Ergodic capacity of decode-and-forward relay strategies over general fast fading channels

Y. Sun, X. Zhong, X. Chen, S. Zhou and J. Wang

Proposed is a novel ergodic capacity formula for decode-and-forward (DF) relaying over a very general fast fading channel model, i.e. the channel gains of the links follow independent but non-necessarily identically distributed Nakagami-*m* distributions. The proposed formula is validated by Monte Carlo simulations and can be computed efficiently. Since the considered channel model is very general, the proposed formula is appropriate for evaluating the performance of DF relaying in a wide variety of realistic wireless channel environments.

Introduction: Decode-and-forward (DF) relaying [1] is a low-cost technique to extend coverage and combat channel fading in current and future wireless networks. The performance of DF relay protocols over Nakagami-*m* slow fading channels has been analysed intensively in terms of outage probability and error rate (e.g. [2–4]); however, these results are not appropriate for fast fading scenarios. On the other hand, there have been few studies on the ergodic capacity of DF relay strategies over Nakagami-*m* fast fading channels. The Nakagami-*m* channel fading model is very general. It captures a wide variety of realistic wireless channel environments [5]. In [4], the average throughput of adaptive DF relaying over Nakagami-*m* fading channels was studied, which assumed certain modulation type for the source-relay link. In [6], the authors analysed the ergodic capacity of DF relay protocols over general fading channels, which was evaluated using Monte Carlo simulations. Ergodic capacity and average throughput of DF relay strategies over Rayleigh fading channels were considered in [6–9]. In this Letter, we propose a novel ergodic capacity formula for DF relaying over independent but non-necessarily identically distributed (INID) Nakagami-*m* fast fading channels.

Ergodic capacity formula: We consider a half-duplex DF relay strategy where the source node sends a message to the relay or destination in the broadcast (Bc) period, then the source and relay transmit to the destination asynchronously in the multiple-access (MAC) period, as depicted in Fig. 1. The transmission time fractionals of the BC and MAC periods are given by α and $1-\alpha$ ($\alpha \in [0, 1]$), respectively. Let $P_{s,1}$ ($P_{s,2}$) be the transmitted power density of the source node in the BC (MAC) period, P_r represents the transmitted power density of the relay node. The complex channel gains of the source-relay (SR), relay-destination (RD) and source- destination (SD) links are denoted by h_1, h_2 and h_3, respectively. We assume that the channel envelope gain $|h_i|$ ($i \in \{1-3\}$) satisfies INID Nakagami-*m* fast fading. Then, the normalised channel power gain $g_i \triangleq |h_i|^2/N_0$ follows a gamma distribution with parameters (m_i, λ_i) [5], where N_0 is the noise power density, m_i is the Nakagami-*m* fading parameter satisfying $m_i \geq 0.5$ and $\lambda_i = m_i/\mathbb{E}\{g_i\} = m_i N_0/\mathbb{E}\{|h_i|^2\}$. The probability density function (PDF) of g_i is

$$f_{G_i}(g_i) = \frac{\lambda_i^{m_i}}{\Gamma(m_i)} g_i^{m_i-1} e^{-\lambda_i g_i}, \; g_i \geq 0 \qquad (1)$$

where $\Gamma(\cdot)$ is the Euler gamma function [10, equation (8.310.1)].

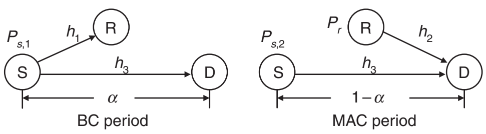

Fig. 1 *Half-duplex relay channel model*

In [6], an ergodic data rate formula of DF relaying is proposed in equation (45) for the case where the instantaneous channel state information (CSI) of all three channel links is known at all the nodes. If the instantaneous CSI is only known at the receivers of corresponding links and the data rate is determined by channel statistics, the ergodic data rate of the above DF relay strategy is described as

$$\begin{aligned} \frac{C_{DF}}{W} = \min\{&\alpha \max\{\mathbb{E}[\ln(1+P_{s,1}g_1)], \mathbb{E}[\ln(1+P_{s,1}g_3)]\} \\ &+ (1-\alpha)\mathbb{E}[\ln(1+P_{s,2}g_3)], \alpha\mathbb{E}[\ln(1+P_{s,1}g_3)] \\ &+ (1-\alpha)\mathbb{E}[\ln(1+P_{s,2}g_3+P_r g_2)]\} \end{aligned} \qquad (2)$$

where W is the bandwidth of each channel link. One can show that (2) is the ergodic capacity of a *degraded* or *reversely degraded* half-duplex relay channel [1, 6–8]. Moreover, it is interesting that (2) is no smaller than the ergodic capacity of the direct transmission (DT) on the SD link, i.e.

$$\frac{C_{DT}}{W} = \alpha\mathbb{E}[\ln(1+P_{s,1}g_3)] + (1-\alpha)\mathbb{E}[\ln(1+P_{s,2}g_3)] \qquad (3)$$

While most terms in (2) can be expressed by the integration $\mathcal{I}(m, \lambda)$ defined by

$$\mathcal{I}(m, \lambda) = \int_0^\infty \ln(1+x) \frac{\lambda^m}{\Gamma(m)} x^{m-1} e^{-\lambda x} dx, \; \lambda > 0 \qquad (4)$$

the last term of (2) involves the sum of two INID gamma random variables.

Let $X_i (i \in \{1, 2, \ldots, N\})$ be INID gamma random variables with parameters (r_i, μ_i). Moschopoulos [11] showed that $Y = \sum_{i=1}^{N} X_i$ follows the mixture of a series of gamma distributions. More specifically, the PDF of Y is determined as

$$f_Y(y) = A \sum_{k=0}^{\infty} \delta_k \frac{\mu^{\rho+k}}{\Gamma(\rho+k)} y^{\rho+k-1} e^{-\mu y}, \; y \geq 0 \qquad (5)$$

where $\mu = \max_i \mu_i$, $A = \prod_{i=1}^{N} (\mu_i/\mu)^{r_i}$, $\rho = \sum_{i=1}^{N} r_i$ and the coefficient $\delta_k > 0$ is determined by the following recursive relations

$$\begin{cases} \delta_0 = 1 \\ \delta_{k+1} = \dfrac{1}{k+1} \displaystyle\sum_{i=1}^{k+1} \left[\sum_{j=1}^{N} r_j \left(1 - \frac{\mu_j}{\mu}\right)^i \right] \delta_{k+1-i} \end{cases} \qquad (6)$$

Therefore, the ergodic data rate of DF relay strategy is given by

$$\begin{aligned} \frac{C_{DF}}{W} = \min\{&\alpha \max[\mathcal{I}(m_1, \lambda_1/P_{s,1}), \mathcal{I}(m_3, \lambda_3/P_{s,1})] \\ &+ (1-\alpha)[\mathcal{I}(m_3, \lambda_3/P_{s,2}), \alpha\mathcal{I}(m_3, \lambda_3/P_{s,1})] \\ &+ (1-\alpha)A \sum_{k=0}^{\infty} \delta_k \mathcal{I}(\rho+k, \mu)\}, \; m_i, \geq 0.5 \end{aligned} \qquad (7)$$

which is derived by using the uniform convergence of (5) [11], and by letting $N = 2$, $(r_1, \mu_1) = (m_3, \lambda_3/P_{s,2})$ and $(r_2, \mu_2) = (m_2, \lambda_2/P_r)$. For all practical purposes, one can use the first l terms of the series in (7), where l is chosen to achieve the required accuracy.

By using [10, equation 9.121.6] and [10, equation 7.522.5], we obtain one expression of $\mathcal{I}(m, \lambda)$, i.e.

$$\mathcal{I}(m, \lambda) = \frac{m}{\lambda} {}_3F_1\left(m+1, 1, 1; 2; -\frac{1}{\lambda}\right), \; \lambda > 0 \qquad (8)$$

where ${}_pF_q$ is the generalised hypergeometric series [10, equation 9.14.1]. We note that (8) holds for any choice of the Nakagami fading parameter $m > 0.5$. One alternative expression of (8) exists in [12, equation (43)].

Equation (7) can be generalised to produce the ergodic capacity formula of the orthogonal multiple DF relay network [2] and the full-duplex multiple DF relay network [8] over Nakagami-*m* fast fading channels. The detailed results are omitted here owing to space limitation.

We now present the numerical results. In practical scenarios, the SR and RD links may have better channel quality than the SD link, because of short communication distance and possible strong line-of-sight (LOS) channel components. In our numerical computations, we assume that $P_{s,1} = P_{s,2}$ and the average signal-to-noise ratios (SNR) of the SR and RD links, i.e. $P_{s,1}\mathbb{E}\{g_1\}$ and $P_r\mathbb{E}\{g_2\}$, are both 10 dB higher than that of the SD link, i.e. $P_{s,1}\mathbb{E}\{g_3\}$. Let the Nakagami fading parameters be $m_1 = 4.3$, $m_2 = 5.6$ and $m_3 = 1$, and the time fraction of BC period be $\alpha = 0.6$. The numerical and Monte Carlo simulation results of C_{DF}/W and C_{DT}/W are illustrated in Fig. 2, where we use the first $l = 22$ terms of the series in (7) to evaluate C_{DF}/W. The proposed formula (7) is validated by our Monte Carlo simulations and can be computed efficiently.

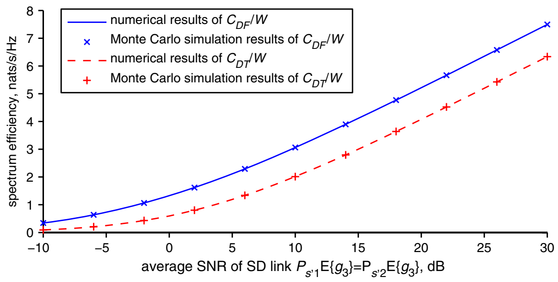

Fig. 2 *Spectrum efficiency of DF relay strategy and direction SD transmission*

Conclusion: A novel ergodic capacity formula is proposed for decode-and-forward (DF) relaying over INID Nakagami-*m* fast fading channels. Since the considered channel model is very general, the proposed formula is appropriate for evaluating the performance of DF relay strategies in a wide variety of realistic wireless channel environments.

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One or more of the Figures in this Letter are available in colour online.

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