqrt{\gamma}) - 1/2 I_0(ab\gamma)e^{-(a^2+b^2)\gamma/2} \tag{1}$$

where $a = \sqrt{2-\sqrt{2}}$, $b = \sqrt{2+\sqrt{2}}$, $Q(\alpha, \beta)$ is the Marcum Q-function, and γ is the bit signal-to-noise ratio (SNR). Recently, very tight bounds of $Q(\alpha, \beta)$ are proposed, which improve the well-known tight bounds proposed in [5]. More precisely, Wang showed that

$$Q(\alpha, \beta) \geq \sqrt{\frac{\pi}{2}}\frac{\beta I_0(\alpha\beta)}{2\sinh(\alpha\beta)}\left[\operatorname{erfc}\left(\frac{\beta-\alpha}{\sqrt{2}}\right) - \operatorname{erfc}\left(\frac{\beta+\alpha}{\sqrt{2}}\right)\right] \tag{2}$$

while Baricz and Sun which was generalised in [6, Theorem 1], proved that

$$Q(\alpha, \beta) \leq \frac{I_0(\alpha\beta)}{2\cosh(\alpha\beta)}\left\{e^{-(\beta-\alpha)^2/2} + e^{-(\beta+\alpha)^2/2} + \alpha\sqrt{\frac{\pi}{2}}\left[\operatorname{erfc}\left(\frac{\beta-\alpha}{\sqrt{2}}\right) - \operatorname{erfc}\left(\frac{\beta+\alpha}{\sqrt{2}}\right)\right]\right\} \tag{3}$$

where both inequalities $\beta \geq \alpha > 0$. Substituting (2) and (3) into (1), we derive

$$P_b(\gamma) \geq I_0(ab\gamma)\left\{\sqrt{\frac{\pi}{2}}\frac{b\sqrt{\gamma}}{2\sinh(ab\gamma)}\left[\operatorname{erfc}\left((b-a)\sqrt{\gamma/2}\right) - \operatorname{erfc}\left((b+a)\sqrt{\gamma/2}\right)\right] - 1/2 e^{-(a^2+b^2)\gamma/2}\right\} \tag{4}$$

$$P_b(\gamma) \leq I_0(ab\gamma)\left\{\sqrt{\frac{\pi}{2}}\frac{a\sqrt{\gamma}}{2\cosh(ab\gamma)}\left[\operatorname{erfc}\left((b-a)\sqrt{\gamma/2}\right) - \operatorname{erfc}\left((b+a)\sqrt{\gamma/2}\right)\right] + 1/2 e^{-(a^2+b^2)\gamma/2}\right\} \tag{5}$$

We now approximate $I_0(x)$, in order to get an approximate BEP involving only exponential and erfc functions. It is know that $I_{0.5}(x)$ and $I_{-0.5}(x)$ have the same asymptote as $I_0(x)$ as $x \to \infty$. They have closed-form expressions, i.e.

$$I_{0.5}(x) = \sqrt{2/\pi x}\sinh(x), \quad I_{-0.5}(x) = \sqrt{2/\pi x}\cosh(x) \tag{6}$$

which can be used to eliminate the denominators of the first terms on the right side of (4) and (5). Substituting $I_0(x)$ with $I_{0.5}(x)$ and $I_{-0.5}(x)$ in (4) and (5), respectively, and summing the results, we have

$$P_b(\gamma) \simeq 1/\sqrt{8\pi ab\gamma}e^{-(a+b)^2\gamma/2} + 1/4\left(\sqrt{a/b} + \sqrt{b/a}\right)\left[\operatorname{erfc}\left((b-a)\sqrt{\gamma/2}\right) - \operatorname{erfc}\left((b+a)\sqrt{\gamma/2}\right)\right] \tag{7}$$

Fig. 1 shows that (7) is more accurate than previous exponential and erfc-type bounds [1, Eqn. 17] and [2, Eqn. 16]. The result of (7) is of similar accuracy with the I_0-type approximation [3, Eqn. 7] which is not proper for further analytical integration.

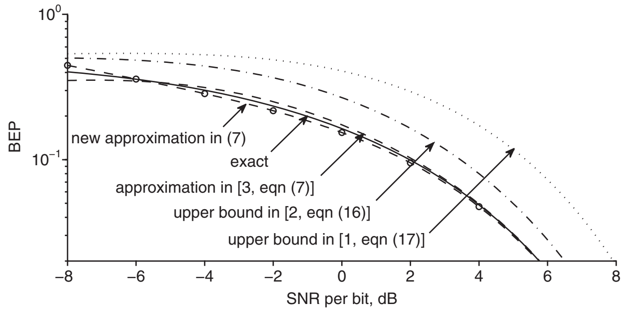

Fig. 1 *Approximations for BEP of DQPSK with Gray coding over AWGN channel*

Approximate average BEP for fading channels: The average BEP for DQPSK with Gray coding over a fading channel is given by

$$\bar{P}_b = \int_0^\infty p_\gamma(\gamma)P_b(\gamma)d\gamma \tag{8}$$

In the following we present approximate results of (8) based on the approximate BEP (7) for an AWGN channel.

For Nakagami-m fading [4, Chap. 2], we can utilise [4, Eqn. (5.18)] and [7, Eqn. 8.310.1] to derive the approximated average BEP, i.e.

$$\bar{P}_b \simeq \frac{(1+c_2)^{1/2-m}\Gamma(m-1/2)}{4\sqrt{\pi c_3}\Gamma(m)} + \frac{\Gamma(m+1/2)}{4\sqrt{\pi}\Gamma(m+1)}\left(\sqrt{a/b}+\sqrt{b/a}\right)\left\{\frac{\sqrt{c_1}}{(1+c_1)^{m+1/2}}\,{}_2F_1\left(1, m+\frac{1}{2}; m+1; \frac{1}{1+c_1}\right) - \frac{\sqrt{c_2}}{(1+c_2)^{m+1/2}}\,{}_2F_1\left(1, m+\frac{1}{2}; m+1; \frac{1}{1+c_2}\right)\right\} \tag{9}$$

where m is the Nakagami-m fading parameter, $c_1 = (b-a)^2\bar{\gamma}/2m$, $c_2 = (b+a)^2\bar{\gamma}/2m$, $c_3 = ab\bar{\gamma}/2m$ and ${}_2F_1$ is the Gaussian hypergeometric function [7, Eqn. 9.100]. We note that our result (9) is valid for any real $m \geq 0.5$. If m is limited to an integer, (9) will reduce to a finite series of algebraic functions by using the integer version of [4, Eqn. (5.18)] and [7, Eqn. 8.339.2].

By using the integral result of [2], we obtain an approximation of (8) for the Rician fading channels [4, Chap. 2]

$$\bar{P}_b \simeq \sqrt{(1+K)/[8ab\bar{\gamma}(1+d_1)]}w(d_1) + 1/2\left(\sqrt{a/b}+\sqrt{b/a}\right)\left\{Q[u(d_2), v(d_2)] - w(d_2)/2\left[1+\sqrt{d_2/(1+d_2)}\right] - Q[u(d_1), v(d_1)] + w(d_1)/2\left[1+\sqrt{d_1/(1+d_1)}\right]\right\} \tag{10}$$

where $K \geq 0$ is the Rician K factor, $d_1 = (b+a)^2\bar{\gamma}/(2+2K)$, $d_2 = (b-a)^2\bar{\gamma}/(2+2K)$,

$$u(x) = \sqrt{K[1+2x-2\sqrt{x(1+x)}]/(2+2x)}$$

$$v(x) = \sqrt{K[1+2x+2\sqrt{x(1+x)}]/(2+2x)}$$

$$w(x) = e^{-K(1+2x)/(2+2x)}I_0[K/(2+2x)]$$

The approximate BEP (7) can also be averaged over the Nakagami-q (Hoyt) fading channel by using [8, Eqn. (21)], and [7, Eqn. 6.621.1] where the Bessel function $J_\nu(x)$ needs to be changed to $i^\nu I_\nu(-ix)$. The final result is omitted here owing to space limitation. The numerical results in Fig. 2 suggest that our approximate average BEP (9) and (10) are much more accurate than the upper bounds given in [1] and [2].

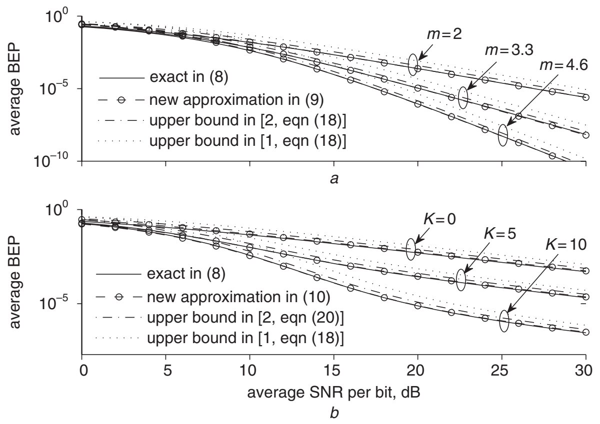

Fig. 2 *Average BEP (8) and our new approximations (9) and (10) against* $\bar{\gamma}$ *for Nakagami fading and Rician fading*

a Nakagami fading
b Rician fading

Conclusion: We have derived novel approximations for the BEP of DQPSK with Gray coding over fading channels. The numerical results show that our expressions are much more accurate than previous results.

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