



## An investigation into the quantitative properties of the specific factors model of international trade

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### Abstract

This paper presents a specification of a fundamental and popular model in the theory of international trade, the specific factors model. Data from 24 sectors of the Japanese economy are used. Simulated comparative static elasticities extend basic trade theory by uncovering and postulating the following quantitative properties:

- (i) near factor price equalization with free trade;
- (ii) high levels of specialization and trade across trading partners; and
- (iii) concentrated benefits and diffused costs of protection.

Free trade can thus be expected to nearly equalize prices of similar inputs across countries and lead to high levels of specialization and trade.

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### 1. Introduction

The specific factors (SF) model of international trade is a general equilibrium model of production in which each sector employs one specific factor

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and shares a common factor with every other sector. The SF model is popular because of its elegant theoretical simplicity and sensible economic character. Its principles have consistently enlightened students of international economics on the income redistribution due to protection and international investment and migration. This study extends the SF model by examining its quantitative nature in a specification using national income data from 24 sectors of the Japanese economy. Three general types of quantitative results with implications for interpreting theory stand out.

First, although factor price equalization (FPE) does not hold in the SF model, factor endowment differences are found to have very small effects on factor prices across competitive trading partners. This result is called near factor price equalization (NFPE). NFPE is also noted in the multifactor general equilibrium model specified by Thompson (1990).

Second, factor endowment differences across trading partners would result in large output differences in the SF model. This result implies a high level of international specialization under free trade, given only that consumers have similar tastes across nations. Free trade in the SF model would lead to nearly equal factor prices and high levels of international trade. International factor movements would have a small impact on factor prices and a large impact on outputs.

Third, the benefits of protection are concentrated heavily and almost entirely on the sector specific factor, with small gains going to the shared factor and losses spread very thinly around other sectors of the economy. This quantitative result bolsters the argument from political economy that the winners from protection would stand to gain from lobbying for political support, while disorganized losers would lack the incentive to lobby for free trade.

The focus of this paper is on the quantitative characteristics of the comparative statics of the SF model. These characteristics have direct implications for the interpretation of a fundamental model of international trade theory. The specification is not a computable general equilibrium (CGE) model of the class surveyed by Shoven and Whalley (1984). CGE models are detailed in their industrial structure, but are not closely related to core theory. The goal of this paper is to begin to create a feel for the quantitative comparative static properties of one of the models which form the theoretical foundation of international trade theory.

## **2. The basic model**

In the SF model popularized by Jones (1971), each industry employs a sector specific factor along with a common or shared factor. Labor is treated as the sector specific factor in the present study, and is assumed to be trained

in a particular production process and unable to readily switch sectors. Also, wage contracts would imply different wages across sectors and would limit intersector labor mobility. The focus in this study is on changes in the demand for labor as reflected by wage adjustments in each sector. Subsequent modelling could allow for intersector labor mobility, various skill groups of labor, wage rigidities and unemployment, and so on.

Firms in each industry also employ homogeneous shared productive capital. This assumption breaks with the textbook picture of sector specific capital machinery which cannot readily be employed in other sectors. Many types of equipment and machinery (cars, buildings, chairs, computers, and so on) are used in virtually every sector. The unit of measure of the theoretical homogeneous capital good in the present model is simply one yen's worth of fixed capital equipment. Investment is assumed to adjust across sectors (in a costless and timeless fashion) as firms in each sector choose their optimal cost minimizing capital input.

The textbook SF model has two sectors, two sector specific factors, and one shared factor. The present specification has 24 sectors, specific labor in each sector, and shared homogeneous capital. Assumptions regarding labor and capital are meant focus on the adjustment in labor demand and output across the economy. Isolating each sector's labor in this manner has useful applications in studying the income redistribution due to protection, the move to free trade, international migration, or international capital movement. The textbook model with sector specific capital would be best suited for examining variation in the sectoral return to capital, reflected perhaps by stock returns.

Essential properties of general equilibrium models of production and trade are developed and reviewed by Jones and Scheinkman (1977), Chang (1979), and Thompson (1987). Two fundamental structural assumptions of these models are full employment and competitive pricing. In the present model,

$$\sum_j^{24} a_{Kj} x_j = v_K, \text{ for productive capital,} \quad (1)$$

$$a_{hj} x_j = v_h, \text{ for every type of labor } h, \text{ and} \quad (2)$$

$$a_{im} w_i + a_{Km} r = p_m, \text{ for every good } m. \quad (3)$$

Factor mix terms  $a_{ij}$  are cost minimizing functions of the relative factor price in each sector, given homothetic production functions with constant returns to scale. Outputs are  $x_j$ , factor endowments  $v_h$ , labor wages  $w_i$ , the price of capital  $r$ , and prices of goods  $p_m$ . Index  $j$  runs across the 24 sectors listed in Table 1. Capital is employed in every sector, which means that  $a_{Kj} \neq 0$  for every  $j$ . Each type of labor is employed only in its sector as

Table 1  
Numbering of sectors

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1	Agriculture, forestry, fishing
2	Mining
3	Construction
4	Food processing
5	Textiles
6	Pulp and paper
7	Chemicals
8	Petroleum and coal
9	Nonmetallic minerals
10	Iron and steel
11	Nonferrous metal
12	Fabricated metals
13	General machinery
14	Electrical machinery
15	Motor vehicles
16	Other transport equipment
17	Precision instrument
18	Other manufacturing
19	Wholesale and retail trade
20	Finance, real estate, insurance
21	Communication
22	Transportation
23	Utilities
24	Business and personal services

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reflected in Eq. (2). Each good is produced with two inputs and is priced competitively as described in Eq. (3).

Jones (1965) and Thompson (1987) show that differentiating Eqs. (1) and (2) leads to:

$$\sum_i^{25} \sigma_h^i \hat{w}_i + \sum_j \lambda_{hj} \hat{x}_j = \hat{v}_h, \text{ for every factor } h. \quad (4)$$

In Eq. (4) the index  $i$  runs across every type of labor and capital. Differentiating Eq. (3) leads to

$$\sum_i^{25} \theta_{ij} \hat{w}_i = \hat{p}_j, \text{ for every good } j. \quad (5)$$

Hats ( $\hat{\phantom{x}}$ ) represent percentage change. Substitution elasticities encompass the economy, describing how the input of factor  $h$  varies with the price of factor  $i$ :

$$\sigma_i^h \equiv \sum_j^{24} \lambda_{hj} \hat{a}_{hj} / \hat{w}_i. \quad (6)$$

The  $\lambda_{hj}$ 's are industry shares, the portion of each factor  $h$  employed in each sector  $j$ . The  $\theta_{ij}$ 's are factor shares, the percentage of income in industry  $j$  going to factor  $i$ . For every factor  $h$ ,  $\sum_j \lambda_{hj} = 1$ . For every good  $m$ ,  $\sum_i \theta_{im} = 1$ . Since each type of labor is used only in its sector,  $\lambda_{ii} = 1$  and  $\lambda_{ih} = \theta_{hi} = 0$  if  $i \neq h$ . Also  $\theta_{ii} + \theta_{Ki} = 1$ .

Arranging Eqs. (4) and (5) into matrix format,

$$\begin{bmatrix} \sigma & \lambda \\ \theta' & 0 \end{bmatrix} \begin{bmatrix} \hat{w} \\ \hat{x} \end{bmatrix} = \begin{bmatrix} \hat{v} \\ \hat{p} \end{bmatrix} \quad (7)$$

Matrices  $\theta$  and  $\gamma$  have dimension  $25 \times 24$  in the present specification, and are derived directly from the data as described in the next section. Derivation of the  $25 \times 25$  matrix  $\sigma$  is described in a subsequent section. Comparative static elasticities of changing factor endowments ( $v$ ) and prices ( $p$ ) on factor payments ( $w$ ) and outputs ( $x$ ) can be calculated when the system matrix in (7) is completely specified.

### 3. The model's factor shares and industry shares

Data leading to matrices  $\theta$  and  $\lambda$  in this SF model have been put on disk by Uno (1987). Factor shares  $\theta_{ii}$  of labor in each sector for Japan in 1980 are presented in Table 2. These factor shares come from the *Annual Report on National Accounts* of the Japanese Economic Planning Agency and the *Census of Manufactures* of the Ministry of International Trade and Industry. The capital share in each sector is the residual,  $\theta_{Ki} = 1 - \theta_{ii}$ .

A hint of the degree of capital intensity across sectors can be obtained from the factor shares. The capital share  $\theta_{Ki}$  is equal to  $ra_{Ki}/p_i$ . Writing the labor share,  $\theta_{ii} = w_i a_{ii}/p_i$ . The ratio of the capital share to the labor share is then  $\theta_{Ki}/\theta_{ii} = ra_{Ki}/w_i a_{ii}$ . The price of capital  $r$  is identical across sectors due to free intersector mobility. To the extent that the wage  $w_i$  is similar across sectors, the ratio  $\theta_{Ki}/\theta_{ii}$  would approximate the factor intensity ratio

Table 2  
Labor factor shares and capital industry shares

	$\theta_{ii}$	$\lambda_{Ki}$		$\theta_{ii}$	$\lambda_{Ki}$		$\theta_{ii}$	$\lambda_{Ki}$
1	0.159	0.142	9	0.571	0.023	17	0.653	0.004
2	0.342	0.005	10	0.250	0.073	18	0.683	0.090
3	0.573	0.040	11	0.431	0.013	19	0.612	0.098
4	0.357	0.028	12	0.683	0.022	20	0.218	0.044
5	0.560	0.018	13	0.697	0.031	21	0.575	0.008
6	0.460	0.018	14	0.499	0.025	22	0.860	0.050
7	0.385	0.050	15	0.481	0.031	23	0.382	0.094
8	0.154	0.016	16	0.918	0.009	24	0.521	0.068

of inputs  $a_{Ki}/a_{ii}$ . The average wage in Japan across sectors is in fact 241,967 yen, with a standard deviation of 50,557 yen.

By this reasoning, petroleum and coal (sector 8) would be the most capital intensive industry, followed by agriculture (1), finance (20), and iron and steel (10). These industries in fact stand out as being capital intensive in the model's comparative statics results. The most labor intensive industries are the other transport equipment sector (16) and transportation (22).

The industry share of capital employed in each sector  $\gamma_{Ki}$  comes from *Gross Fixed Nonresidential Private Business Capital in Japan* of the Economic Planning Agency, and is also presented in Table 2. It is noteworthy that agriculture contains the largest share of capital (14.2%), followed by wholesale and retail trade (9.8%), utilities (9.4%), and other manufacturing (9.0%).

Land is implicitly included with capital in the factor shares  $\theta_{Ki}$ , but excluded from the industry shares  $\gamma_{Ki}$ . This makes the agriculture sector (1) less capital intensive than it appears. This distortion, however, is thought to be slight for a number of reasons. Even in pure farming, the distributional share of land is generally small. In forestry and fishing, land would play even lesser direct roles. It is difficult to separate land's share since all farmland is improved by capital and labor input. At any rate, the large share of capital in agriculture is consistent with its high degree of capital intensity.

#### 4. The model's substitution elasticities

Assuming Cobb-Douglas production functions, economy wide substitution elasticities can be calculated from factor shares and industry shares. The Allen (1938) elasticity of substitution  $S_{hj}^i$  between any pair of factors  $i$  and  $h$  for every good  $j$  equals one with Cobb-Douglas production.

Sato and Koizumi (1973) show that the cross price elasticity  $E_{hj}^i \equiv \hat{a}_{hj}/\hat{w}_i$  between the input of factor  $h$  and the wage of factor  $i$  in sector  $j$  can be written

$$E_{hj}^i = \theta_{ij} S_{hj}^i, \quad (8)$$

given the cost minimizing behavior of firms. The cross price elasticity  $E_{hj}^i$  indicates the effect of a change in the price of factor  $i$  on the input of factor  $h$  in the production of good  $j$ .

According to Eq. (8), the cross price elasticity is a weighted portion of the Allen elasticity. For Cobb-Douglas production,  $E_{hj}^i = \theta_{ij}$ , which allows derivation of cross price elasticities directly from national income data.

Due to constant returns to scale or homogeneity,  $\sum_i E_{hj}^i = 1$ . According to Eq. (6), substitution across the entire economy in matrix  $\sigma$  is a weighted average of the substitution elasticity in each sector,

$$\sigma_h^i = \sum_j \lambda_{hj} E_{hj}^i = \sum_j \lambda_{hj} \theta_{ij}. \quad (9)$$

Table 3  
Derived Cobb-Douglas substitution elasticities

	$\sigma_i^K$	$\sigma_K^i$		$\sigma_i^K$	$\sigma_K^i$		$\sigma_i^K$	$\sigma_K^i$
K		-0.4593	9	0.429	0.0131	17	0.347	0.0026
1	0.841	0.0226	10	0.750	0.0183	18	0.317	0.0615
2	0.658	0.0017	11	0.569	0.0056	19	0.388	0.0600
3	0.427	0.0229	12	0.317	0.0150	20	0.782	0.0096
4	0.643	0.0100	13	0.303	0.0216	21	0.425	0.0046
5	0.440	0.0101	14	0.501	0.0125	22	0.140	0.0430
6	0.540	0.0083	15	0.519	0.0149	23	0.618	0.0359
7	0.615	0.0193	16	0.082	0.0083	24	0.479	0.0354
8	0.846	0.0025						

The derived substitution elasticities for the Japanese economy are presented in Table 3. The first column in Table 3 represents the first column in matrix  $\sigma$  in Eq. (7). The second column in Table 3 represents the first row of  $\sigma$ . The own labor substitution elasticity  $\sigma_i^i$  equals  $-\sigma_K^i$ . Zeroes are predominant in the matrix  $\sigma$  in Eq. (7) since there is no substitution for labor across sectors:  $\sigma_h^i = 0$  when  $i \neq h$ .

Judging by the relative sizes of  $\sigma_i^K$  and  $\sigma_K^i$ , firms can more readily substitute labor for capital than they can substitute capital for labor. When the relative price of capital rises, the unit input of sector specific labor can readily be increased. When the relative price of labor rises, firms in the sector must compete with other sectors for shared capital and little substitution can take place.

Comparative static elasticities of the system in Eq. (7) are found by inverting the system matrix as described by Chang (1979) and Takayama (1982). There are  $49^2 = 2401$  separate comparative static elasticities. While such detail might seem overwhelming, patterns in the results provide insight into important theoretical properties of the SF model.

### 5. Comparative static results of tariffs

The effects of any one of 24 industrial tariffs in this SF model can be traced to the wage of that sector's labor, output in each sector, and the intersector capital payment.

A tariff on steel imported into Japan has been discussed in the face of competition from newly industrializing countries such as South Korea and Brazil. A steel tariff in this model would raise price in the iron and steel sector, redistributing income and reorganizing output. Table 4 reports elasticities of the price of capital and wages of the 24 types of labor with respect to the price of steel.

Table 4  
Elasticities of factor prices with respect to the price of iron and steel

K	0.102	5	-0.080	10	3.70	15	-0.110	20	-0.365
1	-0.538	6	-0.119	11	-0.134	16	-0.009	21	-0.075
2	-0.196	7	-0.162	12	-0.047	17	-0.054	22	-0.017
3	-0.076	8	-0.558	13	-0.044	18	-0.047	23	-0.164
4	-0.183	9	-0.076	14	-0.102	19	-0.064	24	-0.093

If a steel tariff were to raise the domestic price by 10%, wages in iron and steel (sector 10) would rise by 37% and the capital payment would increase by 1.02%. Losses would be spread evenly across wages in the other sectors, with the largest declines in petroleum and coal (-5.58%), agriculture (-5.38%), and finance (-3.65%). Elasticities of outputs in the other sectors with respect to the price of steel mirror the wage elasticity in each sector. Output of iron and steel itself would climb a tremendous 27% with a 10% price increase in steel.

Results in Table 4 illustrate the fundamental principle of political economy that the gains from protection are concentrated on a few while the losses are spread thinly around the economy. Winners from the steel tariff have a strong incentive to lobby for protection, while losers have little individual incentive to oppose it and quite likely little opportunity to organize. The comparative static elasticities in Table 4 are representative of the magnitudes and patterns for price changes across the other sectors of the economy.

Capital owners have very little to gain from protection, as illustrated in Table 4. Elasticities of the price of capital with respect to prices in each sector are not reported, but the largest are in agriculture (0.311), iron and steel (0.102), and utilities (0.086). Capital owners would also have to pay higher prices for finished goods implied by the protection, and their real income could fall.

Table 5 reports the elasticity of each sector's wage with respect to a price increase in its sector. Workers in petroleum and coal (8) have the most to gain from protection, followed by workers in agriculture (1) and finance (20).

Table 5  
Elasticities of each sector's wage with respect to price in that sector

1	4.65	5	1.78	10	3.70	15	2.50	20	4.34
2	2.91	6	2.16	11	2.31	16	1.09	21	1.74
3	1.73	7	2.53	12	1.46	17	1.53	22	1.16
4	2.75	8	6.29	13	1.43	18	1.44	23	2.48
		9	1.74	14	1.99	19	1.60	24	1.88

Table 6  
Elasticities of factor prices with respect to agricultural prices

K	0.311	5	-0.244	10	-0.933	15	-0.335	20	-1.12
1	4.65	6	-0.365	11	-0.410	16	-0.028	21	-0.230
2	-0.598	7	-0.497	12	-0.144	17	-0.165	22	-0.051
3	-0.232	8	-1.71	13	-0.136	18	-0.144	23	-0.503
4	-0.560	9	-0.234	14	-0.312	19	-0.197	24	-0.286

## 6. Effects of trade liberalization

The effects of trade liberalization can also be examined in the context of this model. Opening agriculture to foreign competition would lower the price of agricultural goods in Japan. Table 6 reports the elasticities of factor prices with respect to agricultural prices. A 10% decline in agricultural prices would lower agricultural wages by 46.5% and lower the capital payment by 3.11%. Wages across the rest of the economy would rise, especially in petroleum and coal (17.1%), finance (11.2%), and iron and steel (9.33%). Output elasticities again mirror the wage elasticities in each sector. These results suggest that agriculture in Japan would be hit very hard by a relaxation of agricultural protection.

Table 7 reports the distributional effects of trade liberalization in business services (24). If the US has a comparative advantage relative to Japan in business services, free trade would increase competition and lower the price in Japan. A 10% fall in the price of business services in Japan would lower the wage in that sector by 18.8%, while the payment to capital would fall by a slight 0.45%. Small gains would be spread across labor in the other sectors, with the largest impact on wages in petroleum and coal (2.50%), agriculture (2.40%), finance (1.63%), and iron and steel (1.36%).

## 7. Real income adjustment

These reported  $\partial w/\partial p$  results do not directly address the issue of real income adjustment for labor and capital owners. Changes in real income would depend on the consumption shares of finished goods for each set of

Table 7  
Elasticities of factor prices with respect to the price of business services

K	0.045	5	-0.036	10	-0.136	15	-0.049	20	-0.163
1	-0.240	6	-0.053	11	-0.060	16	-0.004	21	-0.034
2	-0.087	7	-0.073	12	-0.021	17	-0.024	22	-0.007
3	-0.034	8	-0.250	13	-0.020	18	-0.021	23	-0.073
4	-0.082	9	-0.034	14	-0.046	19	-0.029	24	1.88

Table 8  
Elasticities of factor prices with respect to capital supply

K	-0.348	5	0.273	10	1.04	15	0.376	20	1.25
1	1.84	6	0.409	11	0.460	16	0.031	21	0.257
2	0.670	7	0.556	12	0.162	17	0.185	22	0.057
3	0.259	8	1.91	13	0.151	18	0.162	23	0.563
4	0.627	9	0.262	14	0.349	19	0.221	24	0.320

factor owners. Some of the sectors in this specification produce finished goods, while others produce intermediate goods. An input-output scheme would be necessary to trace the effects of sectoral price changes onto the prices of finished goods. Assuming the owners of each factor consume proportional amounts of finished goods, a higher price in a particular sector would likely raise the real income of labor in that sector. Since any price index of finished goods would be higher with such a *ceteris paribus* price increase, the real income of labor in every other sector would have to fall. The change in the real income of capital owners with the price increase would depend on their consumption shares.

#### 8. Effects of foreign investment

Table 8 reports the effects of foreign investment represented by changes in the capital endowment. Since the 1970s, Japan has increased its level of foreign investment abroad. The data reported by Uno (1987) indicates that the annual investment abroad amounts to about 1% of the existing capital stock in Japan. Assuming the opportunity cost of investing abroad is adding to the capital stock at home, the current level of foreign investment is creating a 1% annual decrease in the potential Japanese capital stock.

Results in Table 8 indicate that the return to capital would rise only slightly (0.348%) with such a 1% capital outflow. Wages would fall, with an identical percentage output change in every sector. Wages and output would fall by more than 1% only in petroleum and coal (-1.91%), agriculture (-1.84%), finance (-1.25%), and iron and steel (-1.04%). The immediate income redistribution and output adjustment due to such a level of foreign investment abroad would appear to be negligible and spread thinly around the economy. Over time, however, capital outflows would mount and could represent a sizeable reduction in the stock of capital, wages, and outputs.

#### 9. Effects of changing labor supplies

The income redistributive effects of a changing labor endowment in each sector is examined in Table 9. Consider the effects of a 10% increase in