Age-Optimal Information Updates in Multihop Networks

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***Abstract*—The problem of reducing the age-of-information has been extensively studied in single-hop networks. In this paper, we minimize the age-of-information in general multihop networks. If the packet transmission times over the network links are exponentially distributed, we prove that a preemptive Last Generated First Served (LGFS) policy results in smaller age processes at all nodes of the network (in a stochastic ordering sense) than any other causal policy. In addition, for arbitrary distributions of packet transmission times, the non-preemptive LGFS policy is shown to minimize the age processes at all nodes among all non-preemptive work-conserving policies (again in a stochastic ordering sense). It is surprising that such simple policies can achieve optimality of the joint distribution of the age processes at all nodes even under arbitrary network topologies, as well as arbitrary packet generation and arrival times. These optimality results not only hold for the age processes, but also for any non-decreasing functional of the age processes.**

I. Introduction

There has been a growing interest in applications that require real-time information updates, such as news, weather reports, email notifications, stock quotes, social updates, mobile ads, etc. The freshness of the information is also crucial in other systems, e.g., monitoring systems that obtain information from environmental sensors, wireless systems that need rapid updates of channel state information, etc.

As a metric of data freshness, the *age-of-information*, or simply *age*, was defined in [1]–[4]. At time t, if the freshest update at the destination was generated at time $U(t)$, the age $\Delta(t)$ is defined as $\Delta(t) = t - U(t)$. Hence, age is the time elapsed since the freshest packet was generated.

There are a number of studies that have focused on reducing the age in single-hop networks [4]–[11]. In [4]–[6], the update generation rate was optimized to improve data freshness for First-Come First-Served (FCFS) queueing systems. In [7], [8], it was found that age can be improved by discarding old packets waiting in the queue when a new sample arrives. In [9], [10], the time-average age was characterized for Last-Come First-Served (LCFS) systems with exponential and gamma service time distributions, respectively. The work in [11] analyzed the average peak age in the presence of errors.

Age-optimal generation of update packets was studied for single-hop networks in [12]–[15]. A general class of non-negative, non-decreasing age penalty functions was minimized in [14], [15]. In [16], it was shown that for arbitrary packet generation times, arrival times, and queue buffer size, a preemptive Last Generated First Served (LGFS) policy simultaneously optimizes the age, throughput, and delay in multi-server single-hop networks with exponential service times. The age-of-information was characterized in multihop networks with special topologies in [17]. More recently, a real-time sampling problem of the Wiener process is solved in [18]: If the sampling times are independent of the observed Wiener process, the optimal sampling problem in [18] reduces to an age-of-information optimization problem; otherwise, the optimal sampling policy can use knowledge of the Wiener process to achieve better performance than age-of-information optimization.

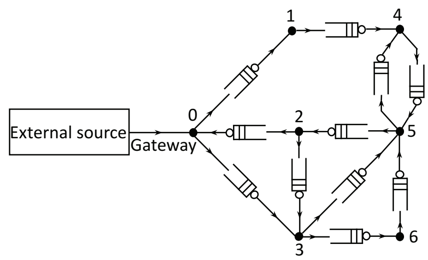

Figure 1: Information updates in a multihop network.

In this paper, we consider a general multihop network, where the update packets are generated at an external source and are then dispersed throughout the network via a gateway node, as shown in Fig. 1. It is well known that delay-optimality is notoriously difficult in multihop networks, except for some special network settings (e.g., tandem networks) [19], [20]. This difficulty stems from the fact that the packet scheduling decisions at each hop are influenced by the decisions on other hops and vise versa. Somewhat to our surprise, it turns out that age minimization has very different features from delay minimization. In particular, we find that some simple policies can achieve optimality of the joint distribution of the age processes at all nodes, even under arbitrary network topologies. The following summarizes our main contributions in this paper:

- We consider a general multihop network where the update packets do not necessarily arrive to the gateway node in the order of their generation times. We prove that, if the packet transmission times over the network links are exponentially distributed, then for arbitrary arrival process, network topology, and buffer sizes, the preemptive LGFS policy minimizes the age processes at all nodes in the

This work has been supported in part by ONR grants N00014-17-1-2417 and N00014-15-1-2166, Army Research Office grants W911NF-14-1-0368 and MURI W911NF-12-1-0385, National Science Foundation grants CNS-1446582, CNS-1421576, and CNS-1518829, and a grant from the Defense Thrust Reduction Agency HDTRA1-14-1-0058.

network among all causal policies in a stochastic ordering sense (Theorem 1). In other words, the preemptive LGFS policy minimizes any *non-decreasing functional of the age processes at all nodes* in a stochastic ordering sense. Note that this age penalty model is very general. Many age penalty metrics studied in the literature, such as the time-average age [4], [5], [7]–[10], [12], [13], average peak age [6], [7], [10]–[12], and average age penalty function [14], [15], are special cases of the general age functional model that we consider in this paper.

- We then prove that, for arbitrary distributions of packet transmission times, the non-preemptive LGFS policy minimizes the age processes at all nodes among all non-preemptive work-conserving policies in the sense of stochastic ordering (Theorem 2). It is interesting to note that age-optimality here can be achieved even if the transmission time distribution differs from one link to another, i.e., the transmission time distributions are heterogeneous.

To the best of our knowledge, these are the first optimal results on minimizing the age-of-information in multihop networks.

II. Model and Formulation

A. Notations and Definitions

For any random variable Z and an event A, let $[Z|A]$ denote a random variable with the conditional distribution of Z for given A, and $\mathbb{E}[Z|A]$ denote the conditional expectation of Z for given A.

Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and $\mathbf{y} = (y_1, y_2, \ldots, y_n)$ be two vectors in $\mathbb{R}^n$, then we denote $\mathbf{x} \leq \mathbf{y}$ if $x_i \leq y_i$ for $i = 1, 2, \ldots, n$. A set $U \subseteq \mathbb{R}^n$ is called upper if $\mathbf{y} \in U$ whenever $\mathbf{y} \geq \mathbf{x}$ and $\mathbf{x} \in U$. We will need the following definitions:

Definition 1. Univariate Stochastic Ordering: [21] Let X and Y be two random variables. Then, X is said to be stochastically smaller than Y (denoted as $X \leq_{\text{st}} Y$), if

$$\mathbb{P}\{X > x\} \leq \mathbb{P}\{Y > x\}, \quad \forall x \in \mathbb{R}.$$

Definition 2. Multivariate Stochastic Ordering: [21] Let $\mathbf{X}$ and $\mathbf{Y}$ be two random vectors. Then, $\mathbf{X}$ is said to be stochastically smaller than $\mathbf{Y}$ (denoted as $\mathbf{X} \leq_{\text{st}} \mathbf{Y}$), if

$$\mathbb{P}\{\mathbf{X} \in U\} \leq \mathbb{P}\{\mathbf{Y} \in U\}, \quad \text{for all upper sets} \quad U \subseteq \mathbb{R}^n.$$

Definition 3. Stochastic Ordering of Stochastic Processes: [21] Let $\{X(t), t \in [0, \infty)\}$ and $\{Y(t), t \in [0, \infty)\}$ be two stochastic processes. Then, $\{X(t), t \in [0, \infty)\}$ is said to be stochastically smaller than $\{Y(t), t \in [0, \infty)\}$ (denoted by $\{X(t), t \in [0, \infty)\} \leq_{\text{st}} \{Y(t), t \in [0, \infty)\}$), if, for all choices of an integer n and $t_1 < t_2 < \ldots < t_n$ in $[0, \infty)$, it holds that

$$(X(t_1), X(t_2), \ldots, X(t_n)) \leq_{\text{st}} (Y(t_1), Y(t_2), \ldots, Y(t_n)), \quad (1)$$

where the multivariate stochastic ordering in (1) was defined in Definition 2.

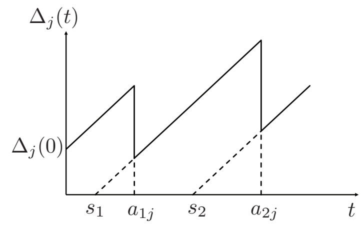

Figure 2: Sample path of the age process $\Delta_j(t)$ at node j.

B. Network Model

We consider a general multihop network represented by a directed graph $\mathcal{G}(\mathcal{V}, \mathcal{L})$ where $\mathcal{V}$ is the set of nodes and $\mathcal{L}$ is the set of links, as shown in Fig. 1. The number of nodes in the network is $|\mathcal{V}| = N$. The update packets are generated at an external source, which is connected to the network through a gateway node 0. The update packets are firstly forwarded to node 0, from which they are dispersed throughout the network. Let $(i, j) \in \mathcal{L}$ denote a link from node i to node j, where i is the origin node and j is the destination node. Once a packet arrives at node i, it is immediately available to all the outgoing links from node i. Each link (i, j) has a queue of buffer size B_{ij} to store the incoming packets. If B_{ij} is finite, the queue buffer may overflow and some packets are dropped. The packet transmission time on each link (i, j) is random.

C. Scheduling Policy

The system starts to operate at time $t = 0$. A sequence of n update packets are generated at the external source, where n can be an arbitrary finite or infinite number. The generation time of the i-th packet is s_i, such that $0 \leq s_1 \leq s_2 \leq \ldots \leq s_n$. We let π denote a scheduling policy that determines when to send the packets on each link and in which order. Define a_{ij} as the arrival time of packet i to node j. Then, $s_i \leq a_{i0} \leq a_{ij}$ for all $j = 1, \ldots, N-1$. The packet generation times $(s_1, s_2, \ldots, s_n)$ and packet arrival times $(a_{10}, a_{20}, \ldots, a_{n0})$ at node 0 are arbitrary given, which are independent of the scheduling policy. Note that the update packets may arrive at node 0 *out of the order* of their generation times. For example, packet $i+1$ may arrive at node 0 earlier than packet i such that $s_i \leq s_{i+1}$ but $a_{i0} \geq a_{(i+1)0}$.

Let Π denote the set of all *causal* policies, in which scheduling decisions are made based on the history and current state of the system. We define several types of policies in Π:

A policy is said to be **preemptive**, if a link can switch to send another packet at any time; the preempted packets will be stored back into the queue if there is enough buffer space and then sent out at a later time when the link is available again. In contrast, in a **non-preemptive** policy, a link must complete sending the current packet before starting to send another packet. A policy is said to be **work-conserving**, if each link is busy whenever there are packets waiting in the queue feeding this link.

D. Age Performance Metric

Let $U_j(t) = \max\{s_i : a_{ij} \leq t\}$ be the generation time of the freshest packet arrived at node j before time t. The *age-of-information*, or simply the *age*, at node j is defined as

$$\Delta_j(t) = t - U_j(t). \quad (2)$$

The initial state $U_j(0^-)$ at time $t = 0^-$ is invariant of the policy $\pi \in \Pi$ for all $j \in \mathcal{V}$. As shown in Fig. 2, the age increases linearly with t but is reset to a smaller value with the arrival of a fresher packet. The age vector of all the network nodes is

$$\boldsymbol{\Delta}(t) = (\Delta_0(t), \Delta_1(t), \ldots, \Delta_{N-1}(t)). \quad (3)$$

The age process of all the network nodes is given by

$$\boldsymbol{\Delta} = \{\boldsymbol{\Delta}(t), t \in [0, \infty)\}. \quad (4)$$

In this paper, we introduce a general *age penalty functional* $g(\boldsymbol{\Delta})$ to represent the level of dissatisfaction for data staleness at all the network nodes.

Definition 4. Age Penalty Functional: Let $\mathbf{V}$ be the set of N-dimensional Lebesgue measurable functions, i.e.,

$$\mathbf{V} = \{f : [0, \infty)^N \mapsto \mathbb{R} \text{ such that } f \text{ is Lebesgue measurable}\}.$$

A functional $g : \mathbf{V} \mapsto \mathbb{R}$ is said to be an *age penalty functional* if g is *non-decreasing* in the following sense:

$$g(\boldsymbol{\Delta}_1) \leq g(\boldsymbol{\Delta}_2), \text{ whenever } \boldsymbol{\Delta}_1(t) \leq \boldsymbol{\Delta}_2(t), \forall t \in [0, \infty). \quad (5)$$

The age penalty functionals used in prior studies include:

- *Time-average age [4], [5], [7]–[10], [12], [13]:* The time-average age of node j is defined as

$$g_1(\boldsymbol{\Delta}) = \frac{1}{T}\int_0^T \Delta_j(t)dt, \quad (6)$$

- *Average peak age [6], [7], [10]–[12]:* The average peak age of node j is defined as

$$g_2(\boldsymbol{\Delta}) = \frac{1}{K}\sum_{k=1}^{K} A_{kj}, \quad (7)$$

  where A_{kj} denotes the k-th peak value of $\Delta_j(t)$ since time $t = 0$.

- *Average age penalty function [14], [15]:* The average age penalty function of node j is

$$g_3(\boldsymbol{\Delta}) = \frac{1}{T}\int_0^T h(\Delta_j(t))dt, \quad (8)$$

  where $h : [0, \infty) \to [0, \infty)$ can be any non-negative and non-decreasing function. As pointed out in [15], a stair-shape function $h(\Delta) = \lfloor \Delta \rfloor$ can be used to characterize the dissatisfaction of data staleness when the information of interests is checked periodically, and an exponential function $h(\Delta)$ is appropriate for online learning and control applications where the desire for data refreshing grows quickly with respect to the age.

III. Age-optimality of LGFS policies

In this section, we present our age-optimality results for general multihop networks. We prove our results in a stochastic ordering sense.

A. Exponential Transmission Time Distributions

We study the age-optimal packet scheduling when the packet transmission times are exponentially distributed, *independent* across the links and *i.i.d.* across time. We consider a LGFS scheduling principle in which the packet being transmitted at each link is generated the latest (i.e., the freshest) one among all packets in the queue; after transmission, the link starts to send the next freshest packet in its queue. We consider a preemptive LGFS (prmp-LGFS) policy at each link $(i, j) \in \mathcal{L}$.

Define a set of parameters $\mathcal{I} = \{n, (s_i, a_{i0})_{i=1}^n, \mathcal{G}(\mathcal{V}, \mathcal{L}), (B_{ij}, (i, j) \in \mathcal{L})\}$, where n is the total number of packets, s_i is the generation time of packet i, a_{i0} is the arrival time of packet i to node 0, $\mathcal{G}(\mathcal{V}, \mathcal{L})$ is the network graph, and B_{ij} is the queue buffer size of link (i, j). Let $\boldsymbol{\Delta}_\pi = \{\boldsymbol{\Delta}_\pi(t), t \in [0, \infty)\}$ be the age processes of all nodes in the network under policy π. The age optimality of prmp-LGFS policy is provided in the following theorem.

Theorem 1. If the packet transmission times are exponentially distributed, *independent* across links and *i.i.d.* across time, then for all $\mathcal{I}$ and $\pi \in \Pi$

$$[\boldsymbol{\Delta}_{\text{prmp-LGFS}}|\mathcal{I}] \leq_{\text{st}} [\boldsymbol{\Delta}_\pi|\mathcal{I}], \quad (9)$$

or equivalently, for all $\mathcal{I}$ and non-decreasing functional g

$$\mathbb{E}[g(\boldsymbol{\Delta}_{\text{prmp-LGFS}})|\mathcal{I}] = \min_{\pi \in \Pi} \mathbb{E}[g(\boldsymbol{\Delta}_\pi)|\mathcal{I}], \quad (10)$$

provided the expectations in (10) exist.

Proof. See Appendix A. □

Theorem 1 tells us that for arbitrary number n, packet generation times $(s_1, s_2, \ldots, s_n)$ and arrival times $(a_{10}, a_{20}, \ldots, a_{n0})$ at node 0, network topology $\mathcal{G}(\mathcal{V}, \mathcal{L})$, and buffer sizes $(B_{ij}, (i, j) \in \mathcal{L})$, the prmp-LGFS policy achieves optimality of the joint distribution of the age processes at the network nodes within the policy space Π. In addition, (10) tells us that the prmp-LGFS policy minimizes any non-decreasing age penalty functional g, including the time-average age (6), average peak age (7), and average age penalty (8).

B. General Transmission Time Distributions

Now, we study the age-optimal packet scheduling for *arbitrary* general packet transmission time distributions which are *independent* across the links and *i.i.d.* across time. We consider the set of non-preemptive work-conserving policies, denoted by $\Pi_{npwc} \subset \Pi$. We propose a non-preemptive LGFS (non-prmp-LGFS) policy. It is important to note that under non-prmp-LGFS policy, the fresh packet replaces the oldest packet in a link's queue when the queue has a finite buffer size and full. We next show that the non-preemptive LGFS policy is age-optimal among the policies in Π_{npwc}.

Theorem 2. If the packet transmission times are *independent* across the links and *i.i.d.* across time, then for all $\mathcal{I}$ and $\pi \in \Pi_{npwc}$

$$[\boldsymbol{\Delta}_{\text{non-prmp-LGFS}}|\mathcal{I}] \leq_{\text{st}} [\boldsymbol{\Delta}_\pi|\mathcal{I}], \quad (11)$$

or equivalently, for all $\mathcal{I}$ and non-decreasing functional g

$$\mathbb{E}[g(\boldsymbol{\Delta}_{\text{non-prmp-LGFS}})|\mathcal{I}] = \min_{\pi \in \Pi_{npwc}} \mathbb{E}[g(\boldsymbol{\Delta}_\pi)|\mathcal{I}], \quad (12)$$

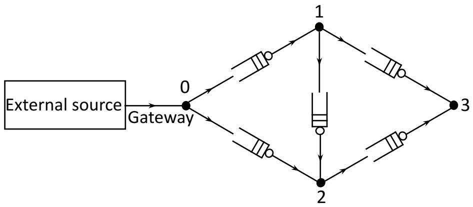

Figure 3: A multihop network.

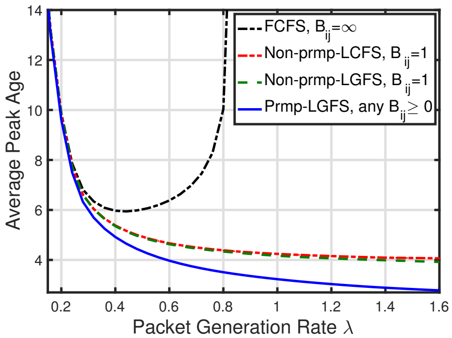

Figure 4: Average peak age at node 2 versus packet generation rate λ for exponential packet transmission times.

provided the expectations in (12) exist.

Proof. The proof of Theorem 2 is similar to that of Theorem 1. The difference is that preemption is not allowed here. See our technical report for more details [22]. □

It is interesting to note from Theorem 2 that, age-optimality can be achieved for arbitrary transmission time distributions, even if the transmission time distribution differs from a link to another.

IV. Numerical Results

We present some numerical results to illustrate the age performance of different policies and validate the theoretical results. We consider the network in Fig. 3. The inter-generation times are *i.i.d.* Erlang-2 distribution with mean $1/\lambda$. The time difference between packet generation and arrival to node 0, i.e., $a_{i0} - s_i$, is either 1 or 100, with equal probability. This means that the update packets may arrive to node 0 out of order of their generation time.

Figure 4 illustrates the average peak age at node 2 versus the packet generation rate λ for the multihop network in Fig. 3. The packet transmission times are exponentially distributed with mean 1 at links $(0, 1)$ and $(1, 2)$, and mean 0.5 at link $(0, 2)$. One can observe that the preemptive LGFS policy achieves a smaller peak age at node 2 than the non-preemptive LGFS policy, non-preemptive LCFS policy (was analyzed for single hop network in [9]), and FCFS policy, where the buffer sizes are either 1 or infinity. It is important to emphasize that the peak age is minimized by preemptive LGFS policy for out of order packet receptions at node 0, and general network topology. This numerical result shows agreement with Theorem 1.

Figure 5 plots the time-average age at node 3 versus the packets generation rate λ for the multihop network in Fig. 3.

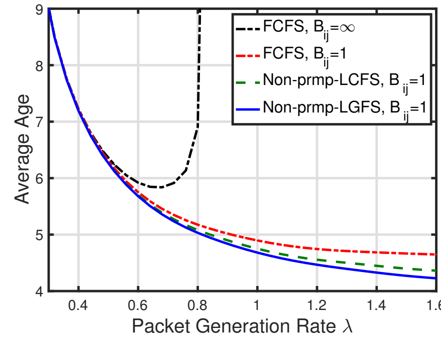

Figure 5: Average age at node 3 versus packet generation rate λ for general packet transmission time distributions.

The plotted policies are FCFS policy, non-preemptive LCFS, and non-preemptive LGFS policy, where the buffer sizes are either 1 or infinity. The packet transmission times at links $(0, 1)$ and $(1, 3)$ follow a gamma distribution with mean 1. The packet transmission times at links $(0, 2)$, $(1, 2)$, and $(2, 3)$ are distributed as the sum of a constant with value 0.5 and a value drawn from an exponential distribution with mean 0.5. We find that the non-preemptive LGFS policy achieves the best age performance among all plotted policies. By comparing the age performance of the non-preemptive LGFS and non-preemptive LCFS policies, we observe that the LGFS scheduling principle improves the age performance when the update packets arrive to node 0 out of the order of their generation times. It is important to note that the non-preemptive LGFS policy minimizes the age among the non-preemptive work-conserving policies even if the packet transmission time distributions are heterogeneous across the links. We also observe that the average age of FCFS policy with $B_{ij} = \infty$ blows up when the traffic intensity is high. This is due to the increased congestion in the network which leads to the delivery of stale packets. Moreover, in case of the FCFS policy with $B_{ij} = 1$, the average age is finite at high traffic intensity, since the fresh packet has a better opportunity to be delivered in a relatively short period compared with FCFS policy with $B_{ij} = \infty$. This numerical result agrees with Theorem 2.

V. Conclusion

In this paper, we made the first attempt to minimize the age-of-information in general multihop networks. We showed that for general system settings, including arbitrary network topology, packet generation times, packet arrival times, and queue buffer sizes, age-optimality can be achieved. These optimality results not only hold for the age processes, but also for any non-decreasing functional of the age processes.

Appendix A
Proof of Theorem 1

Let us define the system state of a policy π:

Definition 5. At any time t, the *system state* of policy π is specified by $\mathbf{U}_\pi(t) = (U_{0,\pi}(t), U_{2,\pi}(t), \ldots, U_{N-1,\pi}(t))$, where $U_{j,\pi}(t)$ is the generation time of the freshest packet that arrived at node j by time t. Let $\{\mathbf{U}_\pi(t), t \in [0, \infty)\}$ be the state process of policy π, which is assumed to be right-continuous. For notational simplicity, let policy P represent the preemptive LGFS policy.

The key step in the proof of Theorem 1 is the following lemma, where we compare policy P with any work-conserving policy π.

Lemma 3. Suppose that $\mathbf{U}_P(0^-) = \mathbf{U}_\pi(0^-)$ for all work conserving policies π, then for all $\mathcal{I}$,

$$[\{\mathbf{U}_P(t), t \in [0,\infty)\}|\mathcal{I}] \geq_{\text{st}} [\{\mathbf{U}_\pi(t), t \in [0,\infty)\}|\mathcal{I}]. \quad (13)$$

We use coupling and forward induction to prove Lemma 3. For any work-conserving policy π, suppose that stochastic processes $\widetilde{\mathbf{U}}_P(t)$ and $\widetilde{\mathbf{U}}_\pi(t)$ have the same distributions with $\mathbf{U}_P(t)$ and $\mathbf{U}_\pi(t)$, respectively. The state processes $\widetilde{\mathbf{U}}_P(t)$ and $\widetilde{\mathbf{U}}_\pi(t)$ are coupled in the following manner: If a packet is delivered from node i to node j at time t as $\widetilde{\mathbf{U}}_P(t)$ evolves in policy P, then there exists a packet delivery from node i to node j at time t as $\widetilde{\mathbf{U}}_\pi(t)$ evolves in policy π. Such a coupling is valid since the transmission time is exponentially distributed and thus memoryless. Moreover, policy P and policy π have identical packet generation times $(s_1, s_2, \ldots, s_n)$ at the external source and packet arrival times $(a_{10}, a_{20}, \ldots, a_{n0})$ to node 0. According to Theorem 6.B.30 in [21], if we can show

$$\mathbb{P}[\widetilde{\mathbf{U}}_P(t) \geq \widetilde{\mathbf{U}}_\pi(t), t \in [0,\infty)|\mathcal{I}] = 1, \quad (14)$$

then (13) is proven.

To ease the notational burden, we will omit the tildes in this proof on the coupled versions and just use $\mathbf{U}_P(t)$ and $\mathbf{U}_\pi(t)$. Next, we use the following lemmas to prove (14):

Lemma 4. Suppose that under policy P, $\mathbf{U}'_P$ is obtained by a packet delivery over the link (i, j) in the system whose state is $\mathbf{U}_P$. Further, suppose that under policy π, $\mathbf{U}'_\pi$ is obtained by a packet delivery over the link (i, j) in the system whose state is $\mathbf{U}_\pi$. If

$$\mathbf{U}_P \geq \mathbf{U}_\pi, \quad (15)$$

then,

$$\mathbf{U}'_P \geq \mathbf{U}'_\pi. \quad (16)$$

Proof. See our technical report [22]. □

Lemma 5. Suppose that under policy P, $\mathbf{U}'_P$ is obtained by the arrival of a new packet to node 0 in the system whose state is $\mathbf{U}_P$. Further, suppose that under policy π, $\mathbf{U}'_\pi$ is obtained by the arrival of a new packet to node 0 in the system whose state is $\mathbf{U}_\pi$. If

$$\mathbf{U}_P \geq \mathbf{U}_\pi, \quad (17)$$

then,

$$\mathbf{U}'_P \geq \mathbf{U}'_\pi. \quad (18)$$

Proof. See our technical report [22]. □

Proof of Lemma 3. For any sample path, we have that $\mathbf{U}_P(0^-) = \mathbf{U}_\pi(0^-)$. This, together with Lemma 4 and Lemma 5, implies that

$$[\mathbf{U}_P(t)|\mathcal{I}] \geq [\mathbf{U}_\pi(t)|\mathcal{I}],$$

holds for all $t \in [0,\infty)$. Hence, (14) holds which implies (13) by Theorem 6.B.30 in [21]. This completes the proof. □

Proof of Theorem 1. According to Lemma 3, we have

$$[\{\mathbf{U}_P(t), t \in [0,\infty)\}|\mathcal{I}] \geq_{\text{st}} [\{\mathbf{U}_\pi(t), t \in [0,\infty)\}|\mathcal{I}],$$

holds for all work-conserving policies π, which implies

$$[\{\mathbf{\Delta}_P(t), t \in [0,\infty)\}|\mathcal{I}] \leq_{\text{st}} [\{\mathbf{\Delta}_\pi(t), t \in [0,\infty)\}|\mathcal{I}],$$

holds for all work-conserving policies π.

Finally, transmission idling only postpones the delivery of fresh packets. Therefore, the age under non-work-conserving policies will be greater. As a result,

$$[\{\mathbf{\Delta}_P(t), t \in [0,\infty)\}|\mathcal{I}] \leq_{\text{st}} [\{\mathbf{\Delta}_\pi(t), t \in [0,\infty)\}|\mathcal{I}],$$

holds for all $\pi \in \Pi$. This completes the proof. □

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