

Variational Principle for Spatial Spreading Speeds and Generalized Wave Solutions in Time Almost Periodic and Space Periodic KPP Models

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Abstract. Spatial spreading and front propagating dynamics is one of most important dynamical issues in KPP models. Such dynamics of KPP models in time independent or periodic media has been widely studied. Recently, the author of the current paper with Huang established some theoretical foundation for the study of spatial spreading and front propagating dynamics of KPP models in time almost periodic and space periodic media. A notion of spreading speed intervals for such models was introduced in the joint paper by the author of this paper and Huang and was shown to be the natural extension of the classical concept of the spreading speeds for time independent or periodic KPP models and can be used for more general time dependent KPP models. A notion of generalized propagating speed intervals of solutions and a notion of generalized wave solutions to time almost periodic and space periodic KPP models were also introduced in the joint paper, which are the generalizations of wave speeds and traveling wave solutions in time independent or periodic KPP models. The current paper is to gain more qualitative and quantitative understanding of the spatial spreading and front propagating dynamics of KPP models in time almost periodic and space periodic media. By applying the principal Lyapunov exponent and principal Floquet bundle theory for time almost periodic parabolic equations, we provide various useful estimates for spreading and generalized propagating speeds for such KPP models. Under some conditions, we establish a variational principle for the spreading speeds and prove the existence of a family of generalized wave solutions. Both the estimates and variational principle provide important and efficient tools for the spreading speeds analysis as well as the spreading speeds computation. Based on the variational principle, the influence of time and space variation of the media on the spreading speeds is also discussed in the paper. It is shown that the time and space variation cannot slow down the spatial spread and it indeed speeds up the spatial spread except certain degenerate cases, which provides deep insights into the understanding of the influence of the inhomogeneity of the underline media on the spatial spread in KPP models.

Key words. Almost periodic KPP model, spreading speed interval, generalized propagating speed interval, traveling wave solution, generalized wave solution, principal Lyapunov exponent, variational principle

Mathematics subject classification. 35K55, 35K57, 35B15, 92D25