Answers to Concepts Review and Critical Thinking Questions

2. Assuming positive cash flows, both the present and the future values will rise.

3. Assuming positive cash flows, the present value will fall and the future value will rise.

4. It’s deceptive, but very common. The basic concept of time value of money is that a dollar today is not worth the same as a dollar tomorrow. The deception is particularly irritating given that such lotteries are usually government sponsored!

5. If the total money is fixed, you want as much as possible as soon as possible. The team (or, more accurately, the team owner) wants just the opposite.

6. The better deal is the one with equal installments.

7. Yes, they should. APRs generally don’t provide the relevant rate. The only advantage is that they are easier to compute, but with modern computing equipment, that advantage is not very important.

8. A freshman does. The reason is that the freshman gets to use the money for much longer before interest starts to accrue. The subsidy is the present value (on the day the loan is made) of the interest that would have accrued up until the time it actually begins to accrue.

Solutions to Questions and Problems

1. The time line is:

```
0 1 2 3 4
PV   $680 $810 $940 $1,150
```

To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

\[
PV = \frac{FV}{(1 + r)^t}
\]

\[
PV@10\% = \frac{680}{1.10} + \frac{810}{1.10^2} + \frac{940}{1.10^3} + \frac{1,150}{1.10^4} = 2,779.30
\]

\[
PV@18\% = \frac{680}{1.18} + \frac{810}{1.18^2} + \frac{940}{1.18^3} + \frac{1,150}{1.18^4} = 2,323.27
\]

\[
PV@24\% = \frac{680}{1.24} + \frac{810}{1.24^2} + \frac{940}{1.24^3} + \frac{1,150}{1.24^4} = 2,054.62
\]
3. The time line is:

```
0 1 2 3 4
$1,225 $1,345 $1,460 $1,590
```

To solve this problem, we must find the FV of each cash flow and add them. To find the FV of a lump sum, we use:

\[ FV = PV(1 + r)^t \]

\[ FV@8\% = \$1,225(1.08)^3 + \$1,345(1.08)^2 + \$1,460(1.08) + \$1,590 = \$6,278.76 \]

\[ FV@11\% = \$1,225(1.11)^3 + \$1,345(1.11)^2 + \$1,460(1.11) + \$1,590 = \$6,543.12 \]

\[ FV@24\% = \$1,225(1.24)^3 + \$1,345(1.24)^2 + \$1,460(1.24) + \$1,590 = \$7,804.09 \]

Notice, since we are finding the value at Year 4, the cash flow at Year 4 is simply added to the FV of the other cash flows. In other words, we do not need to compound this cash flow.

4. To find the PVA, we use the equation:

\[ PVA = \frac{C\left[1 - \left(\frac{1}{1 + r}\right)^t\right]}{r} \]

```
0 1 … 15
PV $5,500 $5,500 $5,500 $5,500 $5,500 $5,500 $5,500 $5,500 $5,500
```

\[ PVA@15\,\text{yrs:} \quad PVA = \$5,500\left[1 - \left(\frac{1}{1.06}\right)^{15}\right] / .06 = \$53,417.37 \]

```
0 1 … 40
PV $5,500 $5,500 $5,500 $5,500 $5,500 $5,500 $5,500 $5,500 $5,500
```

\[ PVA@40\,\text{yrs:} \quad PVA = \$5,500\left[1 - \left(\frac{1}{1.06}\right)^{40}\right] / .06 = \$82,754.63 \]

```
0 1 … 75
PV $5,500 $5,500 $5,500 $5,500 $5,500 $5,500 $5,500 $5,500 $5,500
```

\[ PVA@75\,\text{yrs:} \quad PVA = \$5,500\left[1 - \left(\frac{1}{1.06}\right)^{75}\right] / .06 = \$90,507.16 \]

To find the PV of a perpetuity, we use the equation:

\[ PV = \frac{C}{r} \]

```
0 1 … ∞
PV $5,500 $5,500 $5,500 $5,500 $5,500 $5,500 $5,500 $5,500 $5,500
```

\[ PV = \$5,500 / .06 = \$91,666.67 \]
Notice that as the length of the annuity payments increases, the present value of the annuity approaches the present value of the perpetuity. The present value of the 75-year annuity and the present value of the perpetuity imply that the value today of all perpetuity payments beyond 75 years is only $1,159.50.

5. The time line is:

Here we have the PVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the PVA equation:

\[
PVA = C\left(\frac{1 - \left[\frac{1}{1 + r}\right]^t}{r}\right)
\]

\[
PVA = 38,000 = C\left(\frac{1 - (1 / 1.05815)}{0.058}\right)
\]

We can now solve this equation for the annuity payment. Doing so, we get:

\[
C = \frac{38,000}{9.840437} = 3,861.62
\]

7. Here we need to find the FVA. The equation to find the FVA is:

\[
FVA = C\left(\frac{(1 + r)^t - 1}{r}\right)
\]

\[
FVA\ for\ 20\ years = 4,000\left(\frac{(1.097^{20} - 1)}{.097}\right) = 221,439.14
\]

\[
FVA\ for\ 40\ years = 4,000\left(\frac{(1.097^{40} - 1)}{.097}\right) = 1,631,984.09
\]

Notice that because of exponential growth, doubling the number of periods does not merely double the FVA.

10. The time line is:

This cash flow is a perpetuity. To find the PV of a perpetuity, we use the equation:

\[
PV = \frac{C}{r}
\]

\[
PV = \frac{40,000}{.051} = 784,313.73
\]
12. For discrete compounding, to find the EAR, we use the equation:

\[
\text{EAR} = \left[1 + \left(\frac{\text{APR}}{m}\right)\right]^m - 1
\]

\[
\text{EAR} = \left[1 + \left(\frac{0.07}{4}\right)\right]^4 - 1 = 0.0719, \text{ or } 7.19\%
\]

\[
\text{EAR} = \left[1 + \left(\frac{0.17}{12}\right)\right]^{12} - 1 = 0.1839, \text{ or } 18.39\%
\]

\[
\text{EAR} = \left[1 + \left(\frac{0.13}{365}\right)\right]^{365} - 1 = 0.1388, \text{ or } 13.88\%
\]

To find the EAR with continuous compounding, we use the equation:

\[
\text{EAR} = e^q - 1
\]

\[
\text{EAR} = e^{0.10} - 1 = 0.1052, \text{ or } 10.52\%
\]

20. The time line is:

\[
\begin{array}{cccccccc}
0 & 1 & \cdots & \text{ } & 60 \\
\$79,500 & C & C & C & C & \cdots & C & C & C & C & C & C & C
\end{array}
\]

We first need to find the annuity payment. We have the PVA, the length of the annuity, and the interest rate. Using the PVA equation:

\[
PVA = C\left\{1 - \left[\frac{1}{1 + r}\right]^t\right\} / r
\]

\[
\$79,500 = C\left\{1 - \left[\frac{1}{1 + \left(\frac{0.058}{12}\right)}\right]^{60}\right\} / \left(\frac{0.058}{12}\right)
\]

Solving for the payment, we get:

\[
C = \frac{\$79,500}{51.97521} = 1,529.58
\]

To find the EAR, we use the EAR equation:

\[
\text{EAR} = \left[1 + \left(\frac{\text{APR}}{m}\right)\right]^m - 1
\]

\[
\text{EAR} = \left[1 + \left(\frac{0.058}{12}\right)\right]^{12} - 1 = 0.0596, \text{ or } 5.96\%
\]

21. The time line is:

\[
\begin{array}{cccccccc}
0 & 1 & \cdots & \text{ } & ? \\
-\$18,000 & \$500 & \$500 & \$500 & \$500 & \cdots & \$500 & \$500 & \$500 & \$500 & \$500
\end{array}
\]

Here we need to find the length of an annuity. We know the interest rate, the PV, and the payments. Using the PVA equation:

\[
PVA = C\left\{1 - \left[\frac{1}{1 + r}\right]^t\right\} / r
\]

\[
\$18,000 = \$500\left\{1 - \left(\frac{1}{1 + \left(0.058 / 12\right)}\right)^t\right\} / \left(0.058 / 12\right)
\]

Now we solve for \( t \):
1 / 1.015^t = 1 - \{[(18,000) / (500)](0.015)\}
1 / 1.015^t = 0.46
1.015^t = 1 / 0.46 = 2.174
\( t = \ln 2.174 / \ln 1.015 = 52.16 \) months

24. The time line is:

This problem requires us to find the FVA. The equation to find the FVA is:

\[
FVA = C\left\{\left(1 + \frac{r}{k}\right)^{kt} - 1\right\} / \frac{r}{k}
\]

\[
FVA = 450\left\{\left(1 + \frac{0.10}{12}\right)^{360} - 1\right\} / \frac{0.10}{12}
\]

\[
FVA = 1,017,219.57
\]

26. The time line is:

The cash flows are simply an annuity with four payments per year for four years, or 16 payments. We can use the PVA equation:

\[
PVA = C\left\{1 - \left[1 / (1 + r)^k\right]\right\} / r
\]

\[
PVA = 2,200\left\{1 - \left(1 / (1.0043)^{16}\right)\right\} / 0.0043
\]

\[
PVA = 33,945.97
\]

28. The time line is:

Here the cash flows are annual and the given interest rate is annual, so we can use the interest rate given. We can find the PV of each cash flow and add them together.

\[
PV = 2,480 / 1.0618 + $3,920 / 1.0618^3 + $2,170 / 1.0618^4 = 7,317.47
\]
36. Here we need to compare two cash flows, so we will find the value today of both sets of cash flows. We need to make sure to use the monthly cash flows since the salary is paid monthly. Doing so, we find:

\[
PVA_1 = \frac{75,000}{12} \left( \frac{1 - \frac{1}{1 + (0.07/12)^{24}}}{(0.07/12)} \right) = 139,594.37
\]

\[
PVA_2 = 20,000 + \frac{64,000}{12} \left( \frac{1 - \frac{1}{1 + (0.07/12)^{24}}}{(0.07/12)} \right) = 139,120.53
\]

You should choose the first option since it has a higher PV.

41. The time line is:

Here we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. Using the PVA equation:

\[
PVA = \frac{89,000}{1,850} = \frac{1 - \left[ \frac{1}{(1 + r)^{60}} \right]}{r}
\]

To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate lowers the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

\[r = 0.755\%
\]

The APR is the periodic interest rate times the number of periods in the year, so:

\[APR = 12(0.755\%) = 9.06\%
\]

43. The time line is:

We are given the total PV of all four cash flows. If we find the PV of the three cash flows we know, and subtract them from the total PV, the amount left over must be the PV of the missing cash flow. So, the PV of the cash flows we know are:

\[PV \text{ of Year 1 CF: } 1,300 / 1.09 = 1,192.66
\]
PV of Year 3 CF: $1,950 / 1.09^3 = $1,505.76

PV of Year 4 CF: $2,640 / 1.09^4 = $1,870.24

So, the PV of the missing CF is:

$6,200 – 1,192.66 – 1,505.76 – 1,870.24 = $1,631.34

The question asks for the value of the cash flow in Year 2, so we must find the future value of this amount. The value of the missing CF is:

$1,631.34(1.09)^2 = $1,938.19

45. Here we are finding the interest rate for an annuity cash flow. We are given the PVA, number of periods, and the amount of the annuity. We should also note that the PV of the annuity is the amount borrowed, not the purchase price, since we are making a down payment on the warehouse. The amount borrowed is:

Amount borrowed = .80($2,700,000) = $2,160,000

The time line is:

$2,160,000 $13,400 $13,400 $13,400 $13,400 $13,400 $13,400

Using the PVA equation:

PVA = $2,160,000 = $13,400[{1 – [1 / (1 + r)^{360}]} / r]

Unfortunately this equation cannot be solved to find the interest rate using algebra. To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate lowers the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

r = .527%

The APR is the monthly interest rate times the number of months in the year, so:

APR = 12(.527%) = 6.32%

And the EAR is:

EAR = (1 + .00527)^12 – 1 = .0651, or 6.51%
54. The time line is:

```
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<th>59</th>
<th>60</th>
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<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
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We need to use the PVA due equation, that is:

\[
PVA_{\text{due}} = (1 + r) \times \text{PVA}
\]

Using this equation:

\[
PVA_{\text{due}} = \$68,000 = [1 + (0.064/12)] \times C \left[ \frac{1 - 1 / [1 + (0.064/12)^{60}]}{0.064/12} \right]
\]

\[
\$68,362.67 = C \left[ 1 - \frac{1}{1 + (0.064/12)^{60}} \right] / (0.064/12)
\]

\[
C = \$1,320.27
\]

Notice, to find the payment for the PVA due we simply compound the payment for an ordinary annuity forward one period.
## Calculator Solutions

1. 

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<th>C01</th>
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<td>I = 10</td>
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3. 

Enter 3 8% $1,225

Solve for $1,543.15

Enter 2 8% $1,345

Solve for $1,568.81

Enter 1 8% $1,460

Solve for $1,576.80

FV = $1,543.15 + 1,568.81 + 1,576.80 + 1,590 = $6,278.76

Enter 3 11% $1,225

Solve for $1,675.35

Enter 2 11% $1,345

Solve for $1,657.17

Enter 1 11% $1,460

Solve for $1,620.60

FV = $1,675.35 + 1,657.17 + 1,620.60 + 1,590 = $6,543.12

Enter 3 24% $1,225
Solve for $2,335.61

Enter 2 24% $1,345

Solve for $2,068.07

Enter 1 24% $1,460

Solve for $1,810.40

FV = $2,335.61 + 2,068.07 + 1,810.40 + 1,590 = $7,804.09

4. Enter 15 6% $5,500

Solve for $53,417.37

Enter 40 6% $5,500

Solve for $82,754.63

Enter 75 6% $5,500

Solve for $90,507.16

5. Enter 15 5.8% $38,000

Solve for $3,861.62

7. Enter 20 9.7% $4,000

Solve for $221,439.14

Enter 40 9.7% $4,000

Solve for $1,631,984.09
12. Enter 7% NOM 4 EFF C/Y Solve for 7.19%

Enter 17% NOM 12 EFF C/Y Solve for 18.39%

Enter 13% NOM 365 EFF C/Y Solve for 13.88%

20. Enter 60 N 5.8% / 12 I/Y $79,500 PV Solve for PMT FV $1,529.58

Enter 5.8% NOM 12 EFF C/Y Solve for 5.96%

21. Enter 1.5% N $18,000 I/Y ±$500 PV Solve for PMT FV 52.16

24. Enter 30 × 12 N 10% / 12 I/Y $450 PV Solve for PMT FV $1,017,219.57

26. Enter 4 × 4 N 0.43% I/Y $2,200 PV Solve for PMT FV $33,945.97
28. 

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\[ I = 6.18\% \]

NPV CPT
$7,317.47

36. 

Enter \( 2 \times 12 \) \( 7\% /12 \) \( \text{PV} \) \( \text{PMT} \) \( \text{FV} \)

Solve for \$139,594.37

Enter \( 2 \times 12 \) \( 7\% /12 \) \( \text{PV} \) \( \text{PMT} \) \( \text{FV} \)

Solve for \$119,120.53

\[ 119,120.53 + 20,000 = 139,120.53 \]

41. 

Enter \( 60 \) \( \text{N} \) \( \text{I/Y} \) \( \text{PV} \) \( \pm_{1,850} \)

Solve for \( .755\% \)

\[ .755\% \times 12 = 9.06\% \]

43. 

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\[ I = 9\% \]

NPV CPT
$4,568.66

PV of missing CF = \$6,200 – 4,568.66 = \$1,631.34

Value of missing CF:

Enter \( 2 \) \( \text{N} \) \( 9\% \) \( \text{PV} \) \( \text{PMT} \) \( \text{FV} \)

Solve for \$1,938.19
45. Enter 360 \( .80(\$2,700,000) \) \( \pm \$13,400 \)

Solve for \( .527\% \)

\[
\text{APR} = .527\% \times 12 = 6.32\%
\]

Enter 6.32\% 12

Solve for 6.51\%

54. \( 2^{\text{nd}} \text{BGN} \quad 2^{\text{nd}} \text{SET} \)

Enter 60 6.4\% / 12 \( \$68,000 \)

Solve for \( \$1,320.27 \)