

Existence, Uniqueness, and Stability of Generalized Traveling Waves in Time Dependent Monostable Equations

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Abstract. The current paper is devoted to the study of traveling wave solutions of spatially homogeneous monostable reaction diffusion equations with ergodic or recurrent time dependence, which includes periodic and almost periodic time dependence as special cases. Such an equation has two spatially homogeneous and time recurrent solutions with one of them being stable and the other being unstable. Traveling wave solutions are a type of entire solutions connecting the two spatially homogeneous and time recurrent solutions. Recently, the author of the current paper proved that a spatially homogeneous time almost periodic monostable equation has a spreading speed in any given direction. This result can be easily extended to monostable equations with recurrent time dependence. In this paper, we introduce generalized traveling wave solutions for time recurrent monostable equations and show the existence of such solutions in any given direction with average propagating speed greater than or equal to the spreading speed in that direction and non-existence of such solutions of slower average propagating speed. We also show the uniqueness and stability of generalized traveling wave solutions in any given direction with average propagating speed greater than the spreading speed in that direction. Moreover, we show that a generalized traveling wave solution in a given direction with average propagating speed greater than the spreading speed in that direction is unique ergodic in the sense that its wave profile and wave speed are unique ergodic, and if the time dependence of the monostable equation is almost periodic, it is almost periodic in the sense that its wave profile and wave speed are almost periodic.

Key words. Monostable equation, generalized traveling wave solution, wave profile, wave speed, average propagating speed, spreading speed, existence, uniqueness, stability, sub-solution, super-solution, comparison principle, compact flow, unique ergodic function, almost periodic function, frequency module.

Mathematics subject classification. 35B15, 35B35, 35B40, 35K55, 35K57, 92D25.