

Traveling Waves in Lattice Dynamical Systems

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Abstract

In this paper, we study the existence and stability of traveling waves in lattice dynamical systems, in particular, in lattice ordinary differential equations (lattice ODE's) and in coupled map lattices (CML's). Instead of employing the moving coordinate approach as for partial differential equations, we construct a local coordinate system around a traveling wave solution of a lattice ODE, analogous to the local coordinate system around a periodic solution of an ODE. In this coordinate system the lattice ODE becomes a nonautonomous periodic differential equation, and the traveling wave corresponds to a periodic solution of this equation. We prove the asymptotic stability with asymptotic phase shift of the traveling wave solution under appropriate spectral conditions. We also show the existence of traveling waves in CML's which arise as time-discretizations of lattice ODE's. Finally, we show that these results apply to the discrete Nagumo equation.

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