Dynamical Order in Systems of Coupled Noisy Oscillators

Dedicated to Professor Pavol Brunovsky on the occasion of his 70th birthday

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Abstract. We investigate a dynamical order induced by coupling and/or noise in systems of coupled oscillators. The dynamical order is referred to a one-dimensional topological structure of the global attractor of the system in the context of random skew-product flows. We show that if the coupling is sufficiently strong, then the system exhibits one dimensional dynamics regardless of the strength of noise. If the coupling is weak, then it is shown numerically that the system also exhibits one dimensional dynamics provided the noise is sufficiently strong. We also show that for any coupling and any noise, the system has a unique rotation number and hence all the oscillators tend to oscillate with the same frequency eventually (frequency locking).

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