Asymptotic Dynamics of a Class of Coupled Oscillators Driven by White Noises

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Abstract:
This paper is devoted to the study of the asymptotic dynamics of a class of coupled second order oscillators driven by white noises. It is shown that any system of such coupled oscillators with positive damping and coupling coefficients possesses a global random attractor. Moreover, when the damping and the coupling coefficients are sufficiently large, the global random attractor is a one-dimensional random horizontal curve regardless of the strength of the noises, and the system has a rotation number, which implies that the oscillators in the system tend to oscillate with the same frequency eventually and therefore the so called frequency locking is successful. The results obtained in this paper generalize many existing results on the asymptotic dynamics for a single second order noisy oscillator to systems of coupled second order noisy oscillators. They show that coupled damped second order oscillators with large damping have similar asymptotic dynamics as the limiting coupled first order oscillators as the damping goes to infinite and also that coupled damped second order oscillators have similar asymptotic dynamics as their proper space continuous counterparts, which are of great practical importance.

Keywords: Coupled second order oscillators; white noises; random attractor; random horizontal curve; rotation number; frequency locking

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