Exponential Separation and Principal Lyapunov Exponent/Spectrum for Random/Nonautonomous Parabolic Equations

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Abstract. The purpose of the paper is to extend the principal eigenvalue and principal eigenfunction theory for time independent and periodic parabolic equations to random and general nonautonomous ones. In the random case, a notion of principal Lyapunov exponent serving as an analog of principal eigenvalue is introduced. It is shown that the principal Lyapunov exponent is deterministic and of simple multiplicity. It is also shown that there is a one-dimensional invariant random subbundle corresponding to the solutions that are globally defined and of the same sign, which serves as an analog of principal eigenfunction. In addition, monotonicity of the principal Lyapunov exponent with respect to the zero order terms both in the equation and in the boundary condition is proved. When the second and first order terms are deterministic, it is proved that the principal Lyapunov ex-

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ponent is greater than or equal to the principal eigenvalue of the associated time-averaged equation. In the general nonautonomous case, the concepts of principal spectrum, which serves as an analog of principal eigenvalue, and principal Lyapunov exponents are introduced. As is known, the principal spectrum is a compact interval. It is proved in the paper that the principal spectrum contains all the principal Lyapunov exponents. When the second and first order terms are time independent, a lower estimate of the infimum of the principal spectrum is given in terms of an associated time-averaged equation.