and can be greatly narrowed with approaches such as the turbo detection.

VI. Conclusion

This paper has proposed the FIC algorithm to remove the IRI at the destination in the two-path cooperative relay system. Compared with the original PIC approach, the FIC not only performs better, but it is less complex. It is thus an attractive interference cancellation scheme for the two-path relay system.

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Corrections to "Unified Laguerre Polynomial-Series-Based Distribution of Small-Scale Fading Envelopes"

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***Abstract*—In this correspondence, we point out two typographical errors in Chai and Tjhung's paper, and we offer the correct formula of the unified Laguerre polynomial-series-based cumulative distribution function (cdf) for small-scale fading distributions. A Laguerre polynomial-series-based cdf formula for noncentral chi-square distribution is also provided as a special case of our unified cdf result.**

***Index Terms*—Generalized Laguerre polynomial series expansion, noncentral chi-square distribution, small-scale fading, unified distribution.**

I. Two Typographical Errors in Chai and Tjhung's Paper

In an interesting paper, Chai and Tjhung [1] proposed new unified probability density function (pdf) and cumulative distribution function (cdf) formulas based on the generalized Laguerre polynomial series expansion that cover a wide range of small-scale fading distributions in wireless communications. Many known Laguerre polynomial-series-based small-scale fading distributions are special cases of their results, which include the multiple-waves-plus-diffuse-power fading, $\kappa - \mu$ (noncentral chi), Nakagami–m, Rician (Nakagami–n), Nakagami–q (Hoyt), Rayleigh, Weibull, and $\alpha - \mu$ (Stacy) distributions.

First, we present two typographical errors in [1].

1) Chai and Tjhung stated that the orthogonality relation of generalized Laguerre polynomial is expressed as [1, eq. (2)]

$$\int_0^\infty e^{-x} L_l^\beta(x) L_n^\beta(x) dx = \begin{cases} \Gamma(1+\beta)\binom{n+\beta}{n}, & \text{if } n = l \\ 0, & \text{if } n \neq l \end{cases}$$

where L_n^μ is the generalized Laguerre polynomial, as defined in [2, eq. (22.3.9)]

$$L_n^\mu(x) = \frac{e^x x^{-\mu}}{n!} \frac{d^n}{dx^n}(e^{-x} x^{n+\mu}) = \sum_{k=0}^{n} \binom{n+\mu}{n-k} \frac{(-x)^k}{k!}$$
$$\mu > -1, n = 0, 1, 2, \ldots.$$

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However, there is a typographical error in [1, eq. (2)]. The correct formula is

$$\int_0^\infty x^\beta e^{-x} L_l^\beta(x) L_n^\beta(x)dx = \begin{cases} \Gamma(1+\beta)\binom{n+\beta}{n}, & \text{if } n = l \\ 0, & \text{if } n \neq l. \end{cases}$$

2) The Laguerre polynomial expansion coefficients C_n for the pdf $f_X(x)$ of random variable X was determined by [1, eq. (10)]. The fading envelop random variable R is assumed to satisfy

$$x = r^\alpha / b$$

where α and b are parameters. Chai and Tjhung claimed that [1, p. 3990, line 9]

$$E_X[x^k] = E_R[r^{\alpha k}]/b$$

which contains a small typographical error. The correct expression is given by

$$E_X[x^k] = E_R[r^{\alpha k}]/b^k.$$

Using this and [1, eq. (10)], the coefficients C_n can be expressed by the αkth moment of the fading envelope R.

II. Correct Unified Cumulative Distribution Function Formula

Chai and Tjhung presented a unified cdf formula for the fading envelope R in [1, eq. (13)], i.e.,

$$\begin{aligned} F_R(R) &= \int_0^R f_R(r)dr \\ &= \frac{\alpha}{b^{\beta+1}} \sum_{n=0}^\infty C_n \frac{n!}{\Gamma(n+\beta+1)} \\ &\quad \times \int_0^R \exp\left(-\frac{r^\alpha}{b}\right) r^{\alpha(\beta+1)-1} L_n^\beta\left(\frac{r^\alpha}{b}\right) dr \\ &= \frac{1}{n}\left(\frac{R^\alpha}{b}\right)^{\beta+1} \exp\left(-\frac{R^\alpha}{b}\right) \\ &\quad \times \sum_{n=0}^\infty C_n \frac{n!}{\Gamma(n+\beta+1)} L_{n-1}^{\beta+1}\left(\frac{R^\alpha}{b}\right) \end{aligned} \quad (1)$$

where α, β, and b are some parameters, depending on the type of fading distribution, and are presented in [1, Sec. V] for some special cases.

Unfortunately, the third line of the formula (1) is not valid, because $(1/n)$ is outside of the summation, and the generalized Laguerre polynomial $L_{n-1}^{\beta+1}$ is not defined for $n = 0$. The correct formula of the unified cdf is

$$\begin{aligned} F_R(R) &= \left(\frac{R^\alpha}{b}\right)^{\beta+1} \exp\left(-\frac{R^\alpha}{b}\right) \\ &\quad \times \sum_{n=1}^\infty C_n \frac{\Gamma(n)}{\Gamma(n+\beta+1)} L_{n-1}^{\beta+1}\left(\frac{R^\alpha}{b}\right) \\ &\quad + \frac{1}{\Gamma(\beta+1)} \gamma\left(\beta+1, \frac{R^\alpha}{b}\right) \end{aligned} \quad (2)$$

where we used that $L_0^\mu(x) = 1$ and $C_0 = 1$. Here, $\gamma(\cdot,\cdot)$ is the lower incomplete gamma function defined by [2, eq. (6.5.2)]

$$\gamma(a, x) = \int_0^x t^{a-1} e^{-t} dt.$$

Therefore, the unified Laguerre polynomial-series-based cdf contains one term expressed with the aid of the lower incomplete gamma function.

III. One Special Case of the Unified cdf Formula (2)

We now provide a Laguerre polynomial-series-based cdf formula for noncentral chi-square distribution as a special case of (2). The pdf of noncentral chi-square distribution $\chi^2_{\nu,\lambda}$ with ν degrees of freedom and noncentrality parameter λ is given by [3, eq. (29.4)]

$$f_{\chi^2_{\nu,\lambda}}(r) = \frac{e^{-(r+\lambda)/2}}{2}\left(\frac{r}{\lambda}\right)^{\nu/4-1/2} I_{\nu/2-1}(\sqrt{\lambda r})$$

which is equivalent to the pdf of $\kappa - \mu$ power distribution defined by [4, eq. (2)]. Tiku proposed the Laguerre polynomial expansion of $f_{\chi^2_{\nu,\lambda}}(r)$ in 1965 [5], which is given by (see also [3, eq. (29.11)])

$$f_{\chi^2_{\nu,\lambda}}(r) = \frac{e^{-r/2}}{2}\left(\frac{1}{2}r\right)^{\left(\frac{\nu}{2}-1\right)} \sum_{j=0}^\infty \frac{\left(-\frac{\lambda}{2}\right)^j}{\Gamma\left(\frac{1}{2}\nu + j\right)} L_j^{\frac{\nu}{2}-1}\left(\frac{1}{2}r\right)$$
$$\nu > 0, \lambda \geq 0. \quad (3)$$

Gideon and Gurland obtained a Laguerre polynomial expansion of the cdf of noncentral chi-square distribution in 1977 [6] (see also [3, eq. (29.13)] with the parameter set of L_0), i.e.,

$$\begin{aligned} F_{\chi^2_{\nu,\lambda}}(R) &= \int_0^R f_{\chi^2_{\nu,\lambda}}(r)dr \\ &= \left(\frac{R}{2}\right)^{\frac{\nu}{2}} \exp\left(-\frac{R}{2}\right) \sum_{n=1}^\infty \frac{\left(-\frac{\lambda}{2}\right)^n}{\Gamma(n+\nu/2)n} L_{n-1}^{\frac{\nu}{2}}\left(\frac{R}{2}\right) \\ &\quad + \frac{1}{\Gamma(\nu/2)} \gamma\left(\frac{\nu}{2}, \frac{R}{2}\right), \quad \nu > 0, \lambda \geq 0. \end{aligned} \quad (4)$$

Some other Laguerre polynomial expansions of $F_{\chi^2_{\nu,\lambda}}(R)$ are available in [6] and [3, eq. (29.13)]. We note that the pdf and cdf formulas in (3) and (4) can be derived by choosing $\alpha = 1$, $b = 2$, $\beta = \nu/2 - 1$, and $C_n = (-\lambda/2)^n/n!$ in [1, eq. (11)] and (2) in the previous section.

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On the Power of MIMO Broadcast Systems Under SNR Constraints With Limited Feedback

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***Abstract*—The power problem of joint zero-forcing beamforming and scheduling is addressed in a multiple-input–multiple-output (MIMO) broadcast channel under individual signal-to-noise ratio (SNR) constraints. Perfect channel knowledge is assumed to be available for each single antenna user, whereas the base station (BS) with multiple antennas is provided with partial channel-state information through limited-rate feedback channels. Each user can feed back its quantized channel direction information and needed power information. Only the user with the minimum power is selected to communicate with the BS. Expressions for the average minimum power used to transmit are derived, and tradeoffs between the number of feedback bits, the number of users, and the number of transmit antennas are observed.**

***Index Terms*—Broadcast channel, finite-rate feedback, multiple-input–multiple-output (MIMO) system, multiuser, power.**

I. Introduction

Multiple-input multiple-output (MIMO) with limited feedback has drawn considerable attention in the academic and industrial communities over the past few years and has been considered to be one of the main features of future wireless communications [1]–[3]. Instead of single-user systems [2], multiuser systems nowadays have been the focus point of MIMO theoretical analysis [3].

Regarding multiuser MIMO systems, recent advances show that, with perfect channel-state information at transmitter (CSIT), dirty-paper coding can achieve the whole capacity region of MIMO broadcast channels [4]. However, such a nonlinear coding approach is complicated, and perfect CSIT is generally very difficult to obtain in practice. This motivates various works on MIMO broadcast channels with partial CSIT, e.g., simple yet efficient linear beamforming approaches are deeply analyzed in [5]–[8]. In particular, Yoo *et al.* [6] investigated the sum-rate performance of MIMO broadcast systems with user selection under quantized channel-direction information (CDI) and unquantized channel-quality information (CQI) feedbacks. Shao and Yuan [7] extended the scheme in [6] into a scenario with both CDI and CQI quantization and derived a lower bound to the sum rate. It should be noticed that both [6] and [7] made use of the following pursuing strategy: maximizing the system throughput under a sum power constraint (or individual power constraints). In addition to the two papers, many other MIMO papers, including the latest, i.e., [2], [10], and [11], are also limited to employing this strategy.

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In this paper, we will consider a multiuser MIMO downlink system with both quantized CDI and CQI feedbacks but pursue another optimal strategy: minimizing the total transmit power under individual signal-to-interference-plus-noise-ratio (SINR) constraints (or individual rate constraints) [9]. For simplicity, we assume that only one of all users is selected and allowed to communicate with the base station (BS) at each time. We will use the user power information as CQI. This is obviously different from [6]–[8], in which CQI is based on the user SINR. We will show that multiuser systems have the following two advantages over single-user systems:

As the number of users reaches infinity, 1) the average transmit power with quantized CDI feedback will go to that with unquantized CDI feedback, and 2) the system outage probability will go to zero.

This paper is structured as follows: Section II briefly describes the multiuser MIMO downlink system with two kinds of feedbacks and formulates the user-selection problem. Section III gives the minimum power analysis under quantized CDI and unquantized CQI feedbacks. Section IV considers the minimum power problem under both quantized CDI and quantized CQI feedbacks. Section V provides experimental data. Finally, Section VI draws conclusions.

II. System Model

Consider a single-cell MIMO broadcast channel with M transmit antennas at BS and K users, each having a single receive antenna. We assume that all of these users are homogeneous and experience flat Rayleigh fading. The signal received by a user k can be represented as

$$y_k = \mathbf{h}_k \mathbf{x} + z_k, \qquad k = 1, 2, \ldots, K \tag{1}$$

where $\mathbf{h}_k \in \mathbb{C}^{1\times M}$ is the channel gain vector with zero-mean unit variance independent and identically distributed (i.i.d.) complex Gaussian entries, $\mathbf{x}$ is the transmitted symbol vector containing information symbols to a selected set of users, z_k is the additive Gaussian noise with zero mean and unit variance, and y_k is the symbol received by user k. Denote the set of indices of those selected users as $\mathcal{S}$ and its cardinality $|\mathcal{S}|, 1 \le \phi_{\mathcal{S}} \le M$ as $\phi_{\mathcal{S}}$. The transmit symbol vector $\mathbf{x}$ is related to information symbols $\{s_i : i \in \mathcal{S}\}$ via linear beamforming $\mathbf{x} = \sum_{i\in\mathcal{S}} \mathbf{w}_i s_i$. Therefore, the corresponding received symbol is given by

$$y_k = (\mathbf{h}_k \mathbf{w}_k) s_k + \sum_{i\in\mathcal{S}, i\neq k} (\mathbf{h}_k \mathbf{w}_i) s_i + z_k, \qquad k \in \mathcal{S}. \tag{2}$$

The set of indices of those selected users $\mathcal{S}$ is chosen to minimize the total power. For simplicity, we will only consider the case where $\phi_{\mathcal{S}} = 1$. Our system scheme with user selection is outlined as follows.

1) For the unquantized CQI system, each user first estimates its own needed power and feeds back the unquantized value to the BS, whereas, for the quantized CQI system, the user feeds back a quantized power value only when its needed power is not larger than a given threshold.
2) The BS selects one user with the minimum power to communicate with itself.
3) The selected user feeds back its quantized CDI to the BS.