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Robustness of the Stolper–Samuelson Intensity Price Link

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Chapter Outline

The Stolper–Samuelson theorem isolates conditions under which factor intensity determines the qualitative factor price adjustments to price changes in general equilibrium. The present chapter examines the robustness of this "intensity price link" under relaxations of its sufficient conditions, with parametric specifications of the comparative static model based on neoclassical production, competitive pricing, and full employment.

1 Robustness of the Stolper–Samuelson Intensity Price Link

The Stolper–Samuelson (1941) theorem isolates a set of conditions under which factor intensity is sufficient to determine the qualitative effects of price changes on factor prices. Its novel property is that factor substitution plays no role. A literature evolved pointing out that the theorem does not hold under other conditions, implicitly suggesting a limited scope. The present chapter points out, however, that the Stolper–Samuelson intensity price link is generally robust to parametric relaxations of its sufficient conditions. The scope of the theorem is widened as it is shown to hold under much wider initial conditions than suggested by the list of sufficient conditions. None of the sufficient conditions are necessary for the intensity price link.

The next section reviews the proof of the Stolper–Samuelson theorem. The following sections analyze the intensity price link assuming in turn international factor mobility, nontraded products, factor intensity reversals, elastic factor supply, unemployment, factor market distortions, noncompetitive pricing of outputs, increasing returns, and nonhomothetic production. The intensity price link may hold under any of these conditions and when it is relaxed it is only partly so. Increasing returns are analyzed with a general cost function revealing new patterns of factor price adjustments. A final section summarizes models with many factors and many products, including a high dimensional measure of factor intensity.

2 Proofs of the Intensity Price Link

Proofs of the Stolper–Samuelson theorem follow the work of Koo (1953), Jones (1956), Lancaster (1957), Bhagwati (1959), and Chipman (1966). Its sufficient assumptions include:

- two homogeneous traded products in a small open economy;
- two homogeneous factors, mobile nationally but immobile internationally;
- perfect competition in product and factor markets;
- perfectly inelastic factor supply;
- full employment; and
- linearly homogeneous production functions.

The following sections relax these assumptions using parametric modifications of the algebraic comparative static model. The linearly homogeneous assumption is relaxed with both variable returns and nonhomothetic production. There are other implicit underlying assumptions, including the absence of specific factors, joint production, intermediate products, depletable or renewable resources, and production of capital goods.

The starting point is a 2 × 2 production box, explaining in part the enduring pedagogical popularity of the theorem. Along the contract curve, suppose factor 1 is used intensively in product 1,

$$v_{11}/v_{21} > v_{12}/v_{22} \tag{3.1}$$

where v_{ij} is the input of factor i in the production of product j, $i,j = 1,2$. The contract curve does not cross the diagonal because with homothetic production if a point on the diagonal were on the contract curve all points would have to be. While there can be no factor intensity reversals due to price changes in the economy with linearly homogeneous production, with three factors there could be.

Each endogenous factor price w_i is equal across sectors in the economy and isoquants of the two sectors share a common tangency and the same relative factor price. Exogenous prices p_j for the two traded products determine output levels and corresponding relative factor prices along the contract curve. A higher relative price for product 1 would raise its output and the relative price w_1/w_2 of its intensive

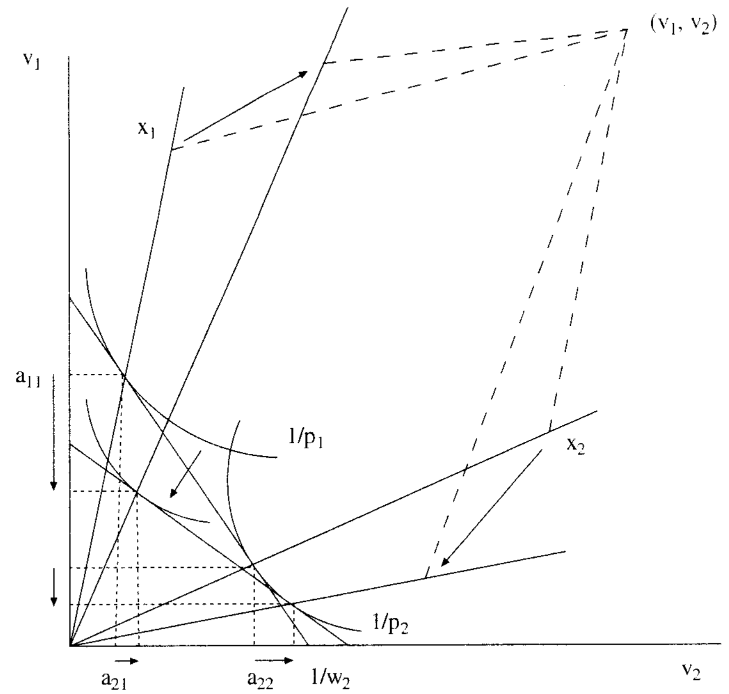

Figure 3.1 Stolper-Samuelson adjustment

factor. Input ratios v_{ij}/v_{2j} would fall in each sector as cost minimizing firms adjust to the new higher relative price of factor 1.

Figure 3.1 presents the corresponding 2 × 2 Lerner-Pearce production diagram. Unit value isoquants $x_j = 1/p_j$ represent the amount of each product worth one unit of numeraire. If dollars are the numeraire, it follows that p_j = \$/product and $1/p_j$ = product/\$. Neoclassical production functions imply concave isoquants with positions of unit value isoquants determined by exogenous prices p_j in the small open economy.

The unique unit isocost line $c_j = 1 = a_{ij}w_1 + a_{2j}w_2$ shows input combinations that cost \$1 and supports the unit isoquants due to cost minimization. Endpoints of the unit value isocost line are $1/w_i$. Firms minimize cost $c_j = \Sigma_i a_{ij} w_i$ where a_{ij} is the cost minimizing amount of factor i used in the production of a unit of product j. Competition ensures $p_j = c_j$, uniquely determining the endogenous w_i at the endpoints of the unit isocost line. The endogenous $a_{ij}(w)$ are functions of the vector of endogenous factor prices w. The factor intensity condition in (3.1) can be stated in terms of relative inputs,

$$a_{11}/a_{21} > a_{12}/a_{22}. \qquad (3.2)$$

as reflected by the steeper expansion path for sector 1.

In Figure 3.1, a ceteris paribus increase in p_1 shifts that unit value isoquant toward the origin as one dollar's worth becomes less of the physical product. The isocost line rotates around isoquant 2, the price of intensive factor w_1 rising while w_2 falls. Production becomes more intensive in relatively cheaper factor 2 as a_{1j} falls and a_{2j} rises. In the matrix of $\delta w_i/\delta p_j \equiv w_{ij}$ results, there is a positive main diagonal with negative elements off the diagonal.

$$\begin{pmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \end{pmatrix} = \begin{pmatrix} + & - \\ - & + \end{pmatrix} \tag{3.3}$$

The algebraic general equilibrium model will be used to introduce parametric relaxations of the various assumptions. Chipman (1966) and Takayama (1982) present the foundations of full employment of factors and competitive pricing of products. Full employment for factor i is stated $v_i = \Sigma_j a_{ij} x_j$ where v_i is the endowment of factor i and x_j the output of product j. Differentiate to find $\mathrm{d}v_i = \Sigma_j a_{ij}\mathrm{d}x_j + \Sigma_j x_j \mathrm{d}a_{ij}$. With homothetic production, cost minimizing unit inputs a_{ij} are functions of factor prices alone and $\mathrm{d}a_{ij} = \Sigma_k(\delta a_{ij}/\delta w_k)\mathrm{d}w_k$. It follows that $\Sigma_j x_j \mathrm{d}a_{ij} = \Sigma_k(\Sigma_j x_j \delta a_{ij}/\delta w_k)\mathrm{d}w_k = \Sigma_k s_{ik}\mathrm{d}w_k$ given the output weighted substitution term $s_{ik} \equiv \Sigma_j x_j \delta a_{ij}/\delta w_k$. Shephard's lemma states that cost minimizing inputs are partial derivatives of cost functions, $a_{ij} = \delta c_j/\delta w_i$ and it follows that $\delta a_{ij}/\delta w_k = \delta^2 c_j/\delta w_i \delta w_k$. Young's theorem on the symmetry of partial derivatives then implies $s_{ik} = s_{ki}$. For notation, $s \equiv s_{12} = s_{21}$. Own substitution terms s_{ii} are negative due to concavity of cost functions. Summing across weighted substitution terms, $\Sigma_i w_i s_{ik} = \Sigma_i w_i \Sigma_j x_j(\delta a_{ij}/\delta w_k) = \Sigma_j x_j \Sigma_i w_i(\delta a_{ij}/\delta w_k) = \Sigma_j x_j \Sigma_i w_i(\delta a_{kj}/\delta w_i) = 0$ by Euler's theorem. Without loss of generality, rescale factors so $w_i = 1$ and it follows that $s = -s_{11} = -s_{22} = s_{12} = s_{21}$. Full employment is stated in the first two equations of the comparative static system (3.4) below.

Competitive pricing for product j is stated $p_j = \Sigma_i a_{ij} w_i$. Differentiate to find $\mathrm{d}p_j = \Sigma_i a_{ij}\mathrm{d}w_i + \Sigma_i w_i \mathrm{d}a_{ij}$. Firms minimize cost, implying the slope of each unit value isoquant $\mathrm{d}a_{1j}/\mathrm{d}a_{2j}$ equals the slope of the isocost line $-w_2/w_1$. The cost minimizing envelope $\Sigma_i w_i \mathrm{d}a_{ij} = 0$ follows, implying $\mathrm{d}p_j = \Sigma_i a_{ij}\mathrm{d}w_i$. Competitive pricing is stated in the second two equations of the 2×2 comparative static factor proportions model,

$$\begin{pmatrix} -s & s & a_{11} & a_{12} \\ s & -s & a_{21} & a_{22} \\ a_{11} & a_{21} & 0 & 0 \\ a_{12} & a_{22} & 0 & 0 \end{pmatrix} \begin{pmatrix} dw_1 \\ dw_2 \\ dx_1 \\ dx_2 \end{pmatrix} = \begin{pmatrix} dv_1 \\ dv_2 \\ dp_1 \\ dp_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ dp_1 \\ dp_2 \end{pmatrix} \tag{3.4}$$

Factor endowments are held constant, $\mathrm{d}v_i = 0$. The factor intensity condition in (3.2) implies $a_{11}a_{22} - a_{12}a_{21} \equiv b > 0$. The positive determinant in (3.4) is $\Delta = b^2$. Factor price equalization occurs inside the production cone of McKenzie (1955) where $\delta w_i/\delta v_k = 0$.

The $\delta w_i/\delta p_j$ or w_{ij} terms are derived from cofactors in the lower left partition of the system matrix using Cramer's rule,

$$\begin{pmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \end{pmatrix} = \begin{pmatrix} a_{22}/b & -a_{12}/b \\ -a_{21}/b & a_{11}/b \end{pmatrix}, \tag{3.5}$$

confirming the intensity price link in (3.3). Note that factor substitution has no effect on the w_{ij} terms. It is a surprise that factors might be perfect substitutes or not substitutes at all and the w_{ij} terms would be identical, a peculiar result that holds for "even" models with the same number of factors and products.

Jones (1965) develops the magnification effect that price changes are weighted averages of factor price changes. In the differentiated competitive pricing condition for product j, $dp_j = \Sigma_i a_{ij} dw_i$, divide both sides by p_j and multiply the left side by w_i/w_i to find $\Sigma_i \theta_{ij} \hat{w}_i = \hat{p}_j$, where ^ represents percentage change and $\theta_{ij} = w_i a_{ij}/p_j$, a factor share. In even models, the intensity price link in elasticity form is determined by properties of the θ matrix in $\theta\hat{w} = \hat{p}$ since $\hat{w}/\hat{p} = \theta^{-1}$. Percentage price changes are weighted averages of factor price changes. In the 2×2 model, if $\hat{p}_1 > \hat{p}_2$ it must be that $\hat{w}_1 > \hat{p}_1 > \hat{p}_2 > \hat{w}_2$. If a single price increases, at least one factor price must rise more in percentage terms and the other factor price falls. For any nonzero vector of price changes, the real income of one factor must rise and the other must fall.

3 International Mobility of Factors and Nontraded Products

Adding internationally mobile factors of production or nontraded products may leave the intensity price link intact. As an example, consider the 3×2 model. The three factors capital, labor, and land provide the foundation for classical economics. Branson and Monoyios (1977) and Thompson (1997b) provide some motivation for trade models with separate skilled and unskilled labor. Batra and Casas (1976), Ruffin (1981), Takayama (1982), Suzuki (1983), Jones and Easton (1983), and Thompson (1985) develop theoretical properties of the 3×2 model. Factors can be unambiguously ranked according to factor intensity,

$$a_{11}/a_{12} > a_{21}/a_{22} > a_{31}/a_{32}. \tag{3.6}$$

Factor 1 is the extreme factor for product 1, factor 3 is extreme for product 2, and factor 2 is the middle factor. Thompson (1985) uncovers the possible sign patterns of w_{ij} terms, which depend on factor intensity as well as factor substitution. Isolating the two extreme factors, the possible sign patterns are

$$\begin{pmatrix} w_{11} & w_{31} \\ w_{12} & w_{32} \end{pmatrix} = \begin{matrix} + & - \\ - & + \\ (a) \end{matrix} \quad \begin{matrix} + & + \\ - & + \\ (b) \end{matrix} \quad \begin{matrix} + & + \\ - & - \\ (c) \end{matrix} \tag{3.7}$$

Sign pattern (3.7a) is the strong result, analogous to the 2×2 model. A higher price of product 1 unambiguously lowers the output of product 2 and demand for extreme factor 3 is expected to fall but in (3.7b) prices of both extreme factors rise. The expanding sector 1 can increase its input of factor 3, releasing complementary middle factor 2. In (3.7c), a higher price for product 2 also lowers the price of its extreme factor. Thompson (1986) isolates various conditions favoring factor price polarization, the separation of international factor prices with a move to free trade. The strong result in (3.7a) cannot be reversed completely as Thompson (1993) notes for the 3×2 magnification effect. Thompson (1995) uses sensitivity analysis in simulations of a 3×2 model of the US economy with skilled and unskilled labor and finds the intensity price link in (3.7a) due to the overwhelming influence of factor intensity.

Internationally mobile factors with factor prices exogenous at world levels can restore the intensity price link. If the middle factor in the 3×2 model is internationally mobile, there is the strong intensity price link in (3.7a). In the $r \times 2$ model when $r > 2$, the w_{ij} matrix has more than a single possible sign pattern and the intensity price link may break down. International mobility of $r - 2$ of the factors, however, would make factor prices exogenous and restore the intensity price link. In the $r \times 2$ model with $r - 2$ of the factors internationally mobile, there is an intensity price link for the internationally immobile factors.

With more products than factors in a small open economy, the comparative static system is overdetermined. Melvin (1968), Travis (1972), and Rader (1979) develop properties of the 2×3 and $2 \times n$ models, $n > 2$. Factor intensity can be unambiguously defined as a ranking of relative inputs across industries when there are two factors. When $n > r = 2$ in a small open economy, however, there are more than two arbitrarily placed unit value isoquants and for almost any set of world prices there is no unique supporting isocost line. Product prices may be assumed to adjust as in Choi (2003) but short of a solution algorithm little more can be said about the intensity price link.

Introducing nontraded products, however, endogenizes prices and can restore the intensity price link. Komiya (1967) and Rivera-Batiz (1982) develop models with nontraded products. In the $2 \times n$ model, if $n - 2$ of the products are nontraded there is a strong intensity price link for the traded products. In the 2×3 model with one nontraded product, Ethier (1972) examines conditions that lead to an intensity price link. Although demand conditions might relax the intensity price link, it is robust to "small" demand elasticities.

4 Factor Intensity Reversals and the Intensity Price Link

Production cones are regions in factor space between expansion paths where all products can be produced with full employment of all factors. Expansion paths are linear with homothetic production. Production cones are generally not unique. Even in the 2×2 model, there are two production cones if the isoquants cross twice. Pearce (1951), James and Pearce (1951), and Harrod (1958) make the point that factor price equalization would not occur with free trade if endowments of the two trading countries lie in different production cones. The country abundant