Stationary Solutions and Spreading Speeds of Nonlocal Monostable Equations in Space Periodic Habitats*

Wenxian Shen and Aijun Zhang
Department of Mathematics and Statistics
Auburn University
Auburn University, AL 36849
U.S.A.

Abstract. The current paper is to investigate positive stationary solutions and spreading speeds of monostable equations with nonlocal dispersal in spatially periodic habitats. The existence and uniqueness of positive stationary solutions and the existence and characterization of spreading speeds of such equations with a symmetric convolution kernel are established in one of the authors’ earlier works for following cases: the nonlocal dispersal is nearly local; the periodic habitat is nearly globally homogeneous or it is nearly homogeneous in a region where it is most conducive to population growth in the zero-limit population. These conditions are to guarantee the existence of the principal eigenvalue of the nonlocal dispersal operator associated to the linearized equation at the trivial solution. Unlike random dispersal operators, such nonlocal dispersal operator may not have a principal eigenvalue. In this paper, we generalize the results mentioned above to general nonlocal monostable equations in spatially periodic habitats. As a consequence, it is seen that the spatial spreading feature is generic for monostable equations with nonlocal dispersal.

Key words. Monostable equation; nonlocal dispersal; random dispersal; periodic habitat; spreading speed; principal eigenvalue; principal eigenfunction; variational principle.

Mathematics subject classification. 45C05, 45G10, 45M20, 47G10, 92D25.

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