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PRODUCTION WITH TWO FACTORS AND MANY GOODS: LARGE FIRMS IN A SMALL OPEN ECONOMY

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A tractable general equilibrium model of a small open economy producing many goods with two primary inputs is developed. Firms are large in that their output decisions affect their costs. One sector produces many different goods under variable costs, which arise through a link between output and the cost of the firm. Comparative static results depend on factor intensity and the degree of increasing costs. Some ambiguities arise in the comparative static adjustments associated with the sector producing the constant cost homogeneous good. [F1]

1. INTRODUCTION

The present paper develops a model of a small open economy with two factors and many goods. One sector produces a homogeneous good, while the other sector produces many goods differentiated by their cost functions. Output effects create variable costs in the production of the different goods. Cost minimizing factor inputs are linked directly to output, and the average cost of the firm increases with its output. Average cost of the firm is thus independent of price, leading to profit. Firms in this sector are large in the small open economy, which may describe the industrial structure of many countries.

Properties of the general equilibrium theory of production and trade depend to a degree on the relative number of productive factors and international markets, as developed by Ethier (1974), Jones and Scheinkman (1976), Chang (1979), Ethier and Svensson (1986), and others. The small open economy assumption exogenizes prices of internationally traded goods at world levels. Domestic demand can be explicitly introduced, as suggested by Dixit and Norman (1980). Nontraded goods also increase the degrees of freedom. Still, the notion of a small price taking economy completely open to international trade has appeal as a paradigm.

Comparative static results in the present model generally follow the pattern suggested by factor intensity, but some ambiguity arises in the Rybczynki-Stolper-Samuelson results. The factor price equalization property does not hold, which implies that migration and foreign investment will affect factoral income distribution.

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The elasticity of factor demand in the general equilibrium depends on long run average cost in the sector producing the different goods. The production possibilities frontier generally remains concave, and the number of different goods varies proportionately with their aggregate output.

2. PRODUCTION AND COST OF THE DIFFERENT GOODS

Cost for one of the different goods is written

$$c_j = c_j(w, x_j) \tag{1}$$

where w is the vector of factor prices and x_j is the output of good j. The price of each factor is the same throughout the economy since factors move freely. By Shephard's lemma, $\partial c_j/\partial w_i = v_{ij}$, where v_{ij} is the input of factor i in good j production. The unit input of factor i is then $a_{ij} = v_{ij}/x_j = (\partial c_j/\partial w_i)/x_j$, and average cost can be written $\alpha_j = \Sigma_i w_i a_{ij} = [\Sigma_i w_i(\partial c_j/\partial w_i))/x_j]$. With constant returns to scale, each of the a_{ij} functions is homogeneous of degree zero in w. Consider the effect of a change in output on a_{ij},

$$\begin{aligned}\partial a_{ij}/\partial x_j &= \partial(v_{ij}/x_j)/\partial x_j = [x_j(\partial v_{ij}/\partial x_j) - v_{ij}]/x_j^2 \\ &= [\partial^2 c_j/\partial w_i \partial x_j - a_{ij}]/x_j = \beta_{ij}.\end{aligned} \tag{2}$$

The cost function includes an interaction term between each factor price and output. For simplicity in the comparative static analysis, assume β_{ij} is constant.

The effect of a change in output on the average cost of the firm can be written

$$\begin{aligned}\partial \alpha_j/\partial x_j &= \partial(\Sigma_i w_i a_{ij})/\partial x_j = \Sigma_i[w_i(\partial a_{ij}/\partial x_j) + a_{ij}(\partial w_i/\partial x_j)] \\ &= \Sigma_i w_i \beta_{ij} = \beta_j,\end{aligned} \tag{3}$$

given $\partial w_i/\partial x_j = 0$. Both w_i and x_j are endogenous variables in the general equilibrium model, and have no a priori links. Average cost of the firm is linear in output. Putting (3) into elasticity form,

$$\beta_j = \Sigma_i (w_i a_i / x_j)(a_{ij} / x_i) = (\hat{p} / \hat{x}) \Sigma_i \theta_{ij} a_{ij} / x_j, \tag{4}$$

where ^ represents percentage change and θ_{ij} is a factor share. Factors with a larger share thus weigh more heavily in the term β_j, which summarizes the link between output and average cost. Production is homothetic in both sectors. In the homogeneous competitive sector, by contrast, $\partial a_{iH} / \partial x_H = 0$ and $\beta_H = 0$. The link between output and factor inputs is a systematic structural friction in the production of the different goods.

Increasing costs occur if the prices of factors used more intensively rise with output, as indicated by (4). Demand for the intensive factor then increases more than supply in the expanding sector. This approach to external effects differs from the variable returns models of Jones (1968), Herberg and Kemp (1969), Kemp (1969), Panagariya (1980), and Ishikawa (1994). Markusen and Schweinberger (1990) introduce industrial production externalities based on output levels and analyze properties of the aggregate cost function.

Equating marginal cost MC_j with marginal revenue $MR_j = p_j$, each firm determines its profit maximizing output x_j. With full employment, aggregate output x_D is the sum of the output of each of the many different goods. Let n be the number of different goods. Thus $x_D = \Sigma_{j=1}^{n} x_j$. For simplicity, rescale the original static output of each good x_j to unity, and $x_D = nx_j = n$. Output of each good can then be written

$$x_j = x_D / n. \tag{5}$$

This simplifying rescaling applies to the original static equilibrium in the comparative static analysis, and does not affect the generality of the qualitative results.

Marginal cost is written $MC_j = \partial c_j / \partial x_j = \partial (x_j \alpha_j) / \partial x_j = \alpha_j + (\partial a_j / \partial x_j) x_j = \alpha_j + \beta_j x_j$. Marginal cost is also linear in output, and has twice the slope of average cost: $\partial MC_j / \partial x_j = \partial \alpha_j / \partial x_j + \beta_j = 2\beta_j$. Profit is maximized where marginal revenue p_j equals marginal cost,

$$p_j = \alpha_j + \beta_j x_j = \alpha_j + \beta_j x_D / n. \tag{6}$$

Firms perceive variable costs, even though their production function has constant returns. The link between output and factor prices is known by the large firm, which explicitly considers the variable costs in (6).

Prices of goods remain at exogenous world levels for the small open economy. Prices of the n different goods are aggregated to a single price p_D. The same aggregator function $\phi(p_1, \ldots, p_n) = p_D$ is used to arrive at an aggregate α and β for the sector: $\phi(\alpha_1, \ldots, \alpha_n) = \alpha_D$ and $\beta_D = \phi(\beta_1, \ldots \beta_n)$. The aggregated p_D could be a simple weighted average of the individual prices, using output shares as weights. The profit maximizing condition in (6) can then be written in aggregate form for the sector as

$$p_D = \alpha_D + \beta_D x_D / n. \tag{7}$$

The level of profit π_j for each good depends directly on the value of β_j, $\pi_j = (p_j - \alpha_j)x_j = (p_j - \alpha_j)x_D/n = (\beta_j/n)x_D/n = \beta_j x_D n^2$, using (6). Profit for the "typical firm" in the sector is found using the aggregate p_D and α_D:

$$\pi_D = (p_D - \alpha_D)x_D/n = \beta_D x_D^2/n^2. \tag{8}$$

When $\beta_D > 0$, there are increasing long run average cost and $\pi_D > 0$.

Residual claimants on average enjoy gains if $\pi_D > 0$. The total value of output is thus not necessarily exhausted among productive inputs. To close the model, any excess income is spent on imported final goods which have no effect on domestic production or the pattern of trade. An example might be foreign vacations. Once again, this assumption may fit the situation of many small countries.

3. THE GENERAL EQUILIBRIUM MODEL

The general equilibrium model with two factors and two goods of Jones (1965) provides the framework for the comparative statics. Aggregate factor price substitution terms

$$s_{hi} = \Sigma_g x_g \partial a_{hg} / \partial w_i \tag{9}$$

between factors h and i summarize how firms respond to changing factor payments as they minimize cost. The index g runs across the different goods j and the homogeneous good H. The own substitution terms s_{KK} and s_{LL} are negative due to concavity of the cost function and Shephard's lemma. Also $s_{KL} = s_{LK}$ due to Young's theorem. With only two inputs, K and L must be substitutes: $s_{LK} > 0$. Full employment of capital and labor leads to the first two equations in the comparative static system (12).

The unit cost of producing one of the different goods is written $\alpha_j = a_{Kj}r + a_{Lj}w$. The aggregator function ϕ is used to arrive at unit inputs for the sector: $a_{KD} = \phi(a_{k1}, \ldots, a_{Kn})$ and $a_{LD} = \phi(a_{L1}, \ldots, a_{Ln})$. Average cost of the aggregated different good is then $\alpha_D = a_{KD}r + a_{LD}w$. Differentiating the unit cost equation and using the cost minimizing envelope result,

$$d\alpha_D = a_{KD}dr + a_{LD}dw. \tag{10}$$

The fourth equation in (12) comes from (10) and differentiating the aggregated profit maximizing condition in (8),

$$p_D = d\alpha_D + (\beta_D/n)dx_D - (\beta_D x_D/n^2)dn. \tag{11}$$

If $\beta_D > 0$, profit is positively associated with output: $\partial\pi_D/\partial x_D = 2\beta_D x_D/n^2 > 0$. Note also that if $\beta_D > 0$, $\partial\pi_D/\partial n = -2\beta_D x_D^2/n^3 < 0$. Fully differentiating (8), $d\pi_D = (2\beta_D x_D/n^2)dx_D - (2\beta x_D/n^3)dn$. Profit increases with exit and falls with entry when $\beta_D > 0$.

For simplicity, assume comparative static changes lead to no change in π_D. Entry and exit, in other words, occur to keep π_D constant. Then $d\pi_D = 0$ and $ndx_D - x_D dn = 0$, the last equation in the comparative static system in (12). Note that dx_D and dn will always have the same sign. Total output of the different goods increases (decreases) if and only if the number of firms increases (decreases). An alternative assumption which leads to identical qualitative outcomes is that profit has a constant relationship with the number of firms: $\partial\pi_D/\partial n$ is some negative constant. Then π_D has a positive relationship with x_D and replaces n in the comparative statics.

Competitive pricing in the homogeneous industry leads to the third equation in (12):

$$\begin{bmatrix} s_{KK} & s_{LK} & a_{KH} & a_{KD} & 0 \\ s_{LK} & s_{LL} & a_{LH} & a_{LD} & 0 \\ a_{KH} & a_{LH} & 0 & 0 & 0 \\ a_{KD} & a_{LD} & 0 & \beta_D/n & -\beta_D x_D/n^2 \\ 0 & 0 & 0 & n & -x_D \end{bmatrix} \begin{bmatrix} dr \\ dw \\ dx_H \\ dx_D \\ dn \end{bmatrix} = \begin{bmatrix} dK \\ dL \\ dp_H \\ dp_D \\ 0 \end{bmatrix} \tag{12}$$

Clearly isolated are exogenous changes in factor endowments (dK and dL) and prices (dp_D and dp_H). These exogenous changes cause comparative static adjustment from one equilibrium to another. Cramer's rule is used to find the comparative static partial derivatives.

4. THE COMPARATIVE STATIC RESULTS

Factor intensity is captured by the sign of $b \equiv a_{KH}a_{LD} - a_{KD}a_{LH}$. If $b < 0$, $a_{KH}/a_{LH} > a_{KD}/a_{LD}$ and the sector producing the different goods is capital intensive. If $b > 0$, production of the homogeneous good is capital intensive. Output of the homogeneous good adjusts to maintain full employment. Cost minimization determines unit factor inputs a_{KH} and a_{LH}, which are homogeneous of degree zero in factor prices.

Comparative static results are reported in Table 1, with this notation:

$$\begin{aligned}
s_j &\equiv 2a_{Lj}a_{Kj}s_{LK}^2 - a_{Kj}s_{LL} - a_{Lj}s_{KK} > 0, j = H, D \\
s &\equiv (a_{KD}a_{LH} + a_{LD}a_{KH})s_{LK} - a_{KH}a_{KD}s_{LL} - a_{LH}a_{LD}s_{KK} > 0 \\
s_L &\equiv a_{LH}s_{LK} - a_{KH}s_{LL} > 0 \\
s_K &\equiv a_{KH}s_{LK} - a_{LH}s_{KK} > 0 \\
s_2 &\equiv s_{KK}s_{LL} - s_{LK}^2 > 0.
\end{aligned} \tag{13}$$

The determinant of the system in (12) is $\Delta = (\beta_D s_H/n) + b^2$. If $\beta_D > 0$, then $\Delta > 0$. With strong enough decreasing costs, the production frontier would be convex, a familiar result.

Table 1. Comparative Static Results

	∂r	∂w	∂x_H	∂x_D	∂n
∂K	$-a_{LH}\beta_D^2/n\Delta$	$a_{KH}a_{LH}\beta_D/n\Delta$	$[(\beta_D s_L/n) + a_{LD}b]/\Delta$	$-a_{LH}b/\Delta$	$-nba_{LH}/x_D\Delta$
∂L	.	$-a^2_{KH}\beta_D/n\Delta$	$[(\beta_D s_K/n) - a_{KD}b]/\Delta$	$a_{KH}b/\Delta$	$nba_{LH}/x_D\Delta$
∂p_H	.	.	$[(\beta_D s_2/n) + s_D]/\Delta$	$-s/\Delta$	$-2ns/x_D\Delta$
∂p_D	.	.	.	s_H/Δ	$2ns_H/x_D\Delta$

Table 2 presents the sign pattern of comparative static results with increasing cost. The production frontier is locally concave. Factor intensity plays a role in the Rybczynski effects of changing endowments on outputs and the symmetric Stolper-Samuelson effects of changing prices on factor prices.

Table 2. Comparative Static Results With Capital (Labor) Intensive Different Goods

	∂r	∂w	∂x_H	$\partial x_D, n$
∂K	–	+	?(+)	+(–)
∂L	+	–	+(?)	–(+)
∂p_H	?(+)	+(?)	+	–
∂p_D	+(–)	–(+)	–	+

A changing endowment of either capital or labor would lower the price of that factor because of its increased supply, while raising the price of the other factor through increased productivity. Factor endowment changes thus affect factor prices, unlike competitive models with factor price equalization. Panagariya (1983) finds this same result in a model with variable returns to scale. A country with a relative abundance of labor would have relatively low wages even with free trade. The same can be said for capital and rents. This property is perhaps more intuitive than factor price equalization and its identical factor prices with free trade.

Aggregate output of the different goods is positively associated with the endowment of its intensive factor, and negatively associated with the other factor endowment. A change in the aggregate price of the different goods affects factor prices as expected according to factor intensity. These two sets of results are similar to the standard Heckscher-Ohlin model.

It is surprising that some ambiguity arises concerning the homogeneous good, a property out of line with standard results. An change in its price always has an ambiguous effect on the price of a factor. For instance, when the homogeneous good is labor intensive, the term $\partial r/\partial p_H$ may not be negative as anticipated. An increase in p_H pulls resources into the production of the homogeneous good, and both factor prices may be bid up in the process. Generally, a rising price causes some factor price to fall. Greater substitution (a larger s_L) favors a positive sign for $\partial r/\partial p_H$. The higher p_H causes the wage to rise, and firms substitute toward capital. A higher degree of substitution means that the demand for capital in the expanding sector increases enough to actually increase the price of capital.

When the homogeneous good uses capital intensively, ambiguity switches to the effects on wages. The implication is that the Stolper-Samuelson effect linking the price of a good with the price of the factor used intensively in its production is relaxed. A tariff on imports might not, in other words, raise the price of the factor used intensively in the import competing sector. The presence of the variable costs in (6) means the standard intuition regarding the effects of tariffs on income redistribution must be modified. In particular circumstances, characteristics of the underlying cost functions must be examined.

The projected pattern of trade is also ambiguous. Suppose there are increasing costs in the capital intensive sector producing different goods, and identical homothetic preferences across two countries. Let the two countries start with identical endowments, and let one unit of capital move from the foreign to the home country as in Ruffin (1977). The capital abundant home country would have a higher output x_D, but might not produce a lower ratio x_H/x_D. The home country could export the labor intensive homogeneous good. The direction of change in the output ratio x_H/x_D has the same sign as the expression $nbL + x_D\beta_D s_L/n$.

When the sector producing the different goods is capital intensive, an increase in K raises the number of different goods. Output x_D clearly rises as entry occurs when the endowment of K rises. When the different goods are labor intensive, $\partial n/\partial K < 0$. Output x_D would then fall due to exit when K rises. The $\partial n/\partial L$ results can be similarly analyzed.

The number of different goods is positively associated with their aggregate price: $\partial n/\partial p_D > 0$. A higher price for the homogeneous good attracts resources and causes the number of different goods to fall: $\partial n/\partial p_H < 0$.

5. CONCLUSION

The present tractable model of production for a small open economy with two primary factors and many goods offers an efficient way to add a richness of structure to the fundamental theory of competitive production and trade. A firm in the increasing cost industry is large in that its output affects factor markets, and this "externality" is part of each firm's profit maximizing decision making. The cost function in the present model has a straightforward empirical interpretation, namely interaction terms between output and factor prices. Aggregation is based on prices of the different goods, which allows each good a separate identity and keeps the model relatively simple.

An advantage of this model over high dimensional general equilibrium models is that many goods can be produced with only two factors of production. Introducing domestic demand would yield a tractable model, but utility functions would have to be specified. The advantage of the present approach over the continuum approach of Dornbusch, Fischer, and Samuelson (1980) is that there are no "flats" in the production frontier and the number of goods is determinate. For application, the critical issue is empirical. If cost functions of the firm are found to include a significant interaction term between their output and factor prices, the production structure in the present paper becomes relevant.

One implication of the present model is that factor price equalization does not hold. Labor (capital) abundant countries would maintain low wages (rents) even in the presence of free trade. Stolper-Samuelson and Rybczynki effects might also be relaxed. A tariff would not necessarily raise the real price of the factor used intensively in its production, and endowment levels would not necessarily predict trade flows.

Computable general equilibrium models can embody qualifications to the core results of competitive trade theory. A feel for general equilibrium models will develop through empirical applications to different countries. The main issues in international trade remain general equilibrium in nature. Trade theory may be developing to the point that empirical investigation will provide core scientific developments: dropping some assumptions, revising parts of the theory, and accepting a range of values for some parameters. The present model of variable costs in general equilibrium is meant to provide one avenue for such empirical investigation.

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