CHAPTER 5
INTRODUCTION TO VALUATION: THE TIME VALUE OF MONEY

Answers to Concepts Review and Critical Thinking Questions

1. The four parts are the present value (PV), the future value (FV), the discount rate \((r)\), and the life of the investment \((t)\).

2. Compounding refers to the growth of a dollar amount through time via reinvestment of interest earned. It is also the process of determining the future value of an investment. Discounting is the process of determining the value today of an amount to be received in the future.

3. Future values grow (assuming a positive rate of return); present values shrink.

4. The future value rises (assuming it’s positive); the present value falls.

Solutions to Questions and Problems

1. The time line for the cash flows is:

   ![](image)

   The simple interest per year is:

   \[
   \$9,000 \times .08 = \$720
   \]

   So after 7 years you will have:

   \[
   \$720 \times 7 = \$5,040 \text{ in interest.}
   \]

   The total balance will be \$9,000 + 5,040 = \$14,040

   With compound interest we use the future value formula:

   \[
   FV = PV(1 + r)^t
   \]

   \[
   FV = \$9,000(1.08)^7 = \$15,424.42
   \]

   The difference is:

   \[
   \$15,424.42 - 14,040 = \$1,384.42
   \]
2. To find the FV of a lump sum, we use:

\[ FV = PV(1 + r)^t \]

\[ FV = \$1,975 \]

\[ FV = \$1,975(1.13)^{11} = \$7,575.83 \]

\[ FV = \$6,734 \]

\[ FV = \$6,734(1.09)^7 = \$12,310.02 \]

\[ FV = \$81,346 \]

\[ FV = \$81,346(1.12)^{14} = \$397,547.04 \]

\[ FV = \$192,050 \]

\[ FV = \$192,050(1.06)^{8} = \$306,098.52 \]

3. To find the PV of a lump sum, we use:

\[ PV = \frac{FV}{(1 + r)^t} \]

\[ PV = \frac{\$15,451}{(1.09)^{13}} = \$5,039.79 \]

\[ PV = \frac{\$51,557}{(1.07)^{4}} = \$39,332.59 \]

\[ PV = \frac{\$886,073}{(1.24)^{29}} = \$1,730.78 \]

\[ PV = \frac{\$550,164}{(1.35)^{40}} = \$3.37 \]
6. The time line is:

\[ \begin{array}{c}
0 & \cdots & 18 \\
-67,000 & \cdots & 320,000
\end{array} \]

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[ FV = PV \left(1 + \frac{r}{t}\right)^t \]

Solving for \( r \), we get:

\[ r = \left(\frac{FV}{PV}\right)^{1/t} - 1 \]
\[ r = \left(\frac{320,000}{67,000}\right)^{1/18} - 1 \]
\[ r = .0908, \text{ or } 9.08\% \]

9. The time line is:

\[ \begin{array}{c}
0 & \cdots & ? \\
-45,000 & \cdots & 225,000
\end{array} \]

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[ FV = PV \left(1 + \frac{r}{t}\right)^t \]

Solving for \( t \), we get:

\[ t = \ln(FV / PV) / \ln(1 + r) \]
\[ t = \ln \left(\frac{225,000}{45,000}\right) / \ln 1.048 = 34.33 \text{ years} \]

13. The time line is:

\[ \begin{array}{c}
0 & \cdots & 119 \\
150 & \cdots & 1,620,000
\end{array} \]

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[ FV = PV \left(1 + \frac{r}{t}\right)^t \]

Solving for \( r \), we get:

\[ r = \left(\frac{FV}{PV}\right)^{1/t} - 1 \]
\[ r = \left(\frac{1,620,000}{150}\right)^{1/119} - 1 = .0812, \text{ or } 8.12\% \]

To find the FV of the first prize in 2040, we use:
FV = PV(1 + r)^t
FV = $1,620,000(1.0812)^{26} = $12,324,441.95

14. The time line is:

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + r)^t

Solving for r, we get:

r = (FV / PV)^{1/t} - 1
r = ($10,311,500 / $12,377,500)^{1/4} - 1 = -4.46%

Notice that the interest rate is negative. This occurs when the FV is less than the PV.

15. The time line from minting to the first sale is:

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + r)^t

Solving for r, we get:

r = (FV / PV)^{1/t} - 1
r = ($430,000 / $15)^{1/192} - 1 = .0549, or 5.49%

The time line from the first sale to the second sale is:

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + r)^t
Solving for \( r \), we get:

\[
r = (FV / PV)^{1/t} - 1
\]

\[
r = (\$4,582,500 / \$430,000)^{1/35} - 1 = .0699, \text{ or } 6.99\%
\]

The time line from minting to the second sale is:

\[
\begin{array}{c}
0 \\
-\$15 \\
\hline
\hline
227 \\
\$4,582,500
\end{array}
\]

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[
FV = PV(1 + r)^t
\]

Solving for \( r \), we get:

\[
r = (FV / PV)^{1/t} - 1
\]

\[
r = (\$4,582,500 / \$15)^{1/227} - 1 = .0572, \text{ or } 5.72\%
\]

16. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[
FV = PV(1 + r)^t
\]

Solving for \( r \), we get:

\[
r = (FV / PV)^{1/t} - 1
\]

\[
a. \text{ The time line is:}
\]

\[
\begin{array}{c}
0 \\
-\$50 \\
\hline
\hline
20 \\
\$100
\end{array}
\]

\[
r = (FV / PV)^{1/t} - 1
\]

\[
r = (\$100 / \$50)^{1/20} - 1
\]

\[
r = .0353, \text{ or } 3.53\%
\]

\[
b. \text{ The time line is:}
\]

\[
\begin{array}{c}
0 \\
-\$50 \\
\hline
\hline
10 \\
FV
\end{array}
\]

\[
FV = PV(1 + r)^t
\]

\[
FV = \$50(1 + .001)^{10}
\]

\[
FV = \$50.50
\]
c. The time line is:

\[
\begin{array}{c|c|c}
0 & \$50.50 & 10 \\
\hline
& \$100 & \\
\end{array}
\]

\[
r = \left( \frac{FV}{PV} \right)^{1/t} - 1
\]

\[
r = \left( \frac{\$100}{\$50.50} \right)^{1/10} - 1
\]

\[
r = 0.0707, \text{ or } 7.07\%
\]

18. To find the FV of a lump sum, we use:

\[
FV = PV(1 + r)^t
\]

\[
\begin{array}{c|c|c}
0 & \$5,000 & 45 \\
\hline
& \text{FV} & \\
\end{array}
\]

\[
FV = \$5,000(1.10)^{45} = \$364,452.42
\]

\[
\begin{array}{c|c|c}
0 & \$5,000 & 35 \\
\hline
& \text{FV} & \\
\end{array}
\]

\[
FV = \$5,000(1.10)^{35} = \$140,512.18
\]

Better start early!

20. The time line is:

\[
\begin{array}{c|c|c}
0 & \$15,000 & 2 \\
\hline
& \$75,000 & \\
\end{array}
\]

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

\[
FV = PV(1 + r)^t
\]

Solving for \(t\), we get:

\[
t = \ln(FV / PV) / \ln(1 + r)
\]

\[
t = \ln(\$75,000 / \$15,000) / \ln(1.09) = 18.68
\]

So, the money must be invested for 18.68 years. However, you will not receive the money for another two years. From now, you’ll wait:

\[
2 \text{ years } + 18.68 \text{ years } = 20.68 \text{ years}
\]
**Calculator Solutions**

1. Enter 7 8% $9,000
   
   **Solve for** $15,424.42
   
   $15,424.42 – 14,040 = $1,384.42

2. Enter 11 13% $1,975
   
   **Solve for** $7,575.83

   Enter 7 9% $6,734
   
   **Solve for** $12,310.02

   Enter 14 12% $81,346
   
   **Solve for** $397,547.04

   Enter 8 6% $192,050
   
   **Solve for** $306,098.52

3. Enter 9 7% $15,451
   
   **Solve for** $8,404.32

   Enter 7 13% $51,557
   
   **Solve for** $51,557

   Enter 24 14% $886,073
   
   **Solve for** $886,073

   Enter 35 9% $550,164
   
   **Solve for** $550,164
6. Enter 18 $67,000 $320,000
Solve for N 9.08% I/Y PV PMT FV

9. Enter 4.80% $45,000 $225,000
Solve for N I/Y PV PMT FV

13. Enter 119 ±$150 $1,620,000
Solve for N I/Y PV PMT FV
Enter 26 8.12% $1,620,000
Solve for N I/Y PV PMT FV

14. Enter 4 ±$12,377,500 $10,311,500
Solve for N I/Y PV PMT FV

15. Enter 192 ±$15 $430,000
Solve for N I/Y PV PMT FV
Enter 35 ±$430,000 $4,582,500
Solve for N I/Y PV PMT FV

16. a. Enter 20 ±$50 $100
Solve for N 3.53% I/Y PV PMT FV

16. b. Enter 10 ±$50 $50.50
Solve for N .10% I/Y PV PMT FV
16. c. Enter 10 \( ±\$50.50 \) \$100
Solve for \( N \) \( I/Y \) \( PV \) \( PMT \) \( FV \)
\( 7.07\% \)

18. Enter 45 10\% \$5,000
Solve for \( N \) \( I/Y \) \( PV \) \( PMT \) \( FV \)
\$364,452.42

Enter 35 10\% \$5,000
Solve for \( N \) \( I/Y \) \( PV \) \( PMT \) \( FV \)
\$140,512.18

20. Enter 9\% \$15,000 \$75,000
Solve for \( N \) \( I/Y \) \( PV \) \( PMT \) \( FV \)
18.68

From now, you’ll wait 2 + 18.68 = 20.68 years