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SIMULATING A MULTIFACTOR GENERAL EQUILIBRIUM MODEL OF PRODUCTION AND TRADE

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This study builds and simulates a high dimensional general equilibrium model of production and trade with inputs of capital and eight separate skilled groups of labor. Constant returns to scale cannot be rejected in any sector as a null hypothesis. International capital flows and labor migration are found to have very small effects on income redistribution, a result which is called near factor price equalization. Protectionism, on the other hand, has more substantial effects. Elasticities describing the income redistribution due to protection in manufacturing, agriculture, and services follow a pattern suggested by factor intensities. [410]

1. INTRODUCTION

Broad issues affecting an entire economy, generally considered the realm of macroeconomics, can be studied in more detail through the simulation of general equilibrium models. Computible models are becoming popular tools for policy analysis. Canada, Japan, and the European Community have recently used such models to predict the effects of taxes, trade policy, and international integration. Advantages of modelling based on microeconomics include a solid theoretical foundation and tested empirical techniques associated with estimating production and cost functions.

A productive input is defined by its distinct market. The ideal application would not aggregate different inputs, at least up to practical limitations on data availability and the degree of detail. Current computible general equilibrium models, surveyed by Shoven and Whalley (1984) and exemplified by Deardorff and Stern (1981), are rich in industrial structure but specify only two factors of production, labor and capital. If more were routinely included, the income redistribution due to international migration, international investment, and protectionism could be examined in detail.

Hamermesh and Grant (1979) and others conclude that skilled and unskilled labor are distinct groups. Clark, Hofler, and Thompson (1988) argue that technically there are at least six distinct groups of labor in U.S. manufacturing. The

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present study splits labor into the eight skill groups recorded across states by the U.S. Census. Substitution elasticities are estimated from factor share equations of translog production functions, using a simultaneous Zellner generalized least squares technique.

A production model of the type popularized by Jones (1965) serves as the basis for a simulation. It is a long run model characterized by full employment and competitive pricing. Output is classified in the three broad industrial sectors: agriculture, manufacturing, and services. This paper brings together two strands in the literature, the estimation of substitution elasticities and these general equilibrium models of production and trade. Elasticities of outputs and input payments with respect to changing input supplies and output prices are found. Constant returns to scale, when tested as a null hypothesis, cannot be rejected in any of the sectors.

Migration or growth of labor and international flows of capital are found to have very small effects on the redistribution of income. The comparative static effects of changing factor endowments on factor payments, in other words, are very inelastic. Opposition to the immigration of Latin Americans into the U.S. must not stem primarily from labor groups which ultimately suffer lower wages. With trade equalizing prices of goods across nations, factor endowment differences alone would create only small factor payment differences. This new concept is called near factor price equalization. Effects of protectionism on the payments to capital and the various labor groups are elastic and can be anticipated from factor intensities.

Section 2 presents the technique of estimating substitution elasticities. Section 3 then presents the general equilibrium model of production and trade, which is simulated in Section 4.

2. ESTIMATION OF SUBSTITUTION ELASTICITIES

Estimation of substitution is based on a translog approximation to the production frontier, developed by Christensen, Jorgensen, and Lau (1973), using cross section data from States for 1978. Capital stock figures are available for manufacturing from the *Annual Survey of Manufactures,* and for agriculture from the *Census of Agriculture.* Capital stock in the *Survey of Current Business* is taken as a proxy for capital input. In services, capital stock data is not available by State. Each State is assumed to employ the same amount of capital per worker in services as in manufacturing. Figures on employment by occupation and industry for all sectors in each State are contained in the *1980 Census of Population.* The eight types of labor are: (1) managers/professional; (2) technical/sales; (3) service; (4) farm/forestry/fishing; (5) craft/repair; (6) operators/fabricators; (7)

transport; and (8) handlers/cleaners.

The production function for each sector, $x = x(v_1,..., v_9)$, is approximated by a translog Taylor series expansion,

$$\ln x = \ln \alpha_0 + \Sigma_i^9 \alpha_i \ln v_i + 1/2 \Sigma_i^9 \Sigma_k^9 \gamma_{ik} \ln v_i \ln v_k, \quad (1)$$

where the index runs over capital and the eight types of labor. Assuming competitive input markets, $\partial x/\partial v_i = w_i$. Each factor share s_i equals the output elasticity from (1):

$$\partial \ln x/\partial \ln v_i = (v_i/x)(\partial x/\partial v_i) = w_i v_i/x \equiv s_i, \quad i = 1,..., 9. \quad (2)$$

Factor share equations for each sector come from (1) and (2):

$$s_i = \alpha_i + \Sigma_k^9 \gamma_{ik} \ln v_k, \quad i = 1,..., 9. \quad (3)$$

Between factors i and k in sector j, Allen (1938: 503-505) defines his partial elasticity of substitution as:

$$S_{ij}^k = (x_j/v_{ij}v_{kj})(X_{ik}/X), \quad (4)$$

where X is the determinant of the bordered Hessian matrix of partials and cross partials of the production function and X_{ik} is the cofactor of that particular element.

For the translog production function in (1), $\gamma_{ik} = \partial^2 \ln x/\partial \ln v_i \partial \ln v_k$. Using (2),

$$\gamma_{ik} = (v_i v_k/x)x_{ik} - (v_i v_k/x^2)w_i w_k, \quad (5)$$

where $x_{ik} = \partial^2 x/\partial v_i \partial v_k$. It follows that

$$x_{ik} = (x/v_i v_k)(\gamma_{ik} + s_i s_k). \quad (6)$$

Similarly from (2), $\gamma_{ii} = \partial^2 \ln x/\partial \ln v_i^2 = (v_i/x)x_{ii} - s_i + s_i$, so

$$x_{ii} = (x/v_i^2)(\gamma_{ii} + s_i^2 - s_i). \quad (7)$$

The determinant X and each cofactor X_{ik} of the bordered Hessian matrix of the production function are built with (6) and (7). Allen's definition is then used directly in each sector to obtain the partial elasticities in (4)

Constraints imposed in the estimation of the system of factor share equations in (3) are symmetry, $\gamma_{ik} = \gamma_{ki}$, and homogeneity, $\Sigma_i^9 \alpha_i = 1$, $\Sigma_i^9 \gamma_{ik} = \Sigma_k^9 \gamma_{ik} = 0$. Zellner's generalized least squares technique produces estimates which are asymptotically more efficient than those obtained from ordinary least squares, as described by Theil (1971). Berndt and Christensen (1973) point out that this estimation procedure assumes constant returns to scale and Hicks neutral technical change. As output moves up along a linear expansion path, factor payments and shares are unchanged given the assumption of homotheticity and constant returns. Where x represents output, this assumption is tested in each sector by estimating

$$s_i = \alpha_i + \Sigma_i^9 \gamma_{ik} \ln v_k + \delta_i x, \tag{8}$$

and testing the null hypothesis that $\delta_i = 0$. A Chi square test reveals that this assumption cannot be rejected in any sector at a liberal 90% confidence level.

Cross price elasticities are derived from the Allen partial elasticities found with (4). Following Sato and Koizumi (1973), a first order condition for a firm minimizing cost subject to an output constraint is $w_i = \phi\, \partial x / \partial v_i$, where ϕ is the Lagrangian multiplier representing marginal cost. Differentiating,

$$\partial v_i / \partial w_k = 1/\phi\, (X_{ik}/X). \tag{9}$$

Expressing input payments in units of output, the cross price elasticity between the input of factor i and the payment to factor k (Allen, 1938: 505-509) is written

$$E_{ij}^k = \theta_{kj} S_{ij}^k, \tag{10}$$

where θ_{kj} is the share of factor k in sector j, $w_k v_{kj}/x_j$. This cross price elasticity can be written,

$$E_{ij}^k = \hat{a}_{ij}/\hat{w}_k, \tag{11}$$

where a_{ij} represents the amount of factor i used per unit of good j, and ^ denotes percentage change. Output and other input prices are held constant. From Shepard's lemma and Taylor's formula, $\partial a_{ij}/\partial w_k = \partial a_{kj}/\partial w_i$, so E_{ik} has the same sign as E_{ki}. Due to homogeneity, $\Sigma_k E_{ij}^k = 0$. As shown in Jones and Easton (1983), $E_{ij}^k = (\theta_{kj}/\theta_{ij}) E_{kj}^i$ which implies $\Sigma_k \theta_{kj} E_{kj}^i = 0$.

Economy wide substitution elasticities are weighted averages of the sectoral cross price elasticities,

$$\sigma_{ik} = \Sigma_j \lambda_{ij} E_{ij}^k. \tag{12}$$

Weights are industry shares λ_{ij}, representing the portion of factor i employed in sector j. A sector where more of a factor is employed weighs more heavily in the economy wide substitution elasticity. The next section shows how this elasticity fits into the comparative static model. As with all isoquants, output is constant as firms respond to changing factor payments by altering their factor mix.

Estimates of factor share equations are presented in Tables 1, 2, and 3. Cross price elasticities from the three sectors are aggregated to the matrix σ of economy wide elasticities presented in Table 4. Across its first row appears the percentage change in the input of capital due to a 1% change in the price of each input. Capital input falls by 1.31% with a 1% increase in its own payment. Firms

Table 1. Estimated Factor Share Equations: Agriculture

S_i	α_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}	γ_{iK}
1	0.025	2.537								
	(4.49)	(11.56)								
2	0.200	−0.285	1.739							
	(6.89)	(−2.06)	(11.49)							
3	0.008	−0.065	−0.065	0.180						
	(4.42)	(−1.18)	(−1.20)	(2.87)						
4	−0.018	−0.204	−0.293	−0.079	4.197					
	(−1.09)	(−1.07)	(−3.54)	(−1.85)	(6.94)					
5	0.045	0.112	0.103	0.119	−0.731	2.40				
	(5.21)	(0.65)	(1.26)	(−1.90)	(−2.45)	(7.36)				
6	0.012	−0.258	−0.367	−0.023	0.010	0.328	1.844			
	(2.48)	(−1.65)	(−2.18)	(−0.26)	(0.07)	(2.30)	(4.88)			
7	0.020	−0.284	0.016	0.190	0.071	0.298	−0.260	0.888		
	(2.42)	(−2.29)	(0.14)	(2.10)	(0.56)	(1.53)	(−1.11)	(3.21)		
8	0.008	−0.138	−0.032	0.016	0.021	0.012	−0.052	0.028	0.370	
	(4.34)	(−2.13)	(−0.47)	(0.38)	(0.51)	(0.21)	(−0.36)	(0.30)	(5.57)	
K	0.700	−1.414	−0.815	0.036	−2.992	−2.400	−1.223	−0.962	−0.226	10.068
		(−4.43)	(−5.96)	(−0.75)	(−3.28)	(−4.30)	(−4.53)	(−4.94)	(−3.57)	

Note: Figures in the parentheses are t statistics.

Table 2. Estimated Factor Share Equations: Manufacturing

S_i	α_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}	γ_{iK}
1	0.089	7.051								
	(9.70)	(20.46)								
2	0.057	−1.800	6.345							
	(7.98)	(−4.69)	(13.10)							
3	0.015	−0.228	−0.319	0.283						
	(1.90)	(−1.26)	(−1.97)	(0.85)						
4	0.012	−0.040	0.077	−0.018	0.188					
	(6.24)	(−0.93)	(2.17)	(−0.34)	(8.56)					
5	0.096	−1.775	−1.350	0.781	−0.144	7.631				
	(9.32)	(−7.57)	(−6.32)	(2.74)	(−2.64)	(16.83)				
6	0.004	−1.959	−2.124	−0.671	−0.161	−3.414	10.512			
	(0.21)	(−6.13)	(−7.85)	(−2.02)	(−2.05)	(−8.27)	(15.28)			
7	0.051	−0.378	−0.067	0.416	0.039	−0.502	−0.817	1.611		
	(12.49)	(−3.34)	(−0.61)	(3.58)	(1.28)	(−2.94)	(−4.49)	(11.12)		
8	0.032	−0.304	−0.134	0.022	−0.031	−0.520	−0.444	−0.068	1.680	
	(10.90)	(−2.29)	(−0.85)	(0.02)	(−1.50)	(−3.61)	(−2.90)	(−0.84)	(14.05)	
K	0.644	−0.564	−0.627	−0.266	−0.091	−0.707	−0.922	−0.215	−0.205	3.415
		(−1.67)	(−2.45)	(−1.68)	(1.68)	(−2.01)	(−1.28)	(−1.85)	(−2.49)	

Note: Figures in the parentheses are t statistics.

Table 3. Estimated Factor Share Equations: Services

s_i	α_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}	γ_{iK}
1	0.052	11.237								
	(1.90)	(17.56)								
2	0.018	−2.911	8.682							
	(0.74)	(−5.35)	(13.33)							
3	0.048	−2.150	−1.933	5.467						
	(3.00)	(−2.96)	(−3.00)	(3.71)						
4	0.004	−0.010	−0.048	−0.018	0.095					
	(6.48)	(−0.61)	(−1.57)	(−0.95)	(9.49)					
5	0.063	−1.218	−0.824	−0.842	0.006	4.703				
	(5.23)	(−3.97)	(−2.44)	(−1.72)	(0.33)	(12.41)				
6	0.054	−1.402	−0.073	−1.120	−0.038	−0.188	1.486			
	(5.31)	(−4.62)	(−0.15)	(−0.30)	(−0.73)	(−0.67)	(1.87)			
7	0.025	−0.146	−0.127	0.197	0.040	−0.232	0.868	0.320		
	(2.30)	(−0.44)	(−0.28)	(0.43)	(1.00)	(−0.64)	(1.43)	(0.45)		
8	0.017	−0.369	0.018	−0.125	−0.025	−0.216	0.362	−0.392	1.071	
	(5.33)	(−3.58)	(0.10)	(−0.96)	(−1.10)	(−2.15)	(1.43)	(−1.94)	(7.95)	
K	0.719	−3.050	2.784	−0.475	−0.022	−1.190	−0.895	−0.600	−0.325	3.773
		(−2.67)	(−2.86)	(−0.87)	(−2.16)	(−2.32)	(−2.23)	(−2.23)	(−2.91)	

Note: Figures in the parentheses are t statistics.

Table 4. Factor Substitution Elasticities

	K	1	2	3	4	5	6	7	8
K	−1.31	0.38	0.36	0.08	0.01	0.20	0.15	0.08	0.05
1	0.41	−1.71	0.57	0.09	−0.01	0.28	0.20	0.07	0.10
2	0.49	0.73	−1.81	0.14	0.01	0.32	0.09	0.05	−0.02
3	0.59	0.65	0.80	−2.61	0.01	0.23	0.51	−0.31	0.13
4	0.76	−0.07	0.21	0.04	−1.78	0.49	0.19	−0.04	0.22
5	0.49	0.58	0.53	0.05	0.06	−2.31	0.56	−0.05	0.10
6	0.35	0.41	0.14	0.16	0.02	0.56	−1.75	0.16	−0.06
7	0.64	0.48	0.29	−0.32	−0.02	−0.16	0.55	−1.69	0.28
8	0.68	1.14	−0.18	0.21	0.14	0.58	−0.32	0.49	−2.75

weakly substitute capital for all groups of labor. Capital substitutes best for the white collar workers in groups 1 and 2, followed by the craft workers in group 5. Capital is a better substitute for these skilled groups than for the more unskilled groups.

Reading down the first column in Table 4, an rise in the price of capital would create an increase in every labor input. A 10% increase in the price of capital would raise the input of farm labor by 7.6%, handlers by 6.8%, and transport labor by 6.4%. These are the largest substitution elasticities in Table 4, which means that firms are relatively sensitive to the price they pay for capital input. Reading down the column for labor group 5, a 1% increase in the wages of craft workers results in a 2.31% decrease in their own input. Firms substitute handlers 0.58%, operators 0.56%, farm workers 0.49%, and technical workers 0.32%. The input of capital rises slightly by 0.20%. Craft and transport workers are weak complements. Each column in Table 4 can be analyzed in a similar fashion.

All of the own elasticities along the main diagonal are elastic, the largest in the unskilled groups 3 and 8. The own capital elasticity is the smallest, followed by skilled groups 1 and 2. There is little evidence of complementarity between inputs in this data. Transport workers are weak complements with service and craft workers. Handlers are weak complements with technical workers and operators.

3. THE MODEL

Numerous studies, such as Jones and Scheinkman (1977), Takayama (1982), and Thompson (1987), have developed the general equilibrium model of production. Theoretical properties can be examined in the context of the simulated results of the next section. Primary factors of production, assumed to be in perfectly inelastic supply at their endowment levels v, are used to produce final outputs x. Endogenous factor payments w determine cost minimizing factor mixes, which are functions homogeneous of degree zero in the factor payments. Competitive pricing insures income is exhausted among the productive factors. Full employment and competitive pricing are written

$$\Sigma_j^3 a_{kj}x_j = v_k, \quad \text{for all inputs k, and} \tag{13}$$

$$\Sigma_i^9 a_{ij}w_i = p_j, \quad \text{for all goods j.} \tag{14}$$

Fully differentiating (13) and (14) leads to:

$$\Sigma_i^9 \sigma_{ki}\hat{w}_i + \Sigma_j^3 \lambda_{kj}\hat{x}_j = \hat{v}_k, \quad \text{for all inputs k, and} \tag{15}$$

$$\Sigma_i^9 \theta_{ij}\hat{w}_i = \hat{p}_j, \quad \text{for all goods j.} \tag{16}$$

Industry shares are defined by $\lambda_{ij} \equiv a_{ij}x_j/v_i$, and factor shares by $\theta_{ij} \equiv w_i a_{ij}/p_j$. It is useful to arrange (15) and (16) into

$$\begin{bmatrix} \sigma & \lambda \\ \theta' & 0 \end{bmatrix} \begin{bmatrix} \hat{w} \\ \hat{x} \end{bmatrix} \begin{matrix} = \\ = \end{matrix} \begin{bmatrix} \hat{v} \\ \hat{p} \end{bmatrix}. \tag{17}$$

Since σ has zero row sums and both λ and θ' are nonnegative and row stochastic, the system matrix in (17) is row stochastic. The mapping from (v, p) to (w, x) is locally invertible (one to one and onto) since the system matrix is the Jacobian of the mapping. For ease of reference, consider the inverse of (17).

$$\begin{bmatrix} \hat{w} \\ \hat{x} \end{bmatrix} = \begin{bmatrix} M & N \\ Q & R \end{bmatrix} \begin{bmatrix} \hat{v} \\ \hat{p} \end{bmatrix}. \tag{18}$$

Estimates of the comparative static elasticities in M, N, Q, and R are the goal of this paper. Factor prices are affected by changing endowments with prices constant, as described in the matrix M. Jones and Scheinkman (1977) show M (i) is symmetric; (ii) is negative semidefinite, so changing endowments affect their own prices at least as much as prices of other factors; and (iii) has a negative diagonal, indicating downward sloping demand in the general equilibrium.

Matrix Q captures the effects of changing endowments on outputs, the traditional Rybczynski results. Each row of Q has at least one positive element, so each output is positively related with at least one factor endowment. Ethier (1974) shows each row must have at least one negative element, so each output is negatively related with at least one factor endowment. If sector m is small ($\Sigma_i \lambda_{im} < 1$) at least one element in row m exceeds one, so changes in some endowment more than proportionally affect that output.

Matrix N describes how changing prices affect factor prices, the traditional Stolper-Samuelson results. Reciprocity occurs between these results and the Q matrix: $\partial w_i / \partial p_j = \partial x_j / \partial v_i$. The elasticities in Q and N are thus symmetric in sign. Chang (1979) develops two general properties of the matrix N: (i) for every factor payment, there is some price which is positively related with it; and (ii) every price is positively related with some factor payment. Ethier (1974) shows every column of N has at least one negative element and at least one element greater than one. An increased price due to protection lowers payment to at least one factor, while more than proportionally raising payment to another. Protection must raise the real income of some factor.

A local surface of production possibilities is described by R, which is symmetric and positive semidefinite. Each output is positively related with its own price, while some other output must fall with unchanged factor endowments. Own effects again at least equal cross effects, so changing prices affect their own outputs no less than others.

4. MODEL SIMULATION

Table 5 presents the calculated factor share matrix θ, with G representing agriculture, M manufacturing, and S services. Factor intensity is described by ratios of these factor shares. With a unit of output defined as one dollar's worth, $\theta_{ig}/\theta_{in} = a_{ig}/a_{in}$ for any two goods g and n. Capital is generally the most intensive input in agriculture, receiving 57.6% of the income in that sector. Agri-

Table 5. Factor Shares

	K	1	2	3	4	5	6	7	8
G	.576	.059	.033	.003	.139	.086	.045	.030	.008
M	.214	.148	.115	.008	.002	.167	.286	.031	.028
S	.261	.269	.211	.041	.002	.096	.067	.036	.018

cultural workers receive 13.9%. Skilled labor is used intensively in services, where skilled groups 1 and 2 together receive 48% of the income. The more unskilled labor groups are used relatively intensively in manufacturing, where groups 5 and 6 together receive 45% of the income in manufacturing.

The calculated industry share matrix λ is presented in Table 6. While agriculture is the capital intensive industry, services is so large that it contains 78.6% of all the capital stock. This is the sector where the least data is collected! Groups 1, 2, 3, 7, and 8 are employed mostly in services, again indicating its dominance in the economy. Manufacturing employs 54.4% of all operators and 31.7% of all craft workers. Agriculture employs almost all farm workers and less than 5% of any other group.

Table 6. Industry Shares

	G	M	S
K	.078	.137	.786
1	.014	.134	.852
2	.010	.136	.855
3	.005	.043	.952
4	.876	.033	.091
5	.037	.317	.646
6	.022	.544	.434
7	.047	.200	.754
8	.025	.296	.679

Substitution elasticities, factor shares, and industry shares are combined into the system (17). Inversion yields results corresponding to (18). Table 7 presents the matrix M of elasticities of factor prices w with respect to factor endowments v. These elasticities are very close to zero, suggesting international capital flows and labor migration have little long run impact on income distribution. The model allows complete adjustment of outputs and reflects assumptions of full employment and free factor mobility between sectors. Regional and temporary effects are thus understated. The largest elasticity is the own elasticity for labor group 7, where a huge 10% immigration of transport workers would in the long

Table 7. $\hat{w}/\hat{v}$ Elasticities

	v_K	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
w_K	−0.34	0.14	0.11	0.02	0.02	0.04	−0.01	0.01	0.01
w_1	0.14	−0.28	0.11	0.02	−0.01	0.02	−0.01	0.02	0
w_2	0.13	0.14	−0.31	0.01	−0.01	0.01	−0.01	0.03	0.02
w_3	0.12	0.16	0.09	−0.36	−0.02	−0.01	−0.10	−0.10	0.01
w_4	1.25	−0.55	−0.46	−0.09	−0.10	−0.04	−0.03	0.04	−0.02
w_5	0.11	0.02	0	0	0	−0.31	0.13	0.05	0
w_6	0.03	−0.04	−0.04	−0.03	−0.01	0.12	−0.07	0.01	0.02
w_7	0.08	0.09	0.12	0.09	0.01	0.16	0.06	−0.59	−0.04
w_8	0.13	−0.02	0.14	0.01	0	0.03	0.14	−0.06	−0.37

Table 8. $\hat{w}/\hat{v}$ Elasticities

	v_K	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
x_G	2.71	−1.35	−1.00	−0.18	0.94	0.01	−0.17	0.01	0.04
x_M	−0.37	−0.49	−0.54	−0.10	−0.03	0.65	1.77	0.09	0.02
x_S	0.30	0.52	0.45	0.08	−0.03	−0.03	−0.32	0.02	0.02

run result in only a 5.9% decline in their payment. Service workers would suffer a 1% decline, and handlers a 0.6% decline. Small increases of all other wages occur. Similar results occur for all unskilled labor immigration. With such a small impact on income redistribution, staunch opponents to immigration must base their opinions on short run, cultural, or political criteria. Reflecting on the factor price equalization theorem in trade theory, even when factor prices are not equalized between freely trading countries, they would not be too far apart. Near factor price equalization is a robust result across various aggregation schemes and patterns of substitution, as discussed by Thompson (1989).

The Rybczynski matrix R in Table 8 shows output responses to endowment changes. A 1% increase in the capital stock increases agricultural output 2.71% and service output 0.30%, while lowering manufacturing output 0.37%. Service output is strongly pulled by increases in its intensive labor, skilled groups 1 and 2. Manufacturing output has strong positive associations with its intensive groups, operators and craft workers.

The production possibility matrix R in Table 9 shows the effects of changing prices on outputs. These long run elasticities are quite large. Output is adjusting to maintain full employment and competitive pricing. Disaggregating sectors leads to smaller $\hat{x}/\hat{p}$ results, as shown in Thompson (1989). It is noteworthy that the service sector dominates the others in these results. The resources it pulls away

Table 9. $\hat{w}/\hat{p}$ Elasticities

	p_G	p_M	p_S
x_G	11.9	−0.76	−11.1
x_M	−0.98	13.3	−12.4
x_S	−0.39	−3.53	3.93

Table 10. $\hat{w}/\hat{p}$ Elasticities

	p_G	p_M	p_S
w_K	0.32	−0.45	1.13
w_1	−0.18	−0.42	1.60
w_2	−0.17	−0.60	1.78
w_3	−0.15	−0.75	1.90
w_4	5.94	0.16	−5.11
w_5	0.04	1.31	−0.36
w_6	−0.17	3.44	−2.27
w_7	0.01	0.77	0.22
w_8	0.25	0.28	0.47

from the other sectors when p_s rises are substantial. If the U.S. is specializing internationally into services as casual empiricism suggests, the other sectors should anticipate substantial decline.

The Stolper-Samuelson matrix N in Table 10 shows the percentage change in factor prices due to a 1% price rise for each good. Protection on agricultural goods resulting in a 1% price rise would raise the capital payment by 0.32%, while lowering the payments to groups 1, 2, 3, and 6 by about 0.2%. Wages of farm workers would rise almost 6%, while wages of handlers rise by 0.25%. Capital and farm workers clearly have the most to gain through protection of agriculture.

A 1% price rise in manufacturing due to protection would increase the wages of craft and operators by 1.31% and 3.44%. Other relatively unskilled groups 4, 7, and 8 would gain 0.16%, 0.77%, and 0.28% respectively. These groups must, however, pay higher prices for consumed manufactured goods, so consumption shares would determine whether their real income rises. Capital owners are hurt by the tariff as their payment falls 0.45%. Managers and technical workers, used more intensively in services, suffer 0.42% and 0.60% declines in wages. Service workers are used almost exclusively in services, and suffer a 0.75% decline in wages.

An issue the U.S. is currently perusing in the GATT negotiations is the high levels of protection other countries place on services. If other countries lower their protection of services (business services, banking, telecommunications, information services, and so on) prices in the U.S. would rise with the increased demand. A 1% rise in price would increase payments to labor groups 1, 2, and 3 by 1.6%, 1.78%, and 1.9% respectively. These relatively large gains go to the groups used intensively in the service industry. Farm workers lose 5.1%, as agricultural output declines. Crafts and operators lose 0.36% and 2.27% respectively with the fall in manufacturing output.

Measurement error affects these results, but reasonably expected degrees of error at this level of aggregation will have small effects. The Census data grouping of labor does not represent a perfect partition by skills or human capital. There are varying amounts of wage dispersion within the groups. The approach here presents one way to disaggregate inputs into a multifactor model, and is meant to stir researchers into searching for other ways. A tacit assumption of the model which can be questioned is that the U.S. is a small open economy. Effects of endowment changes are assumed to work their way through the economy with prices of goods constant. Price changes for each sector are considered with prices of the other goods held constant. At this high level of aggregation of goods, it is not too much of a distortion to assume that the U.S. is a price taker. While the U.S. may not be a price taker in automobiles or iron and steel, for instance, it would be in manufactured goods. This issue can be settled by introducing elements of demand and looking into price dispersions nationally and internationally. It seems unlikely that the amount of endogenous price adjustment, especially at this high level of aggregation, would drastically affect the results reported here.

5. CONCLUSION

This simulation provides an example of a multifactor general equilibrium model of production and trade. Disaggregation of labor by skill groups leads to results of income redistribution beyond those which can be obtained assuming a single type of labor. A similar simulation with the sixteen subsectors of manufacturing is feasible. The own substitution elasticities become larger with labor disaggregation, while cross elasticities become smaller, as predicted by Diewart (1977) and illustrated by Thompson (1989). When factors and industries are disaggregated, the estimation uniformly improves while comparative static elasticities generally shrink. Under any aggregation scheme, elasticities of factor prices with respect to factor endowments are very small. Near factor price equalization is a robust result in these competitive general equilibrium models. Sensitivity analysis is performed by Thompson (1989), where various production functions and aggregation schemes indicate that magnitudes of the comparative static elasticities change very