A REVIEW OF ADVANCEMENTS IN THE GENERAL EQUILIBRIUM THEORY OF PRODUCTION AND TRADE*

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Abstract. This review homogenizes much of the work done in general equilibrium trade theory since Chipman's *Econometrica* (1965–6) survey. Examined are properties of the long run Walrasian model of a small open economy with full employment of productive resources and competitive pricing of homogeneous final goods. The three factor model, which includes human capital or natural resources as primary inputs along with traditional capital and labor, is contrasted with the two factor Heckscher-Ohlin-Samuelson model. Magnification effects in the three factor model are completely developed. A review of these production models with international capital mobility is included.

Chipman's (1965–6) survey of international trade gives thorough clear accounts of classical, neoclassical, and modern trade theories. A tremendous amount of research in trade theory has been done in various directions over the past twenty years. Now a comprehensive survey would have to wrestle with a host of approaches from industrial organization, imperfect competition, product differentiation, hedonic pricing, second best analysis, etc. The scope in this review is limited to advances in the general equilibrium modelling of small open economies. These models are based on a long run Walrasian production structure with competitive pricing of homogeneous final goods and full employment of homogeneous primary factors of production.

Basic contributions have historically been developed in the model with two internationally immobile factors and two traded final goods. Stolper-Samuelson and Rybczynski theorems, factor price equalization, and the Heckscher-Ohlin theorem are all based on this 2×2 model. These results form the foundation for common concepts in international economics, even though the simple model is no more than rudimentary.

Studies of larger dimensional even models with the same number of factors and goods determine which results from the 2×2 model generalize. Less has been done with uneven models, those with differing numbers of factors and goods. Recent research, however, has increased what we know about certain uneven

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models. Attention has been given to the specific factors model, where each sector uses two factors, one specific to the sector and another shared between sectors.

Other studies have developed the three factor model where each sector employs all three factors. This 3×2 model is the simplest general equilibrium model in which complementarity in production may occur. It allows consideration of skilled labor (human capital) as a separate primary input. Alternatively, it embodies the "classical" model with land, labor, and capital inputs. Also it may be regarded as the prototype for larger dimensional uneven models, much as the 2×2 model offers the basic insight for all even models. As intuitive understanding of the properties of this three factor model develop, it will supplement the simpler two factor model in the foundations of trade theory.

The present review complements Takayama's (1982) recent survey in this journal. Emphasis lies on integrating and illustrating a wide range of advancements. A complete treatment of magnification effects in the three factor model is included in the present review, as is a section on international capital mobility. This review offers an accesible avenue for those wanting to become familiar with the theoretical foundation of competitive models of production and trade.

Section 1 presents the basic general equilibrium trade model. Section 2 analyzes results of how *ceteris paribus* changes in factor endowments affect outputs. Section 3 examines how a changing price affects factor payments and real incomes, with factor endowments constant. Section 4 looks into effects of a changing endowment on factor payments. Section 5 examines the Heckscher-Ohlin theorem, in situations with both two and three factors. Section 7 presents a brief look at models with internationally mobile capital.

1. THE BASIC MODEL

Clear presentations of the basic properties of general equilibrium trade models can be found in Chang (1979), Jones and Scheinkman (1977), and notably Takayama (1982). Where r and n are positive integers and $r \geq n$, suppose there are r primary factors of production with factor endowment vector v and payment vector w, and n products with output vector x and price vector p. If $r<n$, flat "Ricardian" production surfaces occur so pure production models become intractible.

Two constraints represent full employment and competitive pricing:

$$v = Ax \,, \tag{1}$$

$$p = A'w \,, \tag{2}$$

where matrix A is composed of $r \times n$ cost minimizing unit factor mixes each dependent on w. The amount of factor k used to produce a unit of good m is written $a_{km}(w)$.

Prices of goods are exogenously given to a small open economy by large world markets. One good is thus assumed to be an aggregated exportable, while the other represents aggregated importables. Adding nontraded goods would consequently require a third good, and introduce an endogenous price dependent on domestic demand. While the treatment of nontraded goods poses interesting questions, this vast literature is not considered in this survey.

Factor endowments are exogenous as well, with perfectly inelastic supplies insuring full employment as in (1). The emphasis is upon comparative statics, moving from one long run equilibrium to another. Endowment changes are considered, but short or medium run adjustment processes and the dynamics of growth are not.

Taking the differential of (1),

$$dv = x\,dA + A\,dx\ . \tag{3}$$

Aggregate economy wide substitution terms s_{ih} can be introduced: $s_{ih} \equiv \Sigma_j x_j a_{ij}^h$, where $\partial a_{ij}/\partial w_h \equiv a_{ij}^h$. These substitution terms summarize how cost minimizing firms across the economy alter their input mix in the face of changing factor payments. If s_{ih} is positive (negative), factors i and h are aggregate technical substitutes (complements). For every i, $dAx = \Sigma_k s_{ik}\,dw$, so (3) becomes

$$dv = S\,dw + A\,dx\ . \tag{4}$$

Considering small changes, cost minimizing behavior insures that

$$w\,dA' = 0\ . \tag{5}$$

Using (5) and taking the differential of (2),

$$dp = A'\,dw\ . \tag{6}$$

Putting (4) and (6) together into matrix form,

$$\begin{pmatrix} S & A \\ A' & 0 \end{pmatrix} \begin{pmatrix} dw \\ dx \end{pmatrix} = \begin{pmatrix} dv \\ dp \end{pmatrix}. \tag{7}$$

For reference, the main coefficient matrix is called B.

Uzawa (1964) argues that given a production function for good m, $x_m = f_m(v_m)$, where v_m is the input vector, an associated cost function $c_m = c_m(x_m, w)$ is uniquely determined. Production functions are the fundamental relationships of the model, leading to (1) and (2). The classical treatment of this duality between production and cost is given by Shephard (1953), including a proof of Shephard's lemma, $c_m^k \equiv \partial c_m/\partial w_k = a_{km}$. Intuitively this lemma follows from the envelope result in (5). Given either a cost or a production function, the other can be derived since an identical structure underlies both.

Substitution matrix S in (7) is symmetric, since $\partial c_m^k/\partial w_h = \partial c_m^h/\partial w_k$ for every good m by Taylor's rule. Cost functions are homogeneous of degree one in factor payments, so the $a_{km}(w)$ are homogeneous of degree zero and depend only on relative wages. Due to this homogeneity, multiplying each row of S by the vector of factor wages yields zero, i.e., $Sw=0$. Increasing a factor's wage causes firms to switch away from it and its input to fall, so S has a negative diagonal. More than two factors are thus necessary for technical complementarity to become a possibility. Furthermore negative semidefiniteness of S follows given only that the own substitution effects based on a^i_{im}'s outweigh the cross effects based on a^k_{im}'s $(i \neq k)$.

The entire coefficient matrix B is the Jacobian of Ax and $A'w$ with respect to w and x, since

$$\begin{pmatrix} \partial Ax/\partial w & \partial Ax/\partial x \\ \partial A'w/\partial w & \partial A'w/\partial x \end{pmatrix} = \begin{pmatrix} x\partial A\partial/w & A \\ A' & \partial c/\partial x \end{pmatrix} = \begin{pmatrix} S & A \\ A' & 0 \end{pmatrix}.$$

Thus the mapping in R^{r+n} from (p, v) to (x, w) is locally invertible (one to one and onto) due to the inverse function theorem. All that is necessary for this result is that the input matrix A be of full rank n. This means simply that each input is truly distinct, not being a linear combination of other inputs. Otherwise different levels of outputs and payments could result from the same combination of prices and endowments.

Following Chang (1979), consider the inverse of the system matrix B,

$$B^{-1} \equiv \begin{pmatrix} C & E \\ E' & F \end{pmatrix} = \begin{pmatrix} \partial w/\partial v & \partial w/\partial p \\ \partial x/\partial v & \partial x/\partial p \end{pmatrix}.$$

Where $H \equiv AA' - S$ and $G \equiv A'H^{-1}A$, it follows that $C = H^{-1}AG^{-1}A'H^{-1} - H^{-1}$, $E' = G^{-1}A'H^{-1}$, $E = H^{-1}AG^{-1}$, and $F = G^{-1} - I_n$. Matrix C is seen to be symmetric. Samuelson's reciprocity result is immediately apparent: $E = (E')'$. Matrix F is symmetric as well, with a positive diagonal, reflecting concavity of the transformation surface. Caratheodory (1967) presents an argument concerning reciprocal quadratic forms that C is negative semidefinite with rank $r-n$.

Next consider four homogeneity properties formally proven in Chang (1979) and Takayama (1982). Constant returns to scale imply that outputs are homogeneous of degree one in inputs:

$$E'v = x\,. \tag{8}$$

With endowments fixed, only relative price changes affect outputs. This means that outputs are homogeneous of degree zero in prices:

$$Fp = 0\,. \tag{9}$$

Factor payments are determined by isocost surfaces which support production isoquants. With prices constant and endowments changing proportionally, factor payments are not affected, and thus are homogenous of degree zero in endowments:

$$Cv=0\,. \tag{10}$$

If prices vary proportionally, so do factor payments. This "neutrality" means factor payments are homogeneous of degree one in prices:

$$Ep=w\,. \tag{11}$$

By (8) and (11), every row of E' and E must have at least one positive element. If a factor endowment is positively (negatively) correlated with some industry's output, it can be said to "strengthen" ("weaken") that industry. In E', (i) for any industry, there is some factor which strengthens it, and (ii) every factor strengthens some industry. In E, (i) for every wage, there is some price which raises it, and (ii) every price raises some wage.

A proof by Either (1974) shows that every column of E (row of E') has at least one negative element. By (8), every column of E contains a nonzero element. Since $BB^{-1}=I_{r+n}$, it follows that $A'E=I_n$. Any row i of A' multiplied by column i of E yields 1, and by any other column 0. Given the appropriate Inada or boundary condition, each industry uses at least two factors. Every row of A' will then have at least two positive elements. Mutiplying row j of A' by column k of E, where $j\neq k$, yields 0. Some element in column k of E must then be negative. So every industry is weakened by at least one factor, while every price lowers at least one wage.

More can be gleaned if the model is put into elasticity form as in Jones (1965). From (5) and (6), where factor share $\theta_{km}\equiv w_k a_{km}/p_m$ and industry share $\lambda_{km}\equiv x_m a_{km}/v_k$,

$$\begin{pmatrix}\sigma & \lambda\\ \theta' & 0\end{pmatrix}\begin{pmatrix}\hat{w}\\ \hat{x}\end{pmatrix}=\begin{pmatrix}\hat{v}\\ \hat{p}\end{pmatrix}. \tag{7'}$$

Matrix σ has a negative diagonal and zero row sums. Matrices λ and θ' are non-negative and row stochastic. Thus the system matrix β is row stochastic.

Where

$$\beta^{-1}\equiv\begin{pmatrix}\delta & \varepsilon\\ \rho & \gamma\end{pmatrix},$$

it follows that $\theta'\varepsilon=I_n$ since $\beta\beta^{-1}=I_{r+n}$. Every column of ε has at least one element greater than unity since θ' is row stochastic. So some factor payment is more than proportionally raised by every price. A tariff on one good then must unambiguously raise the real income of at least one factor.

A further result is obtained by considering that $\rho\lambda=I_n$. Industry m uses some fraction λ_{km} of input k, where $0\leq\lambda_{km}\leq 1$. Every industry uses at least two factors, but not every factor is necessarily used in each industry. Jones and Scheinkman (1977) suppose industry m is small: $\sum_i \lambda_{im}<1$. Since $\lambda_{im}<1$ for all i by assumption, there must be an element in every row of ρ greater than unity. If industry

m is small, there must then be some factor which more than proportionally strengthens the output of good m, since some element in that row is greater than one.

2. $\partial x/\partial v$ RESULTS

At least one factor must weaken while another must strengthen each industry. Every factor strengthens some industry. In the 2×2 model, the $\partial x/\partial v$ Rybczynski matrix has a positive diagonal with equal negative elements off the diagonal. Only general $\partial x/\partial v$ conclusions as in Batra and Casas (1976) have been available in higher dimensional models, "without requiring the detailed sign patterns of the $\partial w/\partial p$ and $\partial x/\partial v$ matrices" as put by Chang (1979).

Given Ruffin's (1981) factor intensity ordering in the 3×2 model, $a_{11}/a_{12}>a_{21}/a_{22}>a_{31}/a_{32}$, definite results are obtainable. Factor 1 is most intensive or "extreme" in industry one, factor 3 extreme in industry two, and factor 2 the "middle" factor. Batra and Casas (1976) argue for the necessity of a "strong" Rybczynski pattern where each extreme factor strengthens the output of the good using it most intensively and weakens the output of the other sector. Their conclusion, however, is based on the assumption of weak substitution among inputs and a roundabout approach to factor intensity. Suzuki (1983) points out that it is possible for $\partial w_3/\partial p_1$ or $\partial w_1/\partial p_2$ ($\partial x_1/\partial v_3$ or $\partial x_2/\partial v_1$) to be positive. Let $\partial x_j/\partial v_i$ be represented by the term r_{ji}, and write out the matrix

$$E'=\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \end{pmatrix} .$$

Possible sign patterns in E' are shown by Thompson (1985) to be:

$$\begin{array}{ccccccc} ++- & +-- & ++- & +-+ & ++- & +-+ & -+- \\ -++ & -++ & --+ & -++ & +-+ & -+- & +-+ . \\ \text{(a)} & \text{(b)} & \text{(b}'\text{)} & \text{(c)} & \text{(c}'\text{)} & \text{(d)} & \text{(d}'\text{)} \end{array}$$

Some other patterns are ruled out, based on this theorem: If an industry is weakened by its extreme factor, the other factors cannot both strengthen it. Patterns with primes are obtained by switching the names of extreme factors and then switching the names of goods. Switched patterns are structurally similar to the originals, since naming is arbitrary.

The first three patterns (a), (b) and (b′) demonstrate a strong Rybczynski result, with each industry strengthened by its extreme factor and weakened by the factor which is extreme in the other sector. Patterns (c) and (c′) are characterized by an extreme factor which strengthens both industries. In (d) and (d′), an extreme factor *weakens* the industry which uses it most intensively, and strengthens the other.

This last result is most surprising and important, since so much basic intuition in international trade is based upon a strong link between factors and goods through the intensity relationship. This relative wealth of outcomes in the 3×2 model results from the interplay in production of factor substitution and intensity. Any pair of factors may be complementary. The degree of factor intensity also becomes crucial to the qualitative nature of outcomes.

Takayama (1982) notes that when the extreme factors are complementary, a strong Rybczynski result must hold. There is in fact some empirical evidence that capital and skilled labor are complementary, although the question would seem to remain open. Thompson and Clark (1983) show that skilled labor and capital are the extreme factors in a simple application of the 3×2 model to the US economy, where one sector is manufacturing (plus services) and the other agriculture. Qualitative $\partial x/\partial v$ and reciprocal $\partial w/\partial p$ results can perhaps be anticipated without a complete specification of the model. A *ceteris paribus* increase in the skilled labor endowment through immigration will create a higher output in manufacturing, the sector where it is extreme. Takayama discusses how the three factor model may help explain Leontief's paradox, the US importing capital intensive commodities even with a high capital to labor ratio. As originally suggested by Leontief, the US simply has a greater relative abundance of skilled labor.

3. $\partial w/\partial p$ RESULTS

Stolper and Samuelson (1941) present an argument that in a 2×2 economy, an increased relative price of a good causes (i) more of that good to be produced, (ii) a rise in the payment to the factor used intensively in that industry and a fall in the other factor payment, and (iii) factor intensity ratios to move away from the more expensive factor in each industry.

Jones (1965) develops the magnification effect, $\hat{w}_1 > \hat{p}_1 > \hat{p}_2 > \hat{w}_2$, where factor 1 (2) is used intensively in sector 1 (2). This follows directly from (7′), since $\theta' \hat{w} = \hat{p}$ and θ' is row stochastic. It can be seen that w_2 could rise with an increase in p_1/p_2, if $\hat{p}_1 > \hat{p}_2 > 0$. If the price of a good increases, *ceteris paribus*, there arises a more than proportional increase in the payment to one factor with a decrease in the payment to the other. The real return to one factor must then rise, as the real return to the other falls.

One way to generalize the Stolper-Samuelson theorem is to move up to a higher dimensional even model ($r = n > 2$). Chipman (1969) differentiates this $\partial w/\partial p$ result by whether it is (i) local or global, and (ii) weak or strong. In the local version, for any m there is a k such that given $\hat{p}_m$, $\hat{w}_k = \lambda \hat{p}_m$, where $\lambda > 1$. The global version states that there is a one to one correspondence between goods and factors such that this relation always holds. In the weak version for any m there is a k such that $\hat{w}_k \geq \hat{p}_m$, while the strong version is satisfied if $\hat{w}_k = \lambda \hat{p}_m$ implies $\hat{w}_h < 0$ for all $h \neq k$. The weak version is implied by the strong.

Ethier (1974) notes that Chipman's weak condition requires restrictions on the matrix A, and develops a general property. It is proven that there are associations between goods and factors such that for every m there are an h and k such that (i) $\hat{w}_h/\hat{p}_m<0$, and (ii) $\hat{w}_k/\hat{p}_m>1$. In these even models, assuming only an invertible system, an increase in the price of any good lowers some wage and more than proportionally raises some other wage.

Still within the context of even models, Jones (1976) calls factor k "important" if $\sum_j \theta_{kj}\geq 1$. Summing across goods in matrix θ indicates how large a role some factor plays in the pricing of goods. This summation is positive since each factor must be used in some industry. For every factor which is not important, it is proven that there must be a good whose price raises its real income.

Consider these $\partial w/\partial p$ results in the 3×2 model, with an eye to how real income responds to any exogenous price change. Note that the symmetry between the $\partial w/\partial p$ and $\partial x/\partial v$ results implies that the same sign patterns described in Section 2 will be found. Clearly if $\partial w_k/\partial p_m<0$, the real income of factor k is inversely related with the price of good m. Whether w_k rises sufficiently to offset a higher price of good m when $\partial w_k/\partial p_m>0$ creates an ambiguity examined by Ruffin and Jones (1977). The ratio of the percentage change in w_k to the percentage change in p_m, $\hat{w}_k/\hat{p}_m$, is defined as β_{km}. Where d_m^k is the quantity of good m consumed by the owners of factor k, their consumption share of good m is written $\phi_m^k\equiv(p_m d_m^k)/Y_k$, where Y_k represents factor k's nominal income $w_k v_k$. Note that $\sum_j \phi_j^k=1$. The real income of factor k equals its nominal income Y_k divided by average prices weighted by consumption shares: $y_k\equiv Y_k/(\sum_j \phi_j^k p_j)$. Totally differentiating this expression, dividing by y_k, and considering changes in the price of good m, $\hat{y}_k=\hat{Y}_k-\phi_m^k\hat{p}_m$. When the price of good m changes, the change in factor k's real income is then determined by the expression $\hat{y}_k=(\beta_{km}-\phi_m^k)\hat{p}_m$.

Looking back at the sign patterns of the comparative static outcomes in the 3×2 model, negative signs present no ambiguity since $\beta_{km}<0$. The magnification effect says that each $\hat{p}$ is a weighted average of the $\hat{w}$'s. With $\hat{p}_1>0$ and $\hat{p}_2=0$ in the top row of the sign patterns, it must be that $\hat{w}_k>\hat{p}_1$ for some k. The top rows with one positive sign show the factor whose welfare is raised by an increase in the price of good one. For instance in the top row of (b), it follows that $\hat{w}_1>\hat{p}_1$, so a tariff on good one would raise the real income of factor one. The other top rows have two positive signs which create ambiguity, since a positive $\hat{p}_1$ may be greater than one of the $\hat{w}_k$'s. In that case, it would follow that $\beta_{k1}<1$. If in addition $\phi_1^k<\beta_{k1}$, the real income of factor k would rise. But if $\beta_{k1}<\phi_1^k$, $\hat{y}_k$ would be negative.

Jones and Easton (1983) produce a diagrammatic argument which can lead to magnification effects for the 3×2 model. Using payment to the middle factor two as numeraire, they examine percentage changes in payments to extreme factors resulting from a change in relative prices. Four different locci are derived as in Fig. 1. For $j=1, 2$, the p_j schedule sets $\hat{p}_j=0$. Along either p_j schedule, $\hat{w}_3/\hat{w}_1=-\theta_{1j}/\theta_{3j}$ from (7'). Each p_j schedule is thus negatively sloped. Due to the assumed factor intensity, the p_1 locus is steeper. The p locus sets $\hat{p}_1=\hat{p}_2$, so again from (7′), $\hat{w}_3/\hat{w}_1=(\theta_{11}-\theta_{13})/(\theta_{33}-\theta_{31})$, which is positive. All schedules have a common intersection at the prevailing equilibrium factor payments w_1 and w_3. Full employment occurs along the v locus which passes through the common intersection and by (7′) may have either a positive or negative slope.

An exogenous increase in the price of good one relative to good two causes the p locus to shift to the right. A new equilibrium occurs with full employment, where the new p locus (p') intersects the full employment v locus. Exogenous price changes are arbitrarily chosen so p_1 and p_2 schedules adjust to the new equilibrium. With $\hat{w}_2=0$ and $\hat{p}_1>\hat{p}_2$, signs and magnitudes of $\hat{w}_1$ and $\hat{w}_3$ can be read from the diagram. By the magnification effect, there must be factors h and k such that $\hat{w}_h>\hat{p}_1>\hat{p}_2>\hat{w}_k$.

The v locus may intersect the new locus p' in seven different regions of Fig. 1.

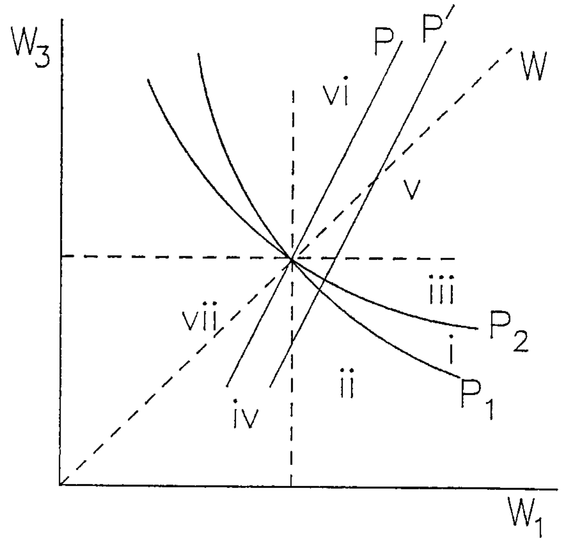

Fig. 1.

Regions i, ii, and iv are considered explicitly by Jones and Easton. The following magnification effects can be read from the diagram:

(i) $\hat{w}_1>\hat{p}_1>\hat{w}_2>\hat{p}_2>\hat{w}_3$,

(ii) $\hat{w}_1>\hat{w}_2>\hat{p}_1>\hat{p}_2>\hat{w}_3$,

(iv_a) $\hat{w}_2>\hat{w}_1>\hat{p}_1>\hat{p}_2>\hat{w}_3$,

(iv_b) $\hat{w}_2>\hat{p}_1>\hat{w}_1>\hat{p}_2>\hat{w}_3$,

and

(iv_c) $\hat{w}_2>\hat{p}_1>\hat{p}_2>\hat{w}_1>\hat{w}_3$.

Payment to middle factor two "floats," as payment to extreme factor one "sinks."

A new equilibrium may also occur in region iii or v. Jones and Easton point out (page 83) that the v locus may be positively sloped and less steep than the p

locus. This creates a solution in region v. Region iii solutions occur where v is negatively sloped, with $-\theta_{12}/\theta_{22}<0$. In regions iii and v, the following magnification effects are found:

(iii) $\hat{w}_1>\hat{p}_1>\hat{p}_2>\hat{w}_2>\hat{w}_3$,
(v$_a$) $\hat{w}_1>\hat{p}_1>\hat{p}_2>\hat{w}_3>\hat{w}_2$,
(v$_b$) $\hat{w}_1>\hat{p}_1>\hat{w}_3>\hat{p}_2>\hat{w}_2$,
and
(v$_c$) $\hat{w}_1>\hat{w}_3>\hat{p}_1>\hat{p}_2>\hat{w}_2$.

Payment to the middle factor sinks, as payment to extreme factor three floats.

Above ray w from the origin, $\hat{w}_3>\hat{w}_1$. Magnification effects in this region can also be derived. Sign patterns of the $\partial w/\partial p$ matrix may be used to derive magnification effects, since one price remains unchanged in either row of the sign patterns. One of the above magnification effects is implied except in the following circumstances. In the top row of sign patterns (c) or (d) where $\hat{p}_1>0$ and $\hat{p}_2=0$, two further results occur:

(vi$_a$) $\hat{w}_3>\hat{w}_1>\hat{p}_1>\hat{p}_2>\hat{w}_2$,
and
(vi$_b$) $\hat{w}_3>\hat{p}_1>\hat{w}_1>\hat{p}_2>\hat{w}_2$.

Payment to factor three floats through region v and "tops out" in region vi, as payment to factor one falls. In the bottom row of sign pattern (c′) or (d′), where $\hat{p}_1=0$ and $\hat{p}_2<0$, these two magnification effects are then found:

(vii$_a$) $\hat{w}_2>\hat{p}_1>\hat{p}_2>\hat{w}_3>\hat{w}_1$,
and
(vii$_b$) $\hat{w}_2>\hat{p}_1>\hat{w}_3>\hat{p}_2>\hat{w}_1$.

Payment to factor one sinks in region iv and here "bottoms out," as payment to factor three floats.

There remain two magnification effects suggested by the diagrammatic technique which are in fact impossible:

(vi$_c$) $\hat{w}_3>\hat{p}_1>\hat{p}_2>\hat{w}_1>\hat{w}_2$,
and
(vii$_c$) $\hat{w}_2>\hat{w}_3>\hat{p}_1>\hat{p}_2>\hat{w}_1$.

These do not arise anywhere in the sign patterns, which cover all possibilities. Stated in terms of extreme factors 1 and 3, the Stolper-Samuelson theorem cannot then be "reversed." This is the bottom line in the three factor model's magnification effects.

4. $\partial w/\partial v$ RESULTS

Lerner (1952) develops sufficient conditions for factor price equalization between trading partners in the 2×2 situation. Samuelson (1949) formalizes the argument, introducing the idea of strong factor intensity: whatever the level of relative endowments or wages, $a_{11}/a_{12} > a_{21}/a_{22}$, good i using factor i intensively ($i=1, 2$). Free trade will equalize prices of goods between trading partners. Identical factor payments and hence factor inputs result in the 2×2 model. It is only required that the general form of each production function is the same in the two trading countries. Samuelson (1953–4) notes that factor payments will not necessarily be equalized between trading partners in uneven models. Jones (1976) directs some attention toward factor price equalization in 3×2 model, noting that the basic concept of factor intensity from the 2×2 model must be modified.

Chipman (1969) develops factor price equalization for even models in terms of a minimum cost function $c_m = c_m(x_m, w)$ and the mapping from w to $p: p=c=\sum A'(w)w \equiv g(w)$. This cost function may be written simply as $c_m = c_m(w)$ when production functions are linearly homogeneous. Note that for any good m, $\partial g_m/\partial w_k = a_{km}$ by Shephard's lemma. The Jacobian of the mapping $p=g(w)$ is the determinant of the factor input matrix $|A|$. Since every industry is different, the columns of A are independent. So the determinant of the square matrix A is nonzero. Thus the inverse function theorem holds: where g is continuous and $g'(w_0)$ is invertible for w_0 in W, there are open sets containing w's and others containing p's such that where $p=g(w_0)$, g is one to one on W and g^{-1} is continuous. The mapping g is locally invertible, or one to one in a neighborhood of the point w_0. If $g(w)=g(w')$, then $w=w'$ for any w and w' in the neighborhood.

Consider the function g as a linear transformation which maps W into P. Where X is a subset of R^n, a linear operator is a linear transformation of X into X. Linear operators are invertible if they are either one to one or onto. In an $r\times n$ model with $r>n$, g is not a linear operator since it maps R^r into R^n. In the 3×2 and any other uneven model, the mapping g from factor payments to goods' prices cannot then be invertible.

This univalence issue can be elucidated with a geometrical argument involving unit value isoquants. In an even model, there are n unit value isoquants $x_n(a_n)=1$ in n dimensional factor space, their exact positions determined by prices of the various goods. These unit value isoquants are supported by a unique unit value isocost hyperplane $A'w=1$. Factor inputs are determined at the intersections of the isoquants and the supporting isocost hyperplane. Factor payments are found where the hyperplane intersects each factor's axis, since the input of factor k costing one unit value would be $1/w_k$ if the inputs of all other factors were zero. Univalence is thus assured by competitive pricing or zero economic profits.

When $r>n$, there are in r dimensional input space only n unit value isoquants. These can be supported by any number of isocost hyperplanes. Factor payments and mixes are then not uniquely determined. The simplest example of this overdetermined situation occurs with two factors and one good, where the supporting isocost line can slide around the single convex isoquant.

Even with the same number of goods and factors, global invertibility of the mapping $p=g(w)$ is neither necessary nor sufficient for complete factor price equalization, as Chipman (1966) points out. Two further conditions sufficient for factor price equalization are (i) factor endowments must lie within the same production cone or "cone of diversification," and (ii) global invertibility must hold. Global invertibility alone is not sufficient. Equalization could occur haphazardly without invertibility. The Gale-Nikaido (1965) theorem offers a sufficient condition for global invertability, namely that all principle minors of factor input matrix be positive.

Factor price equalization is at any rate an unsettling result. It is hardly intuitive that factor endowment differences or changes would not affect factor payments, even in so simple a production model as the 2×2. Such a great amount of emphasis on the factor price equalization result in the history of thought in international economics has lowered our credence in the eyes of many who perceive that factor payments are anything but equal in the real world. Such is the danger of an overapplication of a starkly simple model.

If diagonal terms in the $\partial w/\partial v$ matrix were negative, general equilibrium factor demand curves would slope downward. Of course *ceteris paribus* factor demand curves slope downward due to diminishing marginal returns in the production functions. In the general equilibrium, outputs and other factor payments adjust along with the factor payment in question, so determining the slope of *mutadis mutandis* factor demand would be an open issue.

National income $Y=wv$ is maximized subject to the constraint of the given factor endowment. The first order maximization condition is that $0=\partial Y/\partial v=w+(\partial w/\partial v)v=w+Cv=w$ by (10). The second order condition is $0>\partial^2 Y/\partial v^2=\partial w/\partial v$. General equilibrium diminishing marginal returns and downward sloping demand are thus exhibited for each of the inputs. Samuelson's reciprocity $\partial w_h/\partial v_k=\partial w_k/\partial v_h$ follows from Taylor's theorem.

Two factors h and k are said to be friends (enemies) if $\partial w_h/\partial v_k=\partial w_k/\partial v_h>0$ (<0). Thompson (1983a) shows that friendship is intransitive across factors, while "being enemies" is a transitive relationship. If two factors are friends, in other words, they cannot have a common friend. If a factor has two enemies, they will also be enemies with each other. If a factor is the friend of every other factor, then all other pairs of factors are enemies. Consider finally the factor intensity ordering between any two industries m and n:

$$a_{1m}/a_{1n}>a_{2m}/a_{2n}>\cdots>a_{rm}/a_{rn}\ .$$

It is shown that if all factors adjacent in the ordering are friends and the number of factors r is even (odd), then extreme factors are friends (enemies). These results help picture why consensus on policy affecting factor migration is difficult to attain.

Jones (1985) examines this matrix of $\partial w_h/\partial v_k$ or w_{hk} results with $r>n$. Given the above factor intensity ordering between any two industries m and n, some pair of these terms $(w_{i1}, w_{i+1,1})$ must have the signs $(+-)$. If there is a k such that $w_{i1}<0$ where $i<k$ and $w_{i1}>0$ where $i\geq k$, then

$$\sum_{1=1}^{k-1}\left(\frac{a_{im}}{a_{in}}\right)w_{i1} \quad \text{and} \quad \sum_{i=k}^{r}\left(\frac{a_{im}}{a_{in}}\right)w_{i1}$$

cannot span the origin in (a_{im}, a_{in}) space as required by competitive pricing. There are then at least two sign reversals in the vector $(w_{11}, w_{21}, \cdots, w_{r1})$. Thus an extreme factor cannot be the friend of every other factor.

Batra and Casas (1976) correctly deduce that $\partial w_1/\partial v_2>0$ and $\partial w_2/\partial v_3>0$ in the 3×2 model, but treat factor intensity in a confusing manner. They also show that $\partial w_1/\partial v_3$ and $\partial w_2/\partial v_3$ have opposite signs. More simply this implies that $\partial w_1/\partial v_3<0$. Ruffin (1981) recognizes and elucidates the simplicity and generality of these three factor results. Extreme factors are enemies (increasing the endowment of one lowers the payment to the other) regardless of whether they are technical complements or substitutes. The middle factor is a friend of the other two factors, again regardless of substitutability. In any model with one more type of factor than good, signs in this $\partial w/\partial v$ matrix are independent of substitutability, as argued in Thompson (1983a). Suzuki (1981) arrives at Ruffin's sign pattern result, arguing from diminishing marginal returns.

5. THE HECKSCHER-OHLIN THEOREM

This important theorem states that trading nations or regions effectively export relatively abundant factors through the export of goods which are relatively intensive users of those abundant factors. This statement of the theorem is vague in that "abundant" could refer to either actual endowment levels or factor payment levels. Factor payments determine costs of inputs and hence prices of final goods, which ultimately must be the stimulus for trade.

A curious dialogue in the history of thought involves a seeming paradox of Lenotief (1953), who deduced that the US is revealed by trade to be labor, not capital, abundant relative to its trading partners. Leontief works in the context of a two factor model. Using the foundation laid by Vanek (1968), Leamer (1980) argues that the US is instead capital abundant. Leamer essentially shows that in higher dimensional even models, like the 3×3 example he explicitly develops, a country can be capital abundant and yet have a lower ratio of capital to labor in exports than in imports.

This paradox and Leamer's resolution strictly hold only in even models, since the approach assumes that the matrix of factor inputs is the same in the country and its "world" of trading partners. Free trade would result in factor price equalization, which implies identical inputs across countries. In the 3×2 and other uneven models, factor inputs can vary between trading partners. Leontief's test would have to be redone using the different factor mixes of the countries involved. The Vanek-Leamer resolution would similarly need to account for the likelihood of varying factor payments and mixes between trading partners.

In the context of the 2×2 model, Ruffin (1977) offers a set of assumptions sufficient for a country relatively well endowed with a certain productive factor to export the good intensive in the factor. The home country is called v_1 rich if $v_1/v_2>v_1^*/v_2^*$, where *'s represent variables of the foreign country, and v_1 cheap if $w_1/w_2<w_1^*/w_2^*$. Assume (i) production functions are the same everywhere, (ii) factor one (two) is used intensively in sector one (two), (iii) production surpluses are exported, (iv) some of each good is consumed, and (v) factor owners have identical and homothetic tastes between countries. Suppose then that $v_1/v_2=v_1^*/v_2^*$. With identical production functions, it must be that $x_1/x_2=x_1^*/x_2^*$, so no trade occurs when there are identical tastes. Pretrade production and consumption are summarized in the home country by $x_j=\sum_i d_j^i$, where d_j^i is consumption of good j by the owners of factor i. Since tastes are homothetic, consumers in each economy will consume goods in the same ratio given that prices are equalized by free trade. Suppose one unit of v_2 emigrates from the home country, causing d_2^1 and d_2^2 to fall. By the Rybczynski theorem, $\hat{x}_1>0$. So the v_i rich country must export the v_i intensive good.

Samuelson (1971) works with a 3×2 specific factors model under similar assumptions. In this case the country rich in specific factor k is found to export the good using that specific factor, with resulting changes in factor usage moving the two countries toward factor price equalization. Although there might not be strict equalization with free trade, factor payments in the two countries would be attracted toward each other with the onset of trade.

Suppose the home country is v_1 relative to v_2, and v_2 relative to v_3 rich in the 3×2 model. Then it must also be v_1 relative to v_3 rich. The same statement can be made regarding relative factor cheapness. Suppose one unit of v_3 emigrates. Examining the possible $\partial x/\partial v$ results, in sign pattern (b′) $\hat{x}_1<0$ and $\hat{x}_2<0$. It is not necessary that the home country export good one, although if $\hat{d}_1<\hat{x}_1$, good one would be exported. In case (d′), $\hat{x}_1<0$ and $\hat{x}_2>0$. Since $\hat{d}_2<0$, the home country clearly must export good two. In other more "normal" cases, the home country exports good one, which has factor one as its extreme input.

For equity considerations, it is reassuring to believe that factor payments move toward equality with regions exporting their relatively abundant factors via free commodity trade. Consider, however, a capital rich and a labor rich region. Suppose human capital or skilled labor is the middle factor in the factor intensity ranking, and the two goods produced are machines (which are capital extreme) and food (which is labor extreme). That is, $a_{km}/a_{kf}>a_{hm}/a_{hf}>a_{lm}/a_{lf}$.

It is just possible that the labor rich region would import food with the opening of free trade, as argued by Thompson (1986). Trade would cause relative increases in the demand for and payment to the already dear factor in each region. Payment to the already cheap factor would be lowered. Free trade would *polarize* factor payments, and cannot be assumed *a priori* to help the labor of relatively labor cheap regions. Thus international equity might not be promoted by free trade, even in the context of a long run competitive model with full employment. How much such a consideration should weigh hinges upon observed patterns of factor substitution and factor intensity in the various trading regions. Empirical estimation of the production characteristics of real world economies thus takes on some added importance.

Dixit and Norman (1980, Chapter 4) offer an alternative approach to the Heckscher-Ohlin result in higher dimensional situations where there is no readily apparent way to define factor intensity. A maximum revenue function $r(p, v)$ occurs when firms choose output in the face of exogenous prices p and factor endowments v. Supply functions are the gradients of $r(p, v)$ with respect to p: $x=r_p$. Rybczynski results r_{pv} then offer a "weak" way to define factor intensity: good j is said to be relatively more intensive in factor i than the average when $\partial^2 r/\partial p_j \partial v_i > 0$.

Where p_a represents autarky prices,

$$r(p_a^*, v) \geq r(p_a, v) \quad \text{and} \quad r(p_a^*, v^*) \leq r(p_a, v^*) ,$$

since national revenue or income with any potential free trade price would be greater than revenue in autarky. It follows that

$$[r(p_a, v) - r(p_a^*, v)] - [r(p_a^*, v^*) - r(p_a, v^*)] \leq 0 .$$

If (p_a, v) and (p_a^*, v^*) are close together, this last inequality can be approximated by a second order Taylor series expansion,

$$(p_a - p_a^*) r_{pv} (v - v^*) \leq 0 .$$

Factor endowment differences on average then are negatively correlated with differences in the autarky prices of goods "intensive" in those factors.

In the 3×2 situation, suppose $v_2 = v_2^*$, $V_1 \equiv v_1 - v_1^* < 0$, and $V_3 \equiv v_3 - v_3^* > 0$. Applying the above inequality,

$$(q_1 r_{11} + q_2 r_{21}) V_1 + (q_1 r_{13} + q_2 r_{23}) V_3 \leq 0 ,$$

where $q_j \equiv p_{ja} - p_{ja}^*$, $j=1, 2$. Compare outcomes with sign patterns (a) and (d′) from Section 2. With pattern (a), $q_1 > 0$ and $q_2 < 0$, but with pattern (d′) the opposite must be true. The factor abundance idea holds in a weak sense, but may not hold when stated in terms of extreme and middle factors in the simple factor intensity ranking.

The Dixit-Norman approach has some appeal when there are more than two goods, but it would always be more desirable to define factor intensity independently in terms of relative factor inputs. In addition, autarky prices are typically not observable. Dimensionality of an economy, i.e., the number of primary factors and finished goods, again becomes a crucial issue.

Dimensionality is an essential empirical question which has not been adequately addressed. The Hicks aggregation theorem offers a major clue about how to approach the problem of aggregation. Strictly speaking, goods or factors whose prices or quantities move together can be aggregated without loss of information. Diewert (1974) shows how this theorem can be applied in an approximate sense. Since aggregation results in simpler models, the desire is to aggregate to as few goods and primary factors as possible without lumping together markets which are too distinct.

6. INTERNATIONAL CAPITAL MOBILITY

Throughout this review, factors of production have been "domestic," located inside a country and not mobile internationally. In the comparative static exercises, a factor endowment can change exogenously for some reason outside the scope of the model. Capital for instance could flow into the country due to a higher return domestically than in its other employment in the world. But changes in world prices of goods or changes in endowments of other factors which affect capital payments create no *endogenous* capital flow.

Writers have been intrigued by the picture of internationally mobile capital. Chipman (1971) and Jones and Ruffin (1975) investigate production possibilities when there is capital mobility. Svensson (1984) extends the Dixit and Norman (1980) or Dixit and Woodland (1982) duality approach to the case of capital mobility, finding that capital mobility and trade in goods are generally substitutes, as suggested in the classic treatment of Mundell (1957). Ruffin (1984) has surveyed the literature on international factor mobility.

In the context of a small economy open to "trade" in capital, this input may be produced (not primary), since its manufacture could take place out in the world. Fischer and Frenkel (1972) stress that there can be a difference between investment and the production of capital goods in an economy open to trade in capital. A straightforward approach to capital mobility is to assume that the domestic market for capital is characterized by perfectly elastic demand at the exogenous world capital payment. The level of capital employed in the economy then becomes endogeneous. Capital owners may not migrate with their capital, so maximizing national income and maximizing the welfare of domestic residents can be conflicting policy goals, not to mention conflicts of income distribution.

In the long run, capital is malleable and can be used in either sector. Consider the delima of the science when internationally mobile capital is introduced into the 2×2 model. With both exportables and importables internationally traded, there are three international markets setting prices in an economy with only two inputs. In two dimensional input space, positions of two unit value isoquants and the intersection of the isocost hyperplane with the capital axis are fixed. Clearly the model is overdetermined.

Kemp (1966), Jones (1967), and Ferguson (1978) regard this "classic" situation of complete specialization where one of the two goods is not produced in the general equilibrium. An essentially Ricardian picture of trade and investment is painted. In Ricardo, the number of internationally determined prices (two) is greater than the number of inputs (one), so specialization similarly occurs.

To avoid this Ricardian outcome, the idea of sector specific capital introduced by Caves (1971) has been implemented in a number of studies: Amano (1977), Burgess (1978), Brecher and Findlay (1983), Jones and Dei (1983), Jones, Neary, and Ruane (1983), Srinivasan (1983), Thompson (1985), and others. These results fall outside the long run focus of the present review, but it can be said that specialization is avoided and intuitive properties suggested by the factor proportions model are generally found.

In the context of long run models, Ethier and Svensson (1983) point out that "standard" factor proportion results like factor price equalization are found when the number of international markets m^* equals the number of inputs r, as the above geometric argument suggests. To have a tractable production model, r must be at least as large m^*. If $r > m^*$, there is no factor price equalization, changing endowment affecting factor prices. Thompson (1983b) explicitly develops the 3×2 model with international capital mobility, finding a solid unambiguous factor proportions model.

A general conclusion which emerges from this line of study is that the *mutadis mutandis* demand for capital is downward sloping. Looking back at (7), the system with internationally mobile capital k can be written

$$\begin{pmatrix} -1 & S_{rx(r-1)} & A_{rxn} \\ 0_{(r-1)x1} & & \\ 0_{nx1} & A'_{nx(r-1)} & 0_{rxn} \end{pmatrix} \begin{pmatrix} dv_k \\ dw_{(r-1)x1} \\ dx_{nx1} \end{pmatrix} = \begin{pmatrix} -s_{kk}dw_k^* \\ (dv_i - s_{ik}dw_k^*)_{(r-1)x1} \\ (dp_j - a_{kj}dw_k^*)_{nx1} \end{pmatrix}$$

Remember that r is the total number of factors and n the number of goods. In this general model, dv_k and dw_k^* are scalars indicating the employment of capital and its world price. Dimensions of other matrices and vectors are indicated. There are $r-1$ types of domestic inputs utilized along with the internationally mobile capital. A change in the exogenous world capital payment w_k^* clearly affects the economy. Looking specifically at the effect on capital employment,

$$\partial v_k / \partial w_k^* = -D_r / D_{r-1} ,$$

where D_r (D_{r-1}) is the determinant of the general equilibrium system with r $(r-1)$ domestic inputs. Chang (1979) shows that such determinants have the sign $(-1)^r$, so $\partial v_k / \partial w_k^* < 0$.

A rise (fall) in the world capital payment, in other words, creates capital outflow (inflow) in the full general equilibrium adjustment. This property is not at all insured by diminishing marginal returns, since outputs, other factor payments, and the factor mix are adjusting in the full general equilibrium.

7. CONCLUSION

Ideally, as Chipman points out in his survey, economists should remember that the numbers of factors and goods are variables of a scientific analysis. An intriguing question is what structure of primary factors and finished goods most economically describes a given economy. As aggregation occurs, simplicity is bought with a loss of information. There should be no presumption that different countries or regions would have the same dimensions. This represents a vital area for scientific research in economics.

Practical questions in international economics concern long run patterns of trade and the causes and effects of protection, capital mobility, and labor migration. Policy prescriptions will become more rational with developing knowledge of theoretical implications and empirical actualities. The richness of results in general equilibrium models of production and trade has only just been tasted. While it is inevitable that other avenues of research (the industrial organization approach, the Linder hypothesis, etc.) should develop, students of international economics must recognize the solid foundation laid for the science by the general equilibrium approach.

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REFERENCES

Amano, A. (1977) "Specific Factors, Comparative Advantage and International Investment," *Economica*, **44**, 131–44.

Batra, R. and F. Casas (1976) "A Synthesis of the Heckscher-Ohlin and Neoclassical Models of International Trade," *Journal of International Economics*, **6**, 21–38.

Brecher, R. and R. Findlay (1983) "Tariffs, Foreign Capital and National Welfare with Sector Specific Factors," *Journal of International Economics*, **14**, 277–88.

Burgess, D. (1978) "On the Distributional Effects of Direct Foreign Investment," *International Economic Review*, **19**, 647–64.

Caratheodory, C. (1967) *Calculus of Variations and Partial Differential Equations of the First Order*, Part II, Holden-Day, Inc., 195–7.

Caves, R. (1971) "International Corporations: The Industrial Economics of Foreign Investment," *Economica*, **38**, 1–27.

Chang, W. (1979) "Some Theorems of Trade and General Equilibrium with Many Goods and Factors," *Econometrica*, **47**, 709–26.

Chipman, J. (1965–6) "A Survey of the Theory of International Trade: Part 1, The Classical Theory," *Econometrica*, **33**, 477–519; "Part 2, The Neo-Classical Theory," *Econometrica*, **33**, 685–760; "Part 3, The Modern Theory," *Econometrica*, **34**, 18–76.

Chipman, J. (1969) "Factor Price Equalization and the Stolper Samuelson Theorem," *International Economic Review*, **10**, 399–406.

Chipman, J. (1971) "International Trade with Capital Mobility: A Substitution Theorem," in *Trade, Balance of Payments, and Growth*, edited by J. Bhagwati, R. Jones, R. Mundell, and J. Vanek, North Holland.

Diewert, W. (1974) "A Note on Aggregation and Elasticities of Substitution," *Canadian Journal of Economics*, **7**, 12–20.

Dixit, A. and V. Norman (1980) *Theory of International Trade*, Nisbet and Cambridge.

Dixit, A. and A. Woodland (1982) "The Relationship between Factor Endowments and Commodity Trade," *Journal of International Economics*, **13**, 201–24.

Either, W. (1974) "Some of the Theorems of International Trade with Many Goods and Factors," *Journal of International Economics*, **4**, 199–206.

Ferguson, D. (1978) "International Capital Mobility and Comparative Advantage: The Two-Country, Two-Factor Case," *Journal of International Economics*, **8**, 373–96.

Fischer, S. and J. Frenkel (1972) "Investment, the Two-Sector Model and Trade in Debt and

Capital Goods," *Journal of International Economics*, **2**, 211–33.

Gale, P. and H. Nikaido (1965) "The Jacobian Matrix and Global Univalence of Mappings," *Mathematische Annalen*, **159**, 89–93.

Jones, R. (1965) "The Structure of Simple General Equilibrium Models," *Journal of Political Economy*, **73**, 557–72.

Jones, R. (1967) "International Capital Movements and the Theory of Tariffs and Trade," *The Quarterly Journal of Economics*, **81**, 1–38.

Jones, R. (1976) "Twoness in Trade Theory: Costs and Benefits," Frank Graham lecture mimeograph.

Jones, R. (1985) "A Theorem on Income Distribution in a Small, Open Economy," *Journal of International Economics*, **18**, 171–6.

Jones, R. and F. Dei (1983) "International Trade and Foreign Investment: A Simple Model," *Economic Inquiry*, **21**, 449–64.

Jones, R. and S. Easton (1983) "Factor Intensities and Factor Substitution in General Equilibrium," *Journal of International Economics*, **15**, 65–99.

Jones, R., J. P. Neary, and F. Ruane (1983) "Two-Way Capital Flows: Cross-Hauling in a Model of Foreign Investment," *Journal of International Economics*, **14**, 357–66.

Jones, R. and R. Ruffin (1975) "Trade Patterns with Capital Mobility," in *Current Economic Problems*, edited by M. Parkin and A. Nobay, Cambridge.

Jones, R. and J. Scheinkman (1977) "The Relevance of the Two-Sector Production Model in Trade Theory," *Journal of Political Economy*, **85**, 909–35.

Kemp, M. (1966) "The Gain from International Trade and Investment: A Neo-Heckscher-Ohlin Approach," *The American Economic Review*, **61**, 788–809.

Leamer, E. (1980) "The Leontief Paradox, Reconsidered," *Journal of Political Economy*, **88**, 495–503.

Leontief, W. (1954) "Domestic Production and Foreign Trade: The American Capital Position Re-examined," *Econ. Internazionale*, **7**, 3–32; Reprinted in *Readings in International Economics*, edited by R. Caves and H. Johnson, Irwin, 1968.

Lerner, A. (1952) "Factor Prices and International Trade," *Economica*, **19**, 67–84.

Mundell, R. (1957) "International Trade and Factor Mobility," *American Economic Review*, **47**, 321–35.

Ruffin, R. (1977) "A Note on the Heckscher-Ohlin Theorem," *Journal of International Economics*, **7**, 403–5.

Ruffin, R. (1981) "Factor Movements and Prices with Three Factors and Two Goods," *Economic Letters*, **7**, 177–82.

Ruffin, R. (1984) "International Factor Mobility," Chapter 5 in the *Handbook of International Economics*, ed. by R. Jones and P. Kenen, Elsevier Science Publishers.

Ruffin, R. and R. Jones (1977) "Protection and Real Wages: The Neoclassical Ambiguity," *Journal of Economic Theory*, **14**, 337–48.

Samuelson, P. (1949) "International Factor Price Equalization Once Again," *Economic Journal*, **59**, 182–97.

Samuelson, P. (1953–4) "Prices of Factors and Goods in General Equilibrium," *Review of Economic Studies*, **21**, 1–20.

Samuelson, P. (1971) "Ohlin Was Right," *Scandanavian Journal of Economics* **4**, 365–84.

Shephard, R. (1953) *Cost and Production Functions*, Princeton University Press.

Srinivasan, T. N. (1983) "International Factor Movements, Commodity Trade and Commercial Policy in a Specific Factor Model," *Journal of International Economics*, **14**, 289–312.

Stolper, W. and P. Samuelson (1941) "Protection and Real Wages," *Review of Economic Studies*, 58–73.

Suzuki, K. (1981) "The Extension of International Trade Theory to Threeness," Doctoral Dissertation, University of Rochester.

Suzuki, K. (1983) "A Synthesis of the Heckscher-Ohlin and Neoclassical Models of International Trade: A Comment," *Journal of International Economics*, **14**, 141–4.

Svensson, L. (1984) "Factor Trade and Goods Trade," *Journal of International Economics*, **16**, 365–78.

Takayama, A. (1982) "On Theorems of General Competitive Equilibrium of Production and Trade—A Survey of Some Recent Developments in the Theory of International Trande," *Keio Economic Studies*, **19**, 1–38.

Thompson, H. (1983a) "Factor Migration and Income Redistribution in International Trade," *Keio Economic Studies*, **20**, 65–70.

Thompson, H. (1983b) "Trade and International Factor Mobility," *Atlantic Economic Journal*, **11**, 45–8.

Thompson, H. (1985) "Complementarity in a Simple General Equilibrium Production Model," *Canadian Journal of Economics*, **18**, 616–21.

Thompson, H. (1985) "International Capital Mobility in a Specific Factor Model," *Atlantic Economic Journal*, **13**, 76–9.

Thompson, H. (1986) "Free Trade and Factor Price Polarization," *European Economic Review*, **30**, 419–25.

Thompson, H. and D. P. Clark (1983) "Factor Movements with Three Factors and Two Goods in the U. S. Economy," *Economics Letters*, **12**, 53–60.

Uzawa, H. (1964) "Duality Principles in the Theory of Cost and Production," *International Economic Review*, **5**, 216–20.

Vanek, J. (1968) "The Factor Proportions Theory: The *N*-Factor Case," *Kyklos*, **21**, 749–54.