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# Trade and International Factor Mobility

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## I. Introduction

Free international mobility of a factor implies that its payment will be equalized across national boundaries, just as it is equalized internally by migration between industries. The assumption of (international) factor immobility is in fact the seed which gives rise to international trade as a separate field of inquiry. The trade model of a small, open economy containing two factors and two goods becomes unworkable if one factor is allowed to be mobile. This paper utilizes the three factor, two good (3 x 2) model, where one of the factors is internationally mobile.

The 3 x 2 model, allowing for complementarity in production, has received recent attention in the literature [Batra and Casas, 1976; Ruffin, 1981; Jones and Easton, 1983; Suzuki, 1983; and Thompson and Clark, 1983]. It is known [Mundell, 1957] that free mobility of one factor can substitute for free trade in the imported good. The Lerner Pearce diagram teaches one, however, that a small 2 x 2 country facing world prices and world payment to one factor can levy only one tariff rate, and it may be zero.

Section II presents the general equilibrium trade model, introducing an internationally mobile factor. Section III presents the 3 x 2 version of this mobile factor model. Section IV examines the relationships between factor payments and endowments. Section V studies the effects upon output of an exogenous change in the payment to the mobile factor, or symmetrically the effect of changing world prices upon the endogenously determined employment of the mobile factor.

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## II. The Mobile Factor Model

The general equilibrium trade model is based upon constant returns to scale production and inelastic supplies of immobile factors, as developed in the literature [Jones and Scheinkman, 1976; and Chang, 1979]. Factor endowments are represented by $v_i$, their payments by $w_i$, $i = 1, \ldots r$. The economy's outputs $x_j$ have prices $p_j$, $j = 1, \ldots, n$, determined in world markets; our country is small in its inability to affect world prices. Unit factors mixes, $a_{ij}$'s, describe the amount of factor $i$ used to produce one unit of good $j$, and are homgeneous of degree zero in factor payments. Interfactor technical relationships are summzarized by the terms $s_{hk} = \Sigma_j x_j \partial a_{hj}/\partial w_k$. It is known that $s_{hh} < 0$, and $s_{hk} = s_{kh}$. The homogeneity of the $a_{ij}$'s implies that $\Sigma_i w_i s_{hi} = 0$. Factors are rescaled here to acquire unit factor payments, so that $\Sigma_i s_{hi} = 0$.

Differentiating the statement of full employment, $v_k = \Sigma_j a_{kj} x_j$,

$$dv_k = \Sigma_i s_{ki} dw_i + \Sigma_j a_{kj} dx_j. \quad (1)$$

The cost minimizing behavior of firms insures that $\Sigma_i w_i da_{ij} = 0$. Differentiating the competitive pricing statement, $p_j = \Sigma_i a_{ij} w_i$,

$$dp_j = \Sigma_i a_{ij} dw_i. \quad (2)$$

Equations (1) and (2) constitute the basic general equilibrium trade model, which can be summarized in matrix form,

$$\begin{bmatrix} S & A \\ A & O \end{bmatrix} \begin{bmatrix} dw \\ dx \end{bmatrix} = \begin{bmatrix} dv \\ dp. \end{bmatrix} \quad (3)$$

Endowments and prices are treated as exogenous, the model endogenously determining wages and outputs.

The wage of an internationally mobile factor is exogenously determined in a world market. The country considered is a price taker in the market for the mobile factor, referred to as factor 1. Given $w_1$, our model endogenously determines the level of factor 1 employed, $v_1$. When we isolate exogenous variables, Equation 1 becomes

$$-s_{11}dw_1 = -dv_1 + \Sigma_{i\neq 1} s_{1t}dw_i + \Sigma_j a_{1j}dx_j \,, \tag{4}$$

and

$$dv_h - s_{h1}dw_1 = \Sigma_{i\neq 1} s_{ih}dw_i + \Sigma_j a_{hj}dx_j \,, \quad h \neq 1. \tag{5}$$

From 2 we find

$$dp_j - a_{1j}dw_1 = \Sigma_{i\neq 1} a_{ij}dw_i \,. \tag{6}$$

Equations 4, 5, and 6 can be combined into matrix form as in 3. To solve for the model's comparative statics, the partial derivative of an endogenous variable with respect to some exogenous variable may be found using Cramer's rule. The 2 x 2 mobile factor model exhibits functional dependence, since the system determinant equals zero. The exogenous world payment $w_1$ may be inconsistent with the isocost line uniquely supporting the two unit value isoquants.

## III. The 3 x 2 Mobile Factor Model

With three factors and two goods, the mobile factor model can be written

$$\begin{bmatrix} -1 & s_{12} & s_{13} & a_{11} & a_{12} \\ 0 & s_{22} & s_{23} & a_{21} & a_{22} \\ 0 & s_{23} & s_{33} & a_{31} & a_{32} \\ 0 & a_{21} & a_{31} & 0 & 0 \\ 0 & a_{22} & a_{32} & 0 & 0 \end{bmatrix} \begin{bmatrix} dv_1 \\ dw_2 \\ dw_3 \\ dx_1 \\ dx_2 \end{bmatrix} = \begin{bmatrix} -s_{11}dw_1 \\ dv_2 - s_{12}dw_1 \\ dv_3 - s_{13}dw_1 \\ dp_1 - a_{11}dw_1 \\ dp_2 - a_{12}dw_1 \end{bmatrix} \tag{7}$$

The structure of the technology matrix is captured in the following terms: $b_1 \equiv a_{11}a_{22} - a_{12}a_{21}$, $b_2 \equiv a_{21}a_{32} - a_{22}a_{31}$, $b_3 \equiv a_{11}a_{32} - a_{12}a_{31}$, $c_1 \equiv b_1 + b_2$, $c_2 = b_1 - b_2$, and $c_3 = b_1 + b_3$. The system determinant $D$ of 7 is negative, since $D = -b_2^2$. We impose the factor ordering condition, or foc, (i) $a_{11}/a_{12} > a_{21}/a_{22} > a_{31}/a_{32}$; the mobile factor is most intensive or extreme in industry 1. We shall also consider foc (ii) $a_{21}/a_{22} > a_{11}/a_{12} > a_{31}/a_{32}$, where factor 1 is the middle factor. The notion of extreme and middle factors has been developed in the literature [Ruffin, 1981]. These two conditions exhaust the theoretically interesting possibilities. With (i) $c_1$ and $c_3$ are positive, with the sign of $c_2$ uncertain. Condition (ii) implies that $c_1 > 0$, and $c_2 < 0$, with the sign of $c_3$ uncertain.

Restrictions on the system are necesary to insure stability, i.e., that outputs are positively correlated with their own price, negatively with the price of the other good. If the implicit revenue function $R(p_1, p_2, w_1, v_2, v_3)$ is convex and homogeneous of degree one in prices, the system is stable. Consider the effect of changing endowments of factors 2 and 3 upon the wages of those two factors. The cost minimization envelope property insures, looking at (6), that $\partial w_h/\partial v_k = 0$, where $h, k = 2, 3$.

The effects of (i) changing endowments of the immobile factors upon outputs, and (ii) changing prices upon those factors payments are reciprocal. Solving 7 for these expressions,

$$\begin{aligned} \partial x_1/\partial v_2 &= \partial w_2/\partial p_1 = -a_{32}b_2/D > 0, \\ \partial x_1/\partial v_3 &= \partial w_3/\partial p_1 = a_{22}b_2/D < 0, \\ \partial x_2/\partial v_2 &= \partial w_2/\partial p_2 = a_{31}b_2/D < 0, \text{ and} \\ \partial x_2/\partial v_3 &= \partial w_3/\partial p_2 = -a_{21}b_2/D > 0. \end{aligned} \tag{8}$$

These terms in 8 are readily signed, since $D < 0$, and $b_2 > 0$ in either factor ordering condition. Production of the good using an immobile factor intensively is positively correlated with the endowment of that factor, negatively with the other. The Hecksher Ohlin result that a country exports its abundant factor via commodity trade depends upon this Rybczynski result, and thus holds when stated in terms of the immobile factors. A tariff on either good will lower the real income of the immobile factor used (relatively) intensively in the other sector; the wage, and perhaps real income, of the immobile factor used intensively in the industry receiving the tariff will rise.

## IV. Demand for the Mobile Factor and Factor Interaction

The system determinant $D'$ of the usual 3 x 2 model with all factor payments endogenous is known to be negative [Chang, 1979]. Solving for this determinant, $D' = -c_1^2 s_{12} - c_2^2 s_{13} - c_3^2 s_{23}$. There is a negative relationship between the payment to the mobile factor and its employment, since

$$\partial v_1/\partial w_1 = -D'/D. \tag{9}$$

The result in 9 tells us that the general equilibrium demand curve for the mobile factor slopes downward. A rising world payment to factor 1 causes its outflow, indicating general equilibrium diminishing marginal productivity for the mobile factor.

Also of interest are (i) the effects of a changing wage of the mobile factor upon the payments to the other two factors, and (ii) the effects of changing endowments, due to exogenous growth or migration, of the immobile factors upon the employment of factor 1. The signs of these results are found to be independent of the pattern of factor substitutability. Solving for these effects,

$$\partial w_2/\partial w_1 = -\partial v_1/\partial v_2 = b_2 b_3/D, \text{ and}$$
$$\partial w_3/\partial w_1 = -\partial v_1/\partial v_3 = -b_1 b_2/D. \tag{10}$$

The effect of a changing world payment to the mobile factor upon the payment to factor 2 (3) is the mirror image of the effect of a changing endowment of factor 2 (3) upon the employment of the mobile factor.

The signs of the outcomes in Equation 10 depend upon which factor ordering condition is in effect. Given foc (i), $-b_1 b_2 < 0$; it is positive with foc (ii). In either case, $b_2 b_3 > 0$. If factor 1 is the middle factor, then other factors suffer falling payments (and real incomes, since prices are constant) at the hands of a rising world payment to factor 1. With $w_1$ rising the employment of factor 1 falls. The marginal productivities of both other factors must fall in this case. If factor 1 is extreme, a higher $w_1$ causes the income of the middle factor to fall, while that of the other extreme factor rises. The other extreme factor takes the place of factor 1, its demand and payment rising.

The employment of factor 1 is negatively correlated with the endowment of the other extreme factor in foc (i), while immigration of the middle factor leads to an inflow of factor 1. In foc (ii), an increase in the endowment of either extreme factor causes an inflow of the middle mobile factor. There is only one situation where an immigrating or growing factor displaces factor 1, which must be due to a falling marginal product of factor 1.

## V. The Internationally Mobile Factor and Production

A changing world payment to factor 1 will affect domestic production levels, with technical interfactor relationships playing a role. Also changing world prices of either good, perhaps by a tariff, will affect the employment of the mobile factor. These solutions are seen to be:

$$\partial x_1/\partial w_1 = -\partial v_1/\partial p_1 = (a_{32} c_1 s_{12} + a_{22} c_2 s_{13} + (a_{22} + a_{32}) c_3 s_{23})/D,$$

and (11)

$$\partial x_2/\partial w_1 = -\partial v_1/\partial p_2 = (-a_{31} c_1 s_{12} - a_{21} c_2 s_{13} - (a_{21} + a_{31}) c_3 s_{23})/D.$$

Any one of the substitution terms may be negative, indicating complementarity between those two factors. If a rising payment to the mobile factor increases the output of a sector, then a tariff on that good will cause an outflow of the mobile factor. It can be proven directly from 9 and 11 that a rising (falling) payment to the mobile factor cannot increase (decrease) both outputs.

For simplicity, $d_1 \equiv c_1 s_{13}$, $d_2 \equiv c_2 s_{13}$, and $d_3 \equiv c_3 s_{23}$. Assume that $\partial x_1/\partial w_1 > 0$ and $\partial x_2/\partial w_1 > 0$, which by Equation 11 means that (i) $a_{31}d_1 + a_{21}d_2 + (a_{21} + a_{31})d_3 > 0$, and (ii) $a_{32}d_1 + a_{22}d_2 + (a_{22} + a_{32})d_3 < 0$. From these assumptions it will follow that $D' > 0$, a contradiction. Multiply (i) by $(a_{12} + a_{22})$, and subtract from that the result of multiplying (ii) by $(a_{11} + a_{21})$. Thus (iii) $0 < -c_1d_1 - b_1d_2 - (c_3 + b_2)d_3 = D' + b_2(d_2 - d_3)$. Multiply (i) by $(a_{12} + a_{32})$ and (ii) by $(a_{11} + a_{31})$, and then subtract to find (iv) $0 < -b_3d_1 - c_2d_2 - (c_3 - b_2)d_3 = D' + b_2(d_3 - d_1)$. Finally, multiplying (i) by $a_{12}$ and (ii) by $a_{11}$, and subtracting, (v) $0 < -b_3d_1 - b_1d_2 - c_3d_2 = D' + b_2(d_1 - d_2)$. At least one of the expressions $(d_2 - d_3)$, $(d_3 - d_1)$, and $(d_1 - d_2)$ will be negative. Since $b_2 > 0$, from (iii), (iv), and (v) it follows that $D'$ will always be greater than some positive quantity. QED

Also proven is that $\partial v_1/\partial p_1$ and $\partial v_1/\partial p_2$ cannot both be negative. With $p_1$ rising, for instance, $x_1$ rises and $x_2$ falls. The employment of factor 1 may increase or decrease in this situation. If it were to decrease, a rising $p_2$ would have the opposite effect. Comparing the immobile factors 2 and 3, sector 1 uses factor 2 intensively in either factor ordering condition. An excess demand for factor 2 and supply for factor 3 develop, with $w_2$ rising and $w_3$ falling, as both migrate from sector 2 to 1.

## IX. Conclusion

The comparative statics of the 3 x 2 mobile factor model prove intuitively robust. Major results from the 2 x 2 trade model carry over for the immobile factors into the expanded model. The internationally mobile factor exhibits general equilibrium diminishing marginal returns, while its endogenously determined level of employment responds in an intuitive fashion. Changes in the mobile factor's payment have economically meaningful effects. This model, with its internationally mobile factor, proves a useful extension of the model of a small, open economy.

## MATHEMATICAL APPENDIX

Consider the solution for $\partial x_1/\partial p_1$ as an example of the technique of solution. All exogenous variables except $dp_1$ equal zero. Dividing both sides of (7) by $dp_1$,

$$\begin{bmatrix} -1 & s_{12} & s_{13} & a_{11} & a_{12} \\ 0 & s_{22} & s_{23} & a_{21} & a_{22} \\ 0 & s_{23} & s_{33} & a_{31} & a_{32} \\ 0 & a_{21} & a_{31} & 0 & 0 \\ 0 & a_{22} & a_{32} & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial v_1/\partial p_1 \\ \partial w_2/\partial p_1 \\ \partial w_3/\partial p_1 \\ \partial x_1/\partial p_1 \\ \partial x_2/\partial p_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Using Cramer's rule,

$$D(\partial x_1/\partial p_1) = \begin{vmatrix} -1 & s_{12} & s_{13} & a_{12} \\ 0 & s_{22} & s_{23} & a_{22} \\ 0 & s_{23} & s_{33} & a_{32} \\ 0 & a_{22} & a_{32} & 0 \end{vmatrix}$$

$$= -2a_{22}a_{32}s_{23} + a_{22}^2 s_{33} + a_{32}^2 s_{22}$$

$$= -2a_{22}a_{32}s_{23} - a_{22}^2(s_{13} + s_{23}) - a_{32}^2(s_{12} + s_{23})$$

$$= -a_{32}^2 s_{12} - a_{22}^2 s_{13} - (a_{22} + a_{32})^2 s_{23} \cdot .$$

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