Existence of Generalized Traveling Waves in Time Recurrent and Space Periodic Monostable Equations

Wenxian Shen *
Department of Mathematics and Statistics
Auburn University
Auburn University, AL 36849
U.S.A.

Abstract. This paper is concerned with the extension of the concepts and theories of traveling wave solutions of time and space periodic monostable equations to time recurrent and space periodic ones. It first introduces the concept of generalized traveling wave solutions of time recurrent and space periodic monostable equations, which extends the concept of periodic traveling wave solutions of time and space periodic monostable equations to time recurrent and space periodic ones. It then proves that in the direction of any unit vector ξ, there is $c^*(\xi)$ such that for any $c > c^*(\xi)$, a generalized traveling wave solution in the direction of ξ with averaged propagation speed c exists. It also proves that if the time recurrent and space periodic monostable equation is indeed time periodic, then $c^*(\xi)$ is the minimal wave speed in the direction of ξ and the generalized traveling wave solution in the direction of ξ with averaged speed $c > c^*(\xi)$ is a periodic traveling wave solution with speed c, which recovers the existing results on the existence of periodic traveling wave solutions in the direction of ξ with speed greater than the minimal speed in that direction.

Key words. Monostable equation; generalized traveling wave solution; average propagating speed; spreading speed; sub-solution; super-solution; comparison principle; compact flow; recurrent function.

Mathematics subject classification. 35B15, 35B35, 35B40, 35K55, 35K57, 92D25.

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