

# Spectral theory for non-local dispersal with periodic or almost-periodic time dependence.

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## Abstract

In applications to spatial structure in biology and to the theory of phase transition, it has proved useful to generalize the idea of diffusion to a non-local dispersal with an integral operator replacing the Laplacian. We study the spectral problem for the linear scalar equation

$$u_t(x, t) = \int_{\Omega} K(x, y)u(y, t)dy + h(x, t)u(x, t),$$

and tackle the extra technical difficulties arising because of the lack of compactness for the evolution operator defined by the dispersal. Our aim is firstly to investigate the extent to which the idea of a periodic parabolic principal eigenvalue may be generalized. Secondly, we obtain a lower bound for this in terms of the corresponding averaged spatial problem, and then extend this to the principal Lyapunov exponent in the almost periodic case.

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