Shock wave propagation in composites and ferroic materials

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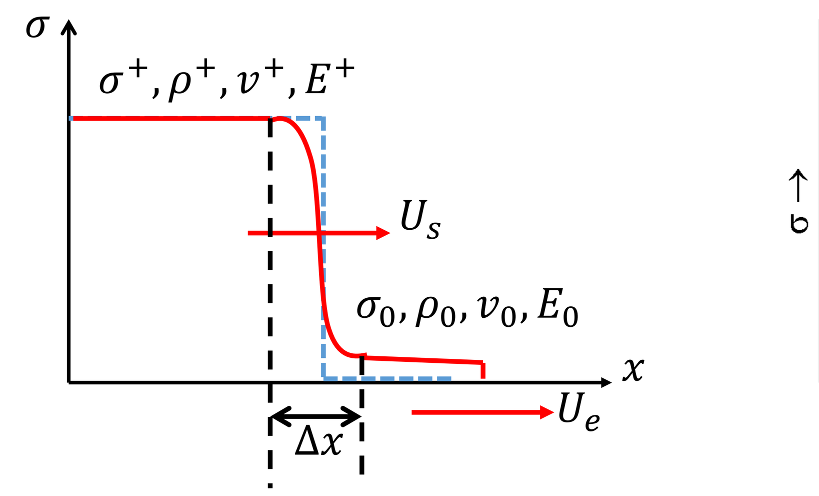
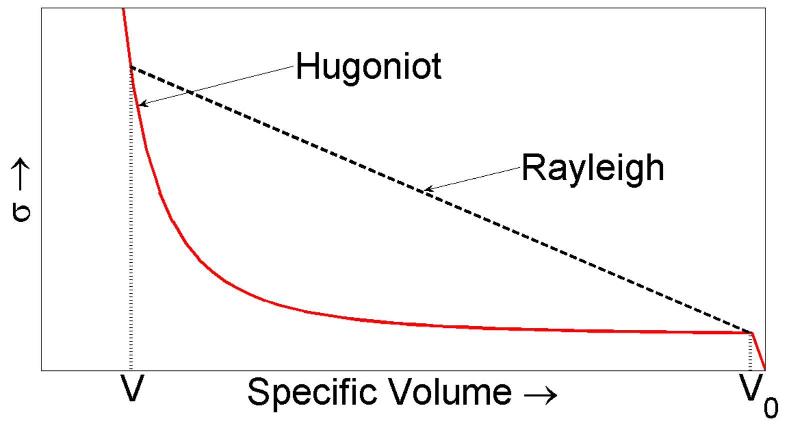
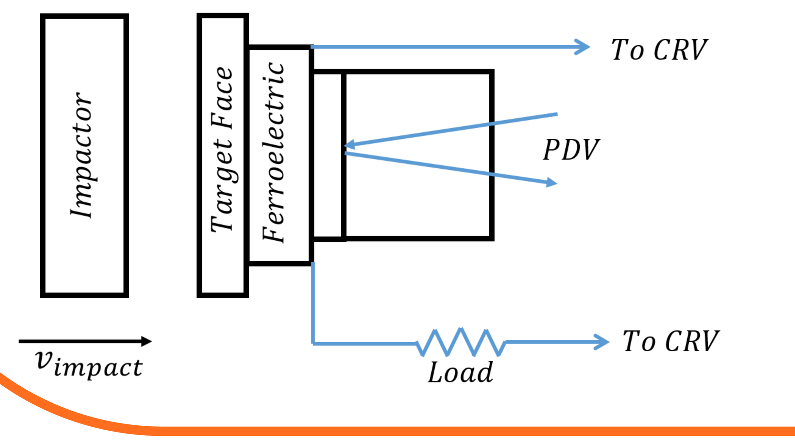
Introduction

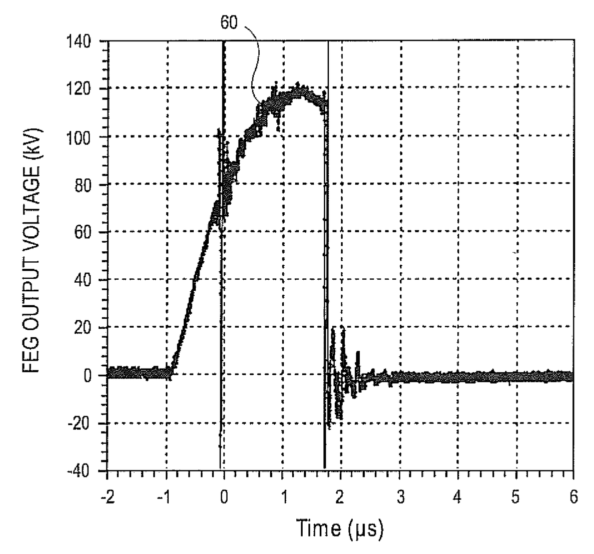
- Shock waves are generated in a material when it is subjected to high pressure high strain rate conditions – typically achieved during high velocity impact.
- Ideally, shock waves are characterized as jump discontinuities.
- In a heterogeneous material, the wave structure evolves to form a structured wave.
- In a ferroelectric material, where electrical and mechanical properties are coupled, we get a huge discharge of current or voltage depending on electrical boundary conditions.
- We study shock wave propagation in composites and ferroelectric materials.





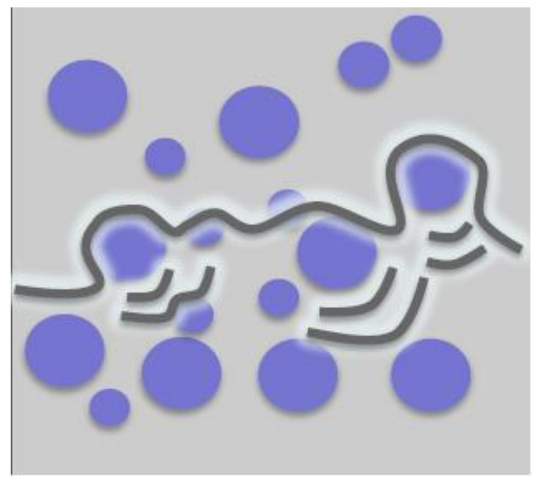
Evolution of structured shock waves (above, left and right). Schematic (below) of impact on ferroelectric material resulting in pulsed voltage output (right).





Front Propagation in a random medium

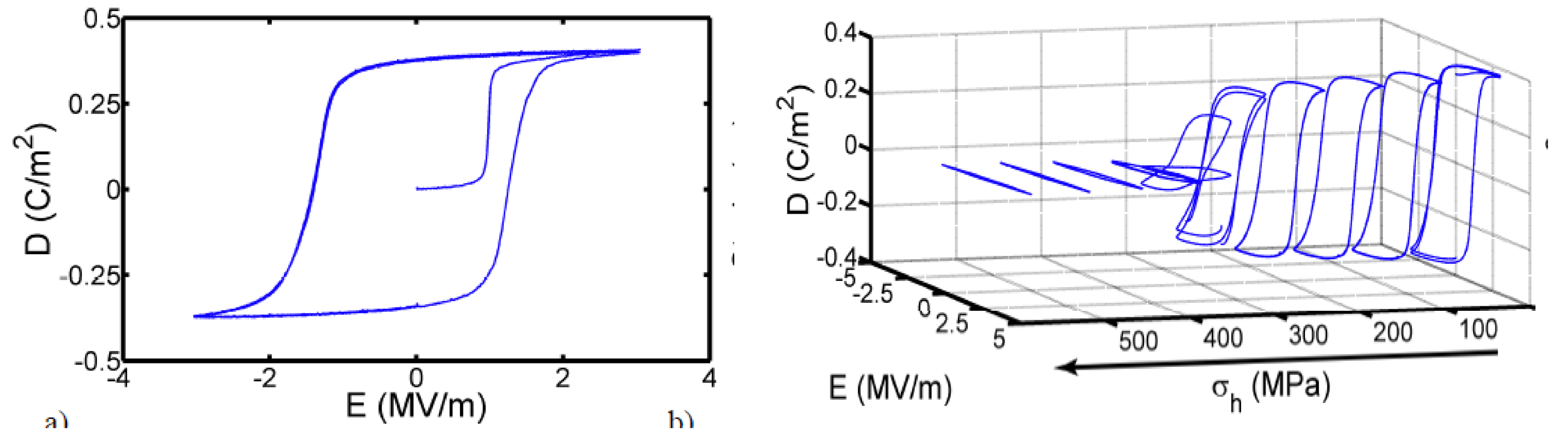
- A wave front traveling in a random medium, interacts with heterogeneities and becomes irregular. We study the evolution of irregularities using the concepts of geometric motion.
- Geometric motion $v_n = f(x)$
- Hamilton Jacobi equation
$$u_t(x,t) + f(x)|\nabla u| = 0$$
- Accounting for randomness and the length scale of heterogeneities
$$u_t^\varepsilon + f\left(x, \frac{x}{\varepsilon}, \omega\right)|\nabla u^\varepsilon| = 0$$
- Under the assumption of stationary ergodic medium, the stochastic Hamilton Jacobi equation can be homogenized to
$$u_t + \bar{H}(x)|\nabla u| = 0$$



- Signed distance function satisfies the stochastic Hamilton Jacobi equation.
- Need $f(x,\omega) \geq 0$ constraint to satisfy empty interior requirement.
- The probability of the local roughness to be r decays as $\exp(-r^5)$
- As an extension, we need a temporal dependence in the normal speed of the front. In case of shock waves, this relation is given by the kinetic relation.

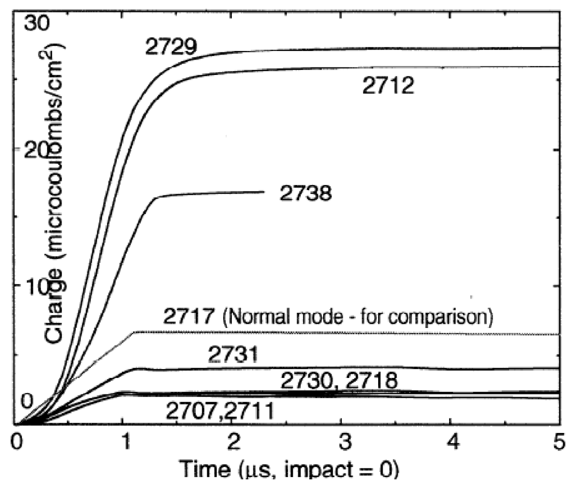
Shock waves in ferroelectric materials

- In a ferroelectric material, the electric and mechanical responses are coupled

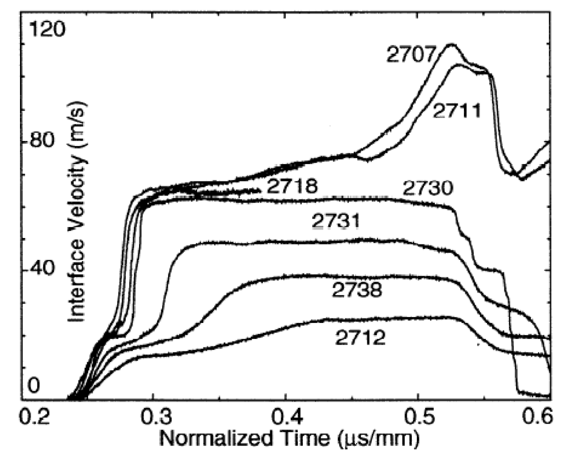


(Above left) Characteristic ($D - E$) response of a ferroelectric material. (Above right) Variation of $D - E$ loop with application of hydrostatic stresses (Valadez, et. al 2013).

- Materials like PZT 95/5 doped with 2% Niobium are most commonly used as ferroelectric generator.
- Various impact experiments have been conducted on different combinations of PZT under different orientations and boundary conditions.

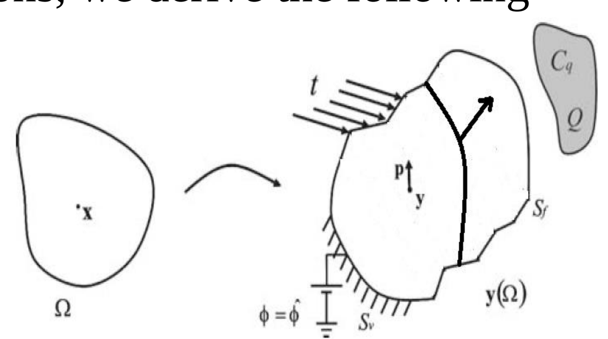


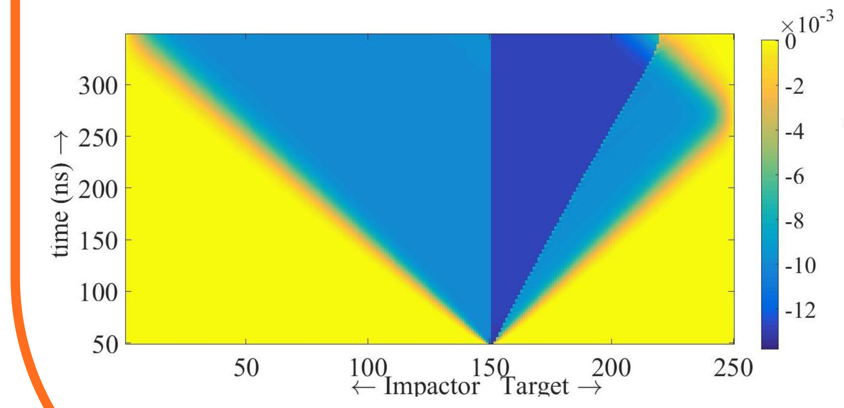
Experimental observations from an impact experiment conduced in Sandia National Lab under short circuit load. Variation of Charge with time (left) shows steady current output. Plot of the particle velocity at target end (right) shows a two wave structure.

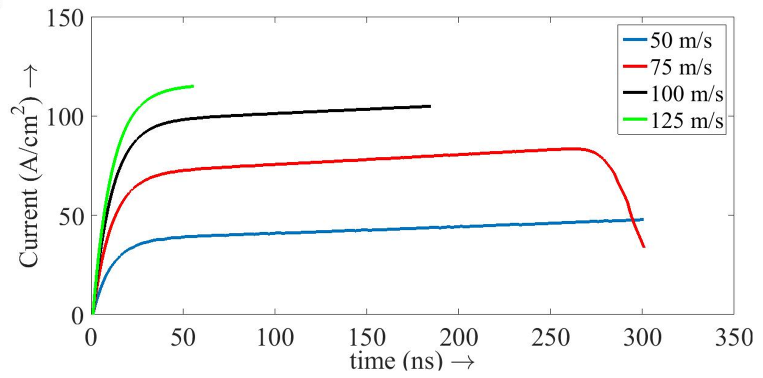


- We explore the exact physics of this process by formulating a set of equations in a continuum setting and then implanting them in a one-dimensional impact problem.
- There is a possibility of dielectric breakdown at high speeds. We account for that by making the phase boundary capable of holding surface charges.
- Using conservation laws and Maxwell equations, we derive the following set of governing equations.

$$-\frac{\partial W_0}{\partial \boldsymbol{p}_0} - \boldsymbol{F}^{-T}\nabla_x\phi = 0 \quad in\ B$$
$$\nabla_y \cdot (\boldsymbol{T} + \boldsymbol{T}_M) - \rho\ddot{\boldsymbol{y}} = 0 \quad in\ \boldsymbol{y}(B,t)$$
$$-\boldsymbol{T}\hat{\boldsymbol{n}} - \boldsymbol{T}_M\hat{\boldsymbol{n}} + \boldsymbol{t}\chi\big(\boldsymbol{y}(\partial B)\big) = 0 \quad on\ \partial\boldsymbol{y}(B)$$
$$\frac{dW_\sigma}{d\sigma} + \phi = 0 \quad on\ S \cap y(B)$$
$$[\![\boldsymbol{T}_0\hat{\boldsymbol{n}}_0]\!] + [\![(\hat{\boldsymbol{n}} \cdot \boldsymbol{T}_M\hat{\boldsymbol{n}})J\boldsymbol{F}^{-T}\hat{\boldsymbol{n}}_0]\!] + (W_\sigma + \phi\sigma)\kappa\langle J\boldsymbol{F}^{-T}\rangle\hat{\boldsymbol{n}}_0 = [\![\boldsymbol{t}_0]\!] \quad on\ S_0 \cap B$$
$$d = [\![H_0 + \langle\eta\rangle\theta]\!] + [\![\nabla_y\phi \cdot \boldsymbol{p}_0]\!] + [\![J(\hat{\boldsymbol{n}} \cdot \boldsymbol{T}_M\hat{\boldsymbol{n}})]\!] + (W_\sigma + \phi\sigma)\langle J\rangle$$
$$-[\![(\boldsymbol{F}\hat{\boldsymbol{n}}_0) \cdot \langle\boldsymbol{T}_0\hat{\boldsymbol{n}}_0 + (\hat{\boldsymbol{n}} \cdot \boldsymbol{T}_M\hat{\boldsymbol{n}})J\boldsymbol{F}^{-T}\hat{\boldsymbol{n}}_0]\!] \quad on\ S_0 \cap B$$



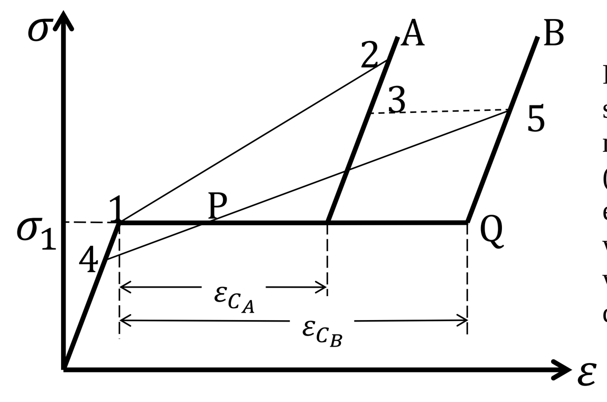




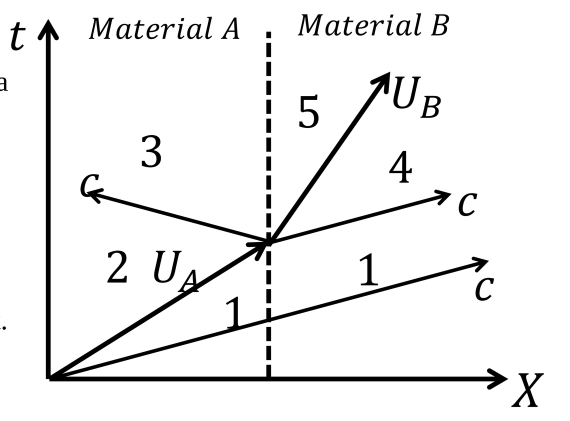
Impact of a linear elastic flyer on a ferroelectric material under short circuit load. The strain map on the X-t plot (left) shows the propagation of waves. The current profile (right) achieves a steady value.

Shock waves in layered composites

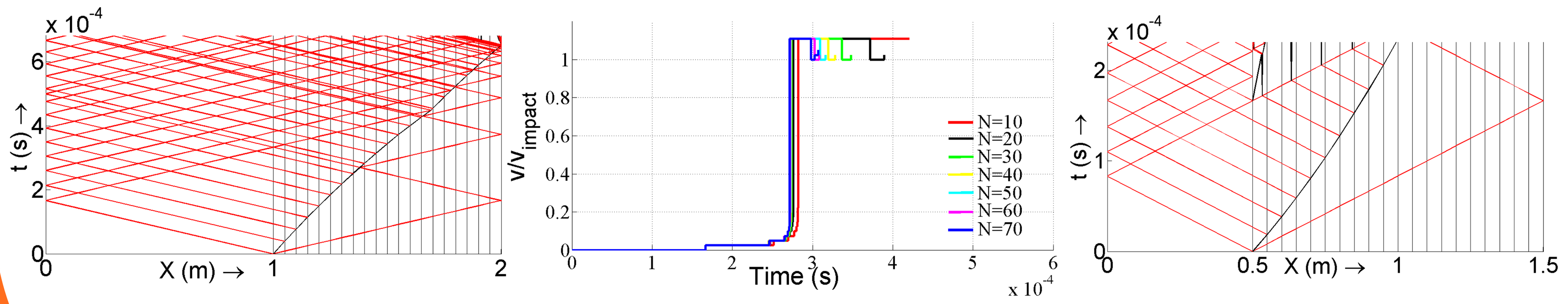
- Shock wave in a layered composite reflects and scatters off the material interfaces
- Structured shock waves form as a result of scattering
- We make a piecewise linear approximation of the Hugoniot.
- Then we classify all the interactions in the medium into set of Riemann problems.



Riemann problem for interaction of a shock wave at the interface of material A and B. The X-t diagram (right) shows the formation of two elastic waves and a shock wave when an incoming shock interacts with a phase boundary. Equivalent diagram in stress-strain space on left.



- For shock wave moving from material A to material B
$$U_B = \frac{rU_A c}{c + (r-1)U_A} \qquad r = \varepsilon_C^A / \varepsilon_C^B$$
- We solve for all the Riemann problems analytically and stich together individual solutions to obtain the general solution
- Layer arrangement should be from stiff to compliant along the shock propagation direction to reduce stresses and shock speeds
- Smooth gradient of properties ensure prevention of spallation



Sample X-t plot for the impact problem (above left). The red lines are elastic waves while the black line is a shock wave. Particle velocity plots at the free edge of the target for layer arrangement of alternate linear and nonlinear layers (above right). Length of impactor is long while N is changed keeping length of target constant. Possibility of spallation shown for impactor of smaller length (right).

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