Asymptotic Dynamics of a Class of Coupled Oscillators Driven by White Noises

Wenxian Shen\textsuperscript{a} \textsuperscript{1}, Zhongwei Shen\textsuperscript{a}, Shengfan Zhou\textsuperscript{b} \textsuperscript{2} \textsuperscript{3}

\textsuperscript{a}Department of Mathematics and Statistics, Auburn University, Auburn 36849, USA
\textsuperscript{b}Department of Applied Mathematics, Shanghai Normal University, Shanghai 200234, PR China

Abstract:
This paper is devoted to the study of the asymptotic dynamics of a class of coupled second order oscillators driven by white noises. It is shown that any system of such coupled oscillators with positive damping and coupling coefficients possesses a global random attractor. Moreover, when the damping and the coupling coefficients are sufficiently large, the global random attractor is a one-dimensional random horizontal curve regardless of the strength of the noises, and the system has a rotation number, which implies that the oscillators in the system tend to oscillate with the same frequency eventually and therefore the so called frequency locking is successful. The results obtained in this paper generalize many existing results on the asymptotic dynamics for a single second order noisy oscillator to systems of coupled second order noisy oscillators. They show that coupled damped second order oscillators with large damping have similar asymptotic dynamics as the limiting coupled first order oscillators as the damping goes to infinite and also that coupled damped second order oscillators have similar asymptotic dynamics as their proper space continuous counterparts, which are of great practical importance.

Keywords: Coupled second order oscillators; white noises; random attractor; random horizontal curve; rotation number; frequency locking

AMS Subject Classification: 60H10, 34F05, 37H10.

\textsuperscript{1}The first author is partially supported by NSF grant DMS-0907752
\textsuperscript{2}The third author is supported by National Natural Science Foundation of China under Grant 10771139, and the Innovation Program of Shanghai Municipal Education Commission under Grant 08ZZ70
\textsuperscript{3}Corresponding Author: zhoushengfan@yahoo.com