

\mathcal{A} -Stability of Global Attractors of Competition Diffusion Systems*

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*Dedicated to Jack K. Hale on the
occasion of his 80th birthday*

Abstract. We study the structural stability of global attractors (\mathcal{A} -stability) for two-species competition diffusion systems with Morse-Smale property. Such systems generate semiflows on positive cones of certain infinite-dimensional Banach spaces (e.g., fractional order spaces). Our main result states that a Morse-Smale two species competition diffusion system is structurally \mathcal{A} -stable, which implies that the set of nonlinearities for which the system possesses Morse-Smale property is open in an appropriate space under the topology of C^2 -convergence on compacta. Moreover, we provide a sufficient condition under which a system has the Morse-Smale property.

Keywords: Competition diffusion system, positive solution, semiflow, critical element, stable and unstable manifold, non-wandering point, global attractor, Morse-Smale property, \mathcal{A} -stability.

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