A REVIEW OF ADVANCEMENTS IN THE GENERAL EQUILIBRIUM THEORY OF PRODUCTION AND TRADE*

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Abstract. This review homogenizes much of the work done in general equilibrium trade theory since Chipman's *Econometrica* (1965–6) survey. Examined are properties of the long run Walrasian model of a small open economy with full employment of productive resources and competitive pricing of homogeneous final goods. The three factor model, which includes human capital or natural resources as primary inputs along with traditional capital and labor, is contrasted with the two factor Heckscher-Ohlin-Samuelson model. Magnification effects in the three factor model are completely developed. A review of these production models with international capital mobility is included.

Chipman's (1965–6) survey of international trade gives thorough clear accounts of classical, neoclassical, and modern trade theories. A tremendous amount of research in trade theory has been done in various directions over the past twenty years. Now a comprehensive survey would have to wrestle with a host of approaches from industrial organization, imperfect competition, product differentiation, hedonic pricing, second best analysis, etc. The scope in this review is limited to advances in the general equilibrium modelling of small open economies. These models are based on a long run Walrasian production structure with competitive pricing of homogeneous final goods and full employment of homogeneous primary factors of production.

Basic contributions have historically been developed in the model with two internationally immobile factors and two traded final goods. Stolper-Samuelson and Rybczynski theorems, factor price equalization, and the Heckscher-Ohlin theorem are all based on this 2×2 model. These results form the foundation for common concepts in international economics, even though the simple model is no more than rudimentary.

Studies of larger dimensional even models with the same number of factors and goods determine which results from the 2×2 model generalize. Less has been done with uneven models, those with differing numbers of factors and goods. Recent research, however, has increased what we know about certain uneven

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models. Attention has been given to the specific factors model, where each sector uses two factors, one specific to the sector and another shared between sectors.

Other studies have developed the three factor model where each sector employs all three factors. This 3×2 model is the simplest general equilibrium model in which complementarity in production may occur. It allows consideration of skilled labor (human capital) as a separate primary input. Alternatively, it embodies the "classical" model with land, labor, and capital inputs. Also it may be regarded as the prototype for larger dimensional uneven models, much as the 2×2 model offers the basic insight for all even models. As intuitive understanding of the properties of this three factor model develop, it will supplement the simpler two factor model in the foundations of trade theory.

The present review complements Takayama's (1982) recent survey in this journal. Emphasis lies on integrating and illustrating a wide range of advancements. A complete treatment of magnification effects in the three factor model is included in the present review, as is a section on international capital mobility. This review offers an accesible avenue for those wanting to become familiar with the theoretical foundation of competitive models of production and trade.

Section 1 presents the basic general equilibrium trade model. Section 2 analyzes results of how *ceteris paribus* changes in factor endowments affect outputs. Section 3 examines how a changing price affects factor payments and real incomes, with factor endowments constant. Section 4 looks into effects of a changing endowment on factor payments. Section 5 examines the Heckscher-Ohlin theorem, in situations with both two and three factors. Section 7 presents a brief look at models with internationally mobile capital.

1. THE BASIC MODEL

Clear presentations of the basic properties of general equilibrium trade models can be found in Chang (1979), Jones and Scheinkman (1977), and notably Takayama (1982). Where r and n are positive integers and $r\geq n$, suppose there are r primary factors of production with factor endowment vector v and payment vector w, and n products with output vector x and price vector p. If $r<n$, flat "Ricardian" production surfaces occur so pure production models become intractible.

Two constraints represent full employment and competitive pricing:

$$v=Ax\,, \tag{1}$$

$$p=A'w\,, \tag{2}$$

where matrix A is composed of $r\times n$ cost minimizing unit factor mixes each dependent on w. The amount of factor k used to produce a unit of good m is written $a_{km}(w)$.

Prices of goods are exogenously given to a small open economy by large world markets. One good is thus assumed to be an aggregated exportable, while the other represents aggregated importables. Adding nontraded goods would consequently require a third good, and introduce an endogenous price dependent on domestic demand. While the treatment of nontraded goods poses interesting questions, this vast literature is not considered in this survey.

Factor endowments are exogenous as well, with perfectly inelastic supplies insuring full employment as in (1). The emphasis is upon comparative statics, moving from one long run equilibrium to another. Endowment changes are considered, but short or medium run adjustment processes and the dynamics of growth are not.

Taking the differential of (1),

$$dv = x\,dA + A\,dx\,. \tag{3}$$

Aggregate economy wide substitution terms s_{ih} can be introduced: $s_{ih} \equiv \Sigma_j x_j a^h_{ij}$, where $\partial a_{ij}/\partial w_h \equiv a^h_{ij}$. These substitution terms summarize how cost minimizing firms across the economy alter their input mix in the face of changing factor payments. If s_{ih} is positive (negative), factors i and h are aggregate technical substitutes (complements). For every i, $dAx = \Sigma_k s_{ik}\,dw$, so (3) becomes

$$dv = S\,dw + A\,dx\,. \tag{4}$$

Considering small changes, cost minimizing behavior insures that

$$w\,dA' = 0\,. \tag{5}$$

Using (5) and taking the differential of (2),

$$dp = A'\,dw\,. \tag{6}$$

Putting (4) and (6) together into matrix form,

$$\begin{pmatrix} S & A \\ A' & 0 \end{pmatrix} \begin{pmatrix} dw \\ dx \end{pmatrix} = \begin{pmatrix} dv \\ dp \end{pmatrix}. \tag{7}$$

For reference, the main coefficient matrix is called B.

Uzawa (1964) argues that given a production function for good m, $x_m = f_m(v_m)$, where v_m is the input vector, an associated cost function $c_m = c_m(x_m, w)$ is uniquely determined. Production functions are the fundamental relationships of the model, leading to (1) and (2). The classical treatment of this duality between production and cost is given by Shephard (1953), including a proof of Shephard's lemma, $c^k_m \equiv \partial c_m/\partial w_k = a_{km}$. Intuitively this lemma follows from the envelope result in (5). Given either a cost or a production function, the other can be derived since an identical structure underlies both.

Substitution matrix S in (7) is symmetric, since $\partial c^k_m/\partial w_h = \partial c^h_m/\partial w_k$ for every good m by Taylor's rule. Cost functions are homogeneous of degree one in factor payments, so the $a_{km}(w)$ are homogeneous of degree zero and depend only on relative wages. Due to this homogeneity, multiplying each row of S by the vector of factor wages yields zero, i.e., $Sw=0$. Increasing a factor's wage causes firms to switch away from it and its input to fall, so S has a negative diagonal. More than two factors are thus necessary for technical complementarity to become a possibility. Furthermore negative semidefiniteness of S follows given only that the own substitution effects based on a^i_{im}'s outweigh the cross effects based on a^k_{im}'s $(i \neq k)$.

The entire coefficient matrix B is the Jacobian of Ax and $A'w$ with respect to w and x, since

$$\begin{pmatrix} \partial Ax/\partial w & \partial Ax/\partial x \\ \partial A'w/\partial w & \partial A'w/\partial x \end{pmatrix} = \begin{pmatrix} x\partial A\partial/w & A \\ A' & \partial c/\partial x \end{pmatrix} = \begin{pmatrix} S & A \\ A' & 0 \end{pmatrix} .$$

Thus the mapping in R^{r+n} from (p, v) to (x, w) is locally invertible (one to one and onto) due to the inverse function theorem. All that is necessary for this result is that the input matrix A be of full rank n. This means simply that each input is truly distinct, not being a linear combination of other inputs. Otherwise different levels of outputs and payments could result from the same combination of prices and endowments.

Following Chang (1979), consider the inverse of the system matrix B,

$$B^{-1} \equiv \begin{pmatrix} C & E \\ E' & F \end{pmatrix} = \begin{pmatrix} \partial w/\partial v & \partial w/\partial p \\ \partial x/\partial v & \partial x/\partial p \end{pmatrix} .$$

Where $H \equiv AA' - S$ and $G \equiv A'H^{-1}A$, it follows that $C = H^{-1}AG^{-1}A'H^{-1} - H^{-1}$, $E' = G^{-1}A'H^{-1}$, $E = H^{-1}AG^{-1}$, and $F = G^{-1} - I_n$. Matrix C is seen to be symmetric. Samuelson's reciprocity result is immediately apparent: $E = (E')'$. Matrix F is symmetric as well, with a positive diagonal, reflecting concavity of the transformation surface. Caratheodory (1967) presents an argument concerning reciprocal quadratic forms that C is negative semidefinite with rank $r-n$.

Next consider four homogeneity properties formally proven in Chang (1979) and Takayama (1982). Constant returns to scale imply that outputs are homogeneous of degree one in inputs:

$$E'v = x . \tag{8}$$

With endowments fixed, only relative price changes affect outputs. This means that outputs are homogeneous of degree zero in prices:

$$Fp = 0 . \tag{9}$$

Factor payments are determined by isocost surfaces which support production isoquants. With prices constant and endowments changing proportionally, factor payments are not affected, and thus are homogenous of degree zero in endowments:

$$Cv=0\,. \tag{10}$$

If prices vary proportionally, so do factor payments. This "neutrality" means factor payments are homogeneous of degree one in prices:

$$Ep=w\,. \tag{11}$$

By (8) and (11), every row of E' and E must have at least one positive element. If a factor endowment is positively (negatively) correlated with some industry's output, it can be said to "strengthen" ("weaken") that industry. In E', (i) for any industry, there is some factor which strengthens it, and (ii) every factor strengthens some industry. In E, (i) for every wage, there is some price which raises it, and (ii) every price raises some wage.

A proof by Either (1974) shows that every column of E (row of E') has at least one negative element. By (8), every column of E contains a nonzero element. Since $BB^{-1}=I_{r+n}$, it follows that $A'E=I_n$. Any row i of A' multiplied by column i of E yields 1, and by any other column 0. Given the appropriate Inada or boundary condition, each industry uses at least two factors. Every row of A' will then have at least two positive elements. Mutiplying row j of A' by column k of E, where $j\neq k$, yields 0. Some element in column k of E must then be negative. So every industry is weakened by at least one factor, while every price lowers at least one wage.

More can be gleaned if the model is put into elasticity form as in Jones (1965). From (5) and (6), where factor share $\theta_{km}\equiv w_k a_{km}/p_m$ and industry share $\lambda_{km}\equiv x_m a_{km}/v_k$,

$$\begin{pmatrix}\sigma & \lambda\\ \theta' & 0\end{pmatrix}\begin{pmatrix}\hat{w}\\ \hat{x}\end{pmatrix}=\begin{pmatrix}\hat{v}\\ \hat{p}\end{pmatrix}. \tag{7'}$$

Matrix σ has a negative diagonal and zero row sums. Matrices λ and θ' are nonnegative and row stochastic. Thus the system matrix β is row stochastic.

Where

$$\beta^{-1}\equiv\begin{pmatrix}\delta & \varepsilon\\ \rho & \gamma\end{pmatrix},$$

it follows that $\theta'\varepsilon=I_n$ since $\beta\beta^{-1}=I_{r+n}$. Every column of ε has at least one element greater than unity since θ' is row stochastic. So some factor payment is more than proportionally raised by every price. A tariff on one good then must unambiguously raise the real income of at least one factor.

A further result is obtained by considering that $\rho\lambda=I_n$. Industry m uses some fraction λ_{km} of input k, where $0\leq\lambda_{km}\leq 1$. Every industry uses at least two factors, but not every factor is necessarily used in each industry. Jones and Scheinkman (1977) suppose industry m is small: $\sum_i \lambda_{im}<1$. Since $\lambda_{im}<1$ for all i by assumption, there must be an element in every row of ρ greater than unity. If industry

m is small, there must then be some factor which more than proportionally strengthens the output of good m, since some element in that row is greater than one.

2. $\partial x/\partial v$ RESULTS

At least one factor must weaken while another must strengthen each industry. Every factor strengthens some industry. In the 2×2 model, the $\partial x/\partial v$ Rybczynski matrix has a positive diagonal with equal negative elements off the diagonal. Only general $\partial x/\partial v$ conclusions as in Batra and Casas (1976) have been available in higher dimensional models, "without requiring the detailed sign patterns of the $\partial w/\partial p$ and $\partial x/\partial v$ matrices" as put by Chang (1979).

Given Ruffin's (1981) factor intensity ordering in the 3×2 model, $a_{11}/a_{12}>a_{21}/a_{22}>a_{31}/a_{32}$, definite results are obtainable. Factor 1 is most intensive or "extreme" in industry one, factor 3 extreme in industry two, and factor 2 the "middle" factor. Batra and Casas (1976) argue for the necessity of a "strong" Rybczynski pattern where each extreme factor strengthens the output of the good using it most intensively and weakens the output of the other sector. Their conclusion, however, is based on the assumption of weak substitution among inputs and a roundabout approach to factor intensity. Suzuki (1983) points out that it is possible for $\partial w_3/\partial p_1$ or $\partial w_1/\partial p_2$ ($\partial x_1/\partial v_3$ or $\partial x_2/\partial v_1$) to be positive. Let $\partial x_j/\partial v_i$ be represented by the term r_{ji}, and write out the matrix

$$E'=\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \end{pmatrix} .$$

Possible sign patterns in E' are shown by Thompson (1985) to be:

(a)	(b)	(b′)	(c)	(c′)	(d)	(d′)
$++-$	$+--$	$++-$	$+-+$	$++-$	$+-+$	$-+-$
$-++$	$-++$	$--+$	$-++$	$+-+$	$-+-$	$+-+$.

Some other patterns are ruled out, based on this theorem: If an industry is weakened by its extreme factor, the other factors cannot both strengthen it. Patterns with primes are obtained by switching the names of extreme factors and then switching the names of goods. Switched patterns are structurally similar to the originals, since naming is arbitrary.

The first three patterns (a), (b) and (b′) demonstrate a strong Rybczynski result, with each industry strengthened by its extreme factor and weakened by the factor which is extreme in the other sector. Patterns (c) and (c′) are characterized by an extreme factor which strengthens both industries. In (d) and (d′), an extreme factor *weakens* the industry which uses it most intensively, and strengthens the other.

This last result is most surprising and important, since so much basic intuition in international trade is based upon a strong link between factors and goods through the intensity relationship. This relative wealth of outcomes in the 3×2 model results from the interplay in production of factor substitution and intensity. Any pair of factors may be complementary. The degree of factor intensity also becomes crucial to the qualitative nature of outcomes.

Takayama (1982) notes that when the extreme factors are complementary, a strong Rybczynski result must hold. There is in fact some empirical evidence that capital and skilled labor are complementary, although the question would seem to remain open. Thompson and Clark (1983) show that skilled labor and capital are the extreme factors in a simple application of the 3×2 model to the US economy, where one sector is manufacturing (plus services) and the other agriculture. Qualitative $\partial x/\partial v$ and reciprocal $\partial w/\partial p$ results can perhaps be anticipated without a complete specification of the model. A *ceteris paribus* increase in the skilled labor endowment through immigration will create a higher output in manufacturing, the sector where it is extreme. Takayama discusses how the three factor model may help explain Leontief's paradox, the US importing capital intensive commodities even with a high capital to labor ratio. As originally suggested by Leontief, the US simply has a greater relative abundance of skilled labor.

3. $\partial w/\partial p$ RESULTS

Stolper and Samuelson (1941) present an argument that in a 2×2 economy, an increased relative price of a good causes (i) more of that good to be produced, (ii) a rise in the payment to the factor used intensively in that industry and a fall in the other factor payment, and (iii) factor intensity ratios to move away from the more expensive factor in each industry.

Jones (1965) develops the magnification effect, $\hat{w}_1>\hat{p}_1>\hat{p}_2>\hat{w}_2$, where factor 1 (2) is used intensively in sector 1 (2). This follows directly from (7′), since $\theta'\hat{w}=\hat{p}$ and θ' is row stochastic. It can be seen that w_2 could rise with an increase in p_1/p_2, if $\hat{p}_1>\hat{p}_2>0$. If the price of a good increases, *ceteris paribus*, there arises a more than proportional increase in the payment to one factor with a decrease in the payment to the other. The real return to one factor must then rise, as the real return to the other falls.

One way to generalize the Stolper-Samuelson theorem is to move up to a higher dimensional even model ($r=n>2$). Chipman (1969) differentiates this $\partial w/\partial p$ result by whether it is (i) local or global, and (ii) weak or strong. In the local version, for any m there is a k such that given $\hat{p}_m$, $\hat{w}_k=\lambda\hat{p}_m$, where $\lambda>1$. The global version states that there is a one to one correspondence between goods and factors such that this relation always holds. In the weak version for any m there is a k such that $\hat{w}_k\geq\hat{p}_m$, while the strong version is satisfied if $\hat{w}_k=\lambda\hat{p}_m$ implies $\hat{w}_h<0$ for all $h\neq k$. The weak version is implied by the strong.

Ethier (1974) notes that Chipman's weak condition requires restrictions on the matrix A, and develops a general property. It is proven that there are associations between goods and factors such that for every m there are an h and k such that (i) $\hat{w}_h/\hat{p}_m<0$, and (ii) $\hat{w}_k/\hat{p}_m>1$. In these even models, assuming only an invertible system, an increase in the price of any good lowers some wage and more than proportionally raises some other wage.

Still within the context of even models, Jones (1976) calls factor k "important" if $\sum_j \theta_{kj}\geq 1$. Summing across goods in matrix θ indicates how large a role some factor plays in the pricing of goods. This summation is positive since each factor must be used in some industry. For every factor which is not important, it is proven that there must be a good whose price raises its real income.

Consider these $\partial w/\partial p$ results in the 3×2 model, with an eye to how real income responds to any exogenous price change. Note that the symmetry between the $\partial w/\partial p$ and $\partial x/\partial v$ results implies that the same sign patterns described in Section 2 will be found. Clearly if $\partial w_k/\partial p_m<0$, the real income of factor k is inversely related with the price of good m. Whether w_k rises sufficiently to offset a higher price of good m when $\partial w_k/\partial p_m>0$ creates an ambiguity examined by Ruffin and Jones (1977). The ratio of the percentage change in w_k to the percentage change in p_m, $\hat{w}_k/\hat{p}_m$, is defined as β_{km}. Where d^k_m is the quantity of good m consumed by the owners of factor k, their consumption share of good m is written $\phi^k_m\equiv(p_m d^k_m)/Y_k$, where Y_k represents factor k's nominal income $w_k v_k$. Note that $\sum_j \phi^k_j=1$. The real income of factor k equals its nominal income Y_k divided by average prices weighted by consumption shares: $y_k\equiv Y_k/(\sum_j \phi^k_j p_j)$. Totally differentiating this expression, dividing by y_k, and considering changes in the price of good m, $\hat{y}_k=\hat{Y}_k-\phi^k_m\hat{p}_m$. When the price of good m changes, the change in factor k's real income is then determined by the expression $\hat{y}_k=(\beta_{km}-\phi^k_m)\hat{p}_m$.

Looking back at the sign patterns of the comparative static outcomes in the 3×2 model, negative signs present no ambiguity since $\beta_{km}<0$. The magnification effect says that each $\hat{p}$ is a weighted average of the $\hat{w}$'s. With $\hat{p}_1>0$ and $\hat{p}_2=0$ in the top row of the sign patterns, it must be that $\hat{w}_k>\hat{p}_1$ for some k. The top rows with one positive sign show the factor whose welfare is raised by an increase in the price of good one. For instance in the top row of (b), it follows that $\hat{w}_1>\hat{p}_1$, so a tariff on good one would raise the real income of factor one. The other top rows have two positive signs which create ambiguity, since a positive $\hat{p}_1$ may be greater than one of the $\hat{w}_k$'s. In that case, it would follow that $\beta_{k1}<1$. If in addition $\phi^k_1<\beta_{k1}$, the real income of factor k would rise. But if $\beta_{k1}<\phi^k_1$, $\hat{y}_k$ would be negative.

Jones and Easton (1983) produce a diagrammatic argument which can lead to magnification effects for the 3×2 model. Using payment to the middle factor two as numeraire, they examine percentage changes in payments to extreme factors resulting from a change in relative prices. Four different locci are derived as in Fig. 1. For $j=1, 2$, the p_j schedule sets $\hat{p}_j=0$. Along either p_j schedule, $\hat{w}_3/\hat{w}_1=-\theta_{1j}/\theta_{3j}$ from (7′). Each p_j schedule is thus negatively sloped. Due to the assumed factor intensity, the p_1 locus is steeper. The p locus sets $\hat{p}_1=\hat{p}_2$, so again from (7′), $\hat{w}_3/\hat{w}_1=(\theta_{11}-\theta_{13})/(\theta_{33}-\theta_{31})$, which is positive. All schedules have a common intersection at the prevailing equilibrium factor payments w_1 and w_3. Full employment occurs along the v locus which passes through the common intersection and by (7′) may have either a positive or negative slope.

An exogenous increase in the price of good one relative to good two causes the p locus to shift to the right. A new equilibrium occurs with full employment, where the new p locus (p') intersects the full employment v locus. Exogenous price changes are arbitrarily chosen so p_1 and p_2 schedules adjust to the new equilibrium. With $\hat{w}_2=0$ and $\hat{p}_1>\hat{p}_2$, signs and magnitudes of $\hat{w}_1$ and $\hat{w}_3$ can be read from the diagram. By the magnification effect, there must be factors h and k such that $\hat{w}_h>\hat{p}_1>\hat{p}_2>\hat{w}_k$.

The v locus may intersect the new locus p' in seven different regions of Fig. 1.

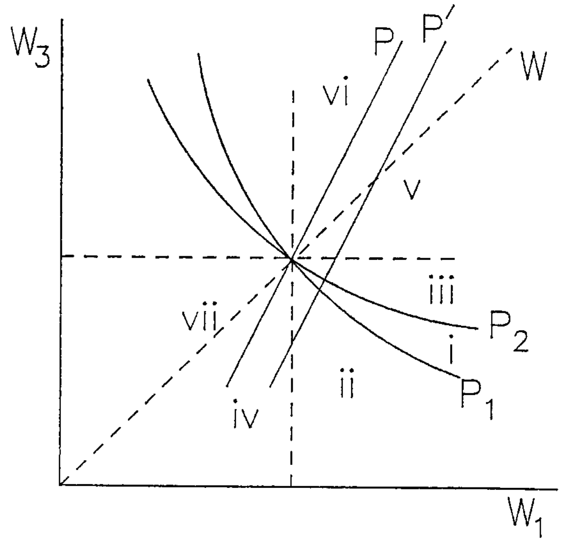

Fig. 1.

Regions i, ii, and iv are considered explicitly by Jones and Easton. The following magnification effects can be read from the diagram:

(i) $\hat{w}_1>\hat{p}_1>\hat{w}_2>\hat{p}_2>\hat{w}_3$,

(ii) $\hat{w}_1>\hat{w}_2>\hat{p}_1>\hat{p}_2>\hat{w}_3$,

(iv$_a$) $\hat{w}_2>\hat{w}_1>\hat{p}_1>\hat{p}_2>\hat{w}_3$,

(iv$_b$) $\hat{w}_2>\hat{p}_1>\hat{w}_1>\hat{p}_2>\hat{w}_3$,

and

(iv$_c$) $\hat{w}_2>\hat{p}_1>\hat{p}_2>\hat{w}_1>\hat{w}_3$.

Payment to middle factor two "floats," as payment to extreme factor one "sinks."

A new equilibrium may also occur in region iii or v. Jones and Easton point out (page 83) that the v locus may be positively sloped and less steep than the p

locus. This creates a solution in region v. Region iii solutions occur where v is negatively sloped, with $-\theta_{12}/\theta_{22}<0$. In regions iii and v, the following magnification effects are found:

(iii) $\hat{w}_1>\hat{p}_1>\hat{p}_2>\hat{w}_2>\hat{w}_3$,
(v$_a$) $\hat{w}_1>\hat{p}_1>\hat{p}_2>\hat{w}_3>\hat{w}_2$,
(v$_b$) $\hat{w}_1>\hat{p}_1>\hat{w}_3>\hat{p}_2>\hat{w}_2$,
and
(v$_c$) $\hat{w}_1>\hat{w}_3>\hat{p}_1>\hat{p}_2>\hat{w}_2$.

Payment to the middle factor sinks, as payment to extreme factor three floats.

Above ray w from the origin, $\hat{w}_3>\hat{w}_1$. Magnification effects in this region can also be derived. Sign patterns of the $\partial w/\partial p$ matrix may be used to derive magnification effects, since one price remains unchanged in either row of the sign patterns. One of the above magnification effects is implied except in the following circumstances. In the top row of sign patterns (c) or (d) where $\hat{p}_1>0$ and $\hat{p}_2=0$, two further results occur:

(vi$_a$) $\hat{w}_3>\hat{w}_1>\hat{p}_1>\hat{p}_2>\hat{w}_2$,
and
(vi$_b$) $\hat{w}_3>\hat{p}_1>\hat{w}_1>\hat{p}_2>\hat{w}_2$.

Payment to factor three floats through region v and "tops out" in region vi, as payment to factor one falls. In the bottom row of sign pattern (c′) or (d′), where $\hat{p}_1=0$ and $\hat{p}_2<0$, these two magnification effects are then found:

(vii$_a$) $\hat{w}_2>\hat{p}_1>\hat{p}_2>\hat{w}_3>\hat{w}_1$,
and
(vii$_b$) $\hat{w}_2>\hat{p}_1>\hat{w}_3>\hat{p}_2>\hat{w}_1$.

Payment to factor one sinks in region iv and here "bottoms out," as payment to factor three floats.

There remain two magnification effects suggested by the diagrammatic technique which are in fact impossible:

(vi$_c$) $\hat{w}_3>\hat{p}_1>\hat{p}_2>\hat{w}_1>\hat{w}_2$,
and
(vii$_c$) $\hat{w}_2>\hat{w}_3>\hat{p}_1>\hat{p}_2>\hat{w}_1$.

These do not arise anywhere in the sign patterns, which cover all possibilities. Stated in terms of extreme factors 1 and 3, the Stolper-Samuelson theorem cannot then be "reversed." This is the bottom line in the three factor model's magnification effects.

4. $\partial w/\partial v$ RESULTS

Lerner (1952) develops sufficient conditions for factor price equalization between trading partners in the 2×2 situation. Samuelson (1949) formalizes the argument, introducing the idea of strong factor intensity: whatever the level of relative