

Variable Employment and Income in General Equilibrium*

HENRY THOMPSON

*Auburn University
Auburn, Alabama*

I. Introduction

Variable employment of labor is introduced into a general equilibrium economy by postulating that the unemployment rate varies inversely with income. *Monsieurs* Walras and Okun are rather crudely introduced in the back alleys of economics. This positive relationship between employment and output is a well established empirical regularity. Why fight it? In the present study, no effort is made to explain why it might hold. By analogy, the law of gravity may prove useful (Why did that apple fall on my head?), although on a microscopic level it becomes irrelevant.

General equilibrium analysis is in practice often based on the simplifying assumption that available resources are fully employed; factor supply is treated as perfectly inelastic. Cassel [1] was the first to explicitly utilize this simple approach to factor markets. Today this full employment assumption is often made international trade, public finance, and other areas.

Walrasian general equilibrium economics is not so restrictive concerning factor supply, which is pictured as sloping upward in the factor payment. Owners of labor are induced by a higher wage to consume less leisure. There would be in the aggregate an army (or a platoon) of voluntarily "unemployed" who would work for higher wages. Kemp and Jones [4] integrate variable labor supply into a general equilibrium setting, where results hinge on the labor supply elasticity, employment remaining "full" in that markets clear.

In the present study, the unemployment rate is inversely related with output, the relationship noted in the real world. Recent theoretical developments (job search, implicit contract, and efficiency wage models) explain how there could exist *some* level of unemployment. Given any unemployment at all, the comparative static link between output and employment is taken here to be straightforward: higher output must mean either more output per firm or more firms, with employment increasing. The desire is to develop the behavior of such an economy, examining it for consistency and stability.

Income and hence employment are endogenously determined in the model's general equilibrium. Comparative static analysis is performed for exogenous changes in prices of goods and

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endowments of capital and "available" labor. An increase in the endowment of available labor is found to lower the unemployment rate due to higher income and increased demand for labor.

II. The Model

Before turning to algebra and comparative statics, intuition can be developed with a geometric argument. Primary factors labor (L) and capital (K) are used to produce manufactures (M) and services (S). Regular isoquants representing unit values of M and S are located by their exogenous prices. Cost minimizing behavior insures that each unit value isoquant is supported by a common unit isocost line, with endpoints $p_j/w = 1/w$ and $p_j/r = 1/r$ ($j = M, S$). Factor inputs, functions of w and r , are thus determined at a_{ij} ($i = L, K$ and $j = M, S$). Production functions exhibit constant returns, and are homothetic, so each expansion path is linear at its input ratio.

Available endowments of capital and labor (K, L) are given. With all available capital and labor employed, outputs of M and S are determined. Employment N , however, depends on income which may be insufficient to employ all available labor. If so, output is biased towards capital intensive manufactures. As income rises, the ratio of services to manufactures increases, another empirical regularity. Production moves along a Rybczynski line as the production possibilities frontier expands.

The algebraic general equilibrium production model has been developed by a number of authors, notably Jones [3], Chang [2], and Takayama [6]; conditions of employment, cost minimizing behavior, and competitive pricing are its basic relationships. Let u represent the unemployment rate: $u \equiv (L - N)/L$; it is supposed that $u > 0$ or $N < L$. Okun's Law relates percentage changes in income with percentage point changes in the unemployment rate. As a variant, changes in the unemployment rate are assumed to be linearly related with changes in the level of income:

$$du = -\alpha dY, \quad \alpha > 0. \quad (1)$$

By straightforward differentiation, $du = (NdL - LdN)/L^2$. Substituting into (1),

$$dN - \alpha LdY = (1 - u)dL. \quad (2)$$

Notice that endogenous changes in employment N are thus related to changes in both endogenous national income and the exogenous endowment of labor. This equation becomes part of the structure of the general equilibrium, constraining behavior of the labor market. Labor moves between sectors in a circumscribed manner.

Employment is written

$$N = a_{LS}x_S + a_{LM}x_M, \quad (3)$$

where x_j represents output ($j = M, S$). Differentiating (3) and using (2) leads to

$$(1 - u)dL + \alpha LdY = \sum_j a_{Lj}dx_j + s_{LL}dw + s_{LK}dr. \quad (4)$$

Substitution terms s_{hk} ($h, k = L, K$) summarize the manner in which firms substitute between factors as factor payments change: $s_{hk} \equiv \sum_j x_j \partial a_{hj} / \partial w_k$. By Shephard's lemma and Taylor's formula, $s_{LK} = s_{KL}$. Full employment of capital yields a similar but simpler relationship,

$$dK = \sum_j a_{Kj} dx_j + s_{LK} dw + s_{KK} dr. \tag{5}$$

By competitive pricing of each good, $p_j = \sum_k w_k a_{kj}$. Differentiating then leads to two more equations of the model,

$$dp_j = a_{Lj} dw + a_{Kj} dr, \quad j = M, S, \tag{6}$$

given a cost minimizing envelope result, $w da_{Lj} + r da_{Kj} = 0$.

National income is written $Y = wN + rK$, and when differentiated yields

$$\Theta dY - K dr - N dw = (1 - u)w dL + rdK, \tag{7}$$

where $\Theta \equiv (1 - \alpha wL)$. Much depends upon the sign of Θ : $\Theta > 0$ iff $1 > \alpha wL$, which implies $\alpha = -du/dy < (1 - u)/wN$. Note that α relates changes in the unemployment rate to changes in the level of income. It thus has a very small magnitude, so the condition could be expected to hold.

An example will help drive this point home. Letting $\hat{}$ represent percentage change, the condition holds whenever $u/Y = -\alpha Y/u > -(1 - u)/u \Theta_N$, where $\Theta_N \equiv wN/Y$, labor's share of income. Suppose u is 5%. The increase in Y needed to decrease u to 4% (a 20% decrease in u) depends upon the size of Θ_N . If $\Theta_N = .75$, the percentage change in Y must be greater than only 0.79% for Θ to be positive. A lower value of Θ_N decreases the minimum necessary size of the percentage change in Y . For reasonable values of u and Θ_N , Θ is positive.

Equations (4) through (7) are collected into matrix form. Exogenous changes are isolated on the right:

$$\begin{bmatrix} s_{LL} & s_{LK} & a_{LS} & a_{LM} & -\alpha L \\ s_{LK} & s_{KK} & a_{KS} & a_{KM} & 0 \\ a_{LS} & a_{KS} & 0 & 0 & 0 \\ a_{LM} & a_{KM} & 0 & 0 & 0 \\ -N & -K & 0 & 0 & \Theta \end{bmatrix} \begin{bmatrix} dw \\ dr \\ dx_S \\ dx_M \\ dY \end{bmatrix} = \begin{bmatrix} (1 - u)dL \\ dK \\ dp_S \\ dp_M \\ (1 - u)w dL + rdK \end{bmatrix}$$

Partial derivatives of each of the five endogenous variables (w, r, x_S, x_M, Y) with respect to any of the four exogenous variables (L, K, p_S, p_M) are found with Cramer's rule. It is assumed that labor is used intensively in services, and capital in manufactures: $b \equiv a_{LS}a_{KM} - a_{LM}a_{KS} > 0$. The determinant D of the system is positive: $D = \Theta b^2$.

Comparative static results are presented in the Appendix, and signs of these results are summarized:

	∂w	∂r	∂x_S	∂x_M	∂Y
∂L	0	0	+	-	+
∂K	0	0	?	?	+
∂p_S	+	-	+	-	+
∂p_M	-	+	?	?	+

III. Results

The gradient of income with respect to any exogenous variable is positive. Since the unemployment rate is negatively related with income; an increase in any exogenous variable lowers the unemployment rate. As employment of labor increases, output adjusts along a Rybczynski line.

When the amount of available labor L increases, income and employment rise. It follows from the results in Table I that

$$\partial u / \partial L = (du/dY)(\partial Y / \partial L) = -\alpha(1-u)w/\Theta < 0.$$

An increase in the amount of available labor increases income and the demand for labor, unambiguously lowering the unemployment rate. This could be anticipated from (7), and is a prime example of the value of general as opposed to partial equilibrium modelling.

Changing factor endowments do not ultimately affect factor payments in the general equilibrium. Two regions or countries with differing endowments but identical prices through free trade will have the same factor payments, the factor price equalization result. With an increase in the amount of available labor, output of labor intensive services expands as output of manufactures declines; labor and capital move into services, as factor payments return to their original level. This equalization result occurs because the model has the same number of factors and international markets. In general, factor substitution plays a role in determining how a changing labor endowment affects factor payments.

Payment to a factor is positively related with the price of the good in the sector where it is used intensively, and negatively related with the other price, the Stolper-Samuelson result. These algebraic results, in fact, are identical to those with full employment. Changing prices affect income and the employment of labor, but any change in employment has no "secondary" effect on factor payments because of factor price equalization.

Output of labor intensive services is positively related, and output of manufactures negatively related, with the labor endowment; this is the expected Rybczynski result. The induced increase in employment of labor N complements the increased labor endowment.

Adjustments in outputs are ambiguous with changes in the endowment of capital. Income and employment of labor are spurred through an increase in the capital endowment; the induced increase in employment pushes the composition of output in the opposite direction. Outputs of both sectors could in fact rise, which implies a partial qualification of the basic Heckscher-Ohlin result. A marginally capital abundant region will not necessarily export the capital intensive good, with identical tastes across regions; Ruffin [5] develops the full employment argument.

Effects of changing prices on outputs are summarized by the $\partial x / \partial p$ results. As the relative price of a good rises with full employment, its output rises and the other falls along the production frontier. With variable employment, expansion or reduction of the frontier occurs, outputs moving along a Rybczynski line. When the price of manufactures rises, its output would rise as services output falls, but the induced increase in employment has just the opposite effect; this creates the noted ambiguity. With an increase in the price of services, the induced increase in employment pushes the composition of output in the same direction as the change in price.

IV. Conclusion

This paper develops a simple way to introduce a variable employment into general equilibrium models of production. It is observed that employment depends on income in real world economies, so the added structure may help mirror reality. An interesting implication is that the unemployment rate is inversely related with the amount of available labor. In other words, immigration lowers the unemployment rate in the long run.

Table I. Comparative Static Results

	∂w	∂r	∂x_S	∂x_M	∂Y
∂L	0*	0*	$(1-u)a_{KM}/\theta b$	$-(1-u)a_{KS}/\theta b$	$(1-u)w/\theta$
∂K	0*	0*	$-a_{LM}/b^* + a_{KM}r\alpha L/\theta b$	$a_{LS}/b^* - a_{KS}r\alpha L/\theta b$	r/θ
∂p_S	a_{KM}/b^*	$-a_{LM}/b^*$	$s_1/b^2 + a_{KM}\beta_M\alpha L/D$	$-s_3/b^2 - a_{KS}\beta_M\alpha L/D$	$\beta_M/\theta b$
∂p_M	$-a_{KS}/b^*$	a_{LS}/b^*	$-s_3/b^2 + a_{KM}\beta_S\alpha L/D$	$s_2/b^2 - a_{KS}\beta_S\alpha L/D$	$\beta_S/\theta b$

*Results from the full employment model.

Appendix

Comparative static results are presented in Table I. The following notation is employed:

$$\beta_M \equiv a_{KM}N - a_{LM}K,$$

$$\beta_S \equiv a_{LS}K - a_{KS}N,$$

$$s_1 \equiv 2a_{LM}a_{KM}s_{LK} - a_{LM}^2s_{KK} - a_{KM}^2s_{LL},$$

$$s_2 \equiv 2a_{LS}a_{KS}s_{LK} - a_{LS}^2s_{KK} - a_{KS}^2s_{LL}, \text{ and}$$

$$s_3 \equiv (a_{LS}a_{KM} + a_{KS}a_{LM})s_{LK} - a_{LS}a_{LM}s_{KK} - a_{KS}a_{KM}s_{LL}.$$

Employment point (K, N) is assumed to be within the appropriate production cone to avoid complete specialization and redundant factors: so $a_{LS}/a_{KS} > N/K > a_{LM}/a_{KM}$. Thus β_M and β_S are positive. Substitution terms s_{KK} and s_{LL} are negative, while s_{LK} is positive; when payment to an input rises, firms use less of it and more of the other inputs. Thus $s_1, s_2,$ and s_3 are all positive, as are all the terms in Table I.

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