Spectral Theory for General Nonautonomous/Random Dispersal Evolution Operators

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Abstract. We investigate the spectral theory of the following general non-autonomous evolution equation
\[ \partial_t u(t, x) = A(u(t, \cdot))(x) + h(t, x)u(t, x), \quad x \in D, \]
where $D$ is a bounded subset of $\mathbb{R}^N$ which can be a smooth domain or a discrete set, $A$ is a general linear dispersal operator (for example a Laplacian operator, an integral operator with positive kernel or a cooperative discrete operator) and $h(t, x)$ is a smooth function on $\mathbb{R} \times \bar{D}$. We first study the influence of time dependence on the principal spectrum of dispersal equations and show that the principal Lyapunov exponent of a time-dependent dispersal equation is always greater than or equal to that of the time-averaged one. Several results about the principal eigenvalue of time-periodic parabolic equations are extended to general time-periodic dispersal ones. Finally, the investigation is generalized to random time-dependent dispersal equations.

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