Journal of Economic Integration
12(2), June 1997; 180–205

Quotas and Quality in an International Duopoly

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Abstract

This paper examines possible adjustments to a change in a binding quota in the context of an international duopoly. Consumers directly value embodied quality of goods, which is chosen simultaneously with quantity, and before quantity in a sequential model. Possible responses to a small change in a binding quota are derived. The same three types of equilibria occur in the simultaneous and sequential models. Foreign quality downgrading can occur if domestic quality falls, and is more likely starting with a low quantity of high quality imports. Domestic quality and quantity respond in opposite directions. Welfare effects are discussed.

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I. Introduction

A virtual folk theorem in international economics is that quotas lead to quality upgrading of imports. Theoretical studies of quotas and quality find that import quality rises in response to a quota: the competitive models of Falvey [1979], Santoni and VanCott [1980], Rodriquez [1979], and Mayer [1982]; the monopoly model of Das and Donnenfeld [1987]; and the duopoly model of Das and Donnenfeld [1989]. Patterson [1966], Meier [1973], and MacPhee [1974] have documented increases in quality with quotas in the steel and textile industries. Similar effects have been noted by Feenstra [1984] in automobiles, Aw and Roberts [1986] in footwear, and Anderson [1985] in cheese.

Krishna [1987] argues that a quota imposed on a foreign monopolist can lead to quality downgrading of imports if the value of quality to the average consumer is less than its value to the marginal consumer, a result directly related to the work of Spence [1975]. In a model of vertical product differentiation, Ries [1993] shows that foreign multiproduct Cournot competitors will not upgrade if they produce relatively lower quality goods. Chen [1992] shows that import quality downgrading may be a signal to sustain oligopoly collusion in a supergame when a quota just binds in the free trade collusive equilibrium.

The present paper evaluates the effects of a small change in a binding quota in an international duopoly of one domestic firm and one foreign firm. As in all comparative static analysis, the presumption is that local analysis can be carried over to global issues such as large quota increases. The willingness of consumers to pay for quality is modeled by the demand specification introduced by Mussa and Rosen [1979], and subsequently utilized by Chiang and Spatt [1982], Das and Donnenfeld [1989], and Beard and Ekelund [1991], among others. Two models of noncooperative behavior are employed. Choices of quality and quantity are made simultaneously in the first model, while choice of quality precedes choice of quantity in a sequential model.

If quality is varied through superficial modifications that can be made at the time of manufacture, firms can be said to choose quality and quantity simultaneously. Examples are the choice of upholstery, tires, and options in automobiles. If quality is varied through costly design changes, firms must commit to a choice of quality before production. Examples are a car's interior room, fuel economy, and handling. The surprising conclusion is reached that it is not critical for the range of possible outcomes whether quality is chosen simultaneously or sequentially. This result suggests that the interpretation of product quality may be relatively unimportant when studying quotas.

Three types of equilibria occur for both the simultaneous and sequential models under the assumption that the domestic firm manufactures the higher quality product. In every case, the domestic firm's quality and quantity respond in opposite directions to a quota. A tighter quota leads to either reduced output or reduced quality for the domestic firm.

A tighter quota may also lead to reduced import quality. Import downgrading can occur only when accompanied by domestic downgrading, and is more likely when the market is characterized by a small quantity of high quality imports.

The paper is divided into five sections and a conclusion. Section II outlines assumptions and specifies payoff functions for the international duopoly. Sections III and IV analyze quota constrained equilibria in the case of simultaneous choice of qualities and quantities. Section V evaluates the sequential model in which quality commitments are made prior to Nash quantity competition. Section VI considers the welfare effects of quota changes.

II. Fundamental Assumptions of the Model

The preference model follows Mussa and Rosen [1979]. Assume a large set of potential customers, each with unit demand for the good, buying either one or zero units. A consumer's maximum willingness to pay for the good depends on the amount of embodied quality X. Assume a consumer of type θ is willing to pay up to θX dollars for one unit of the product of quality X in the absence of a better alternative. When two or more versions of the product are available, a consumer chooses the version that yields the greatest difference between value θX and price. If no version yields value at least as large as price, the consumer does not by the product.[1]

1. Formally, it is assumed that a person of type θ who consumes one unit of the good of quality X along with Y dollars of other commodities has utility $Y + \theta X$. This formulation implies X is measured in utility units yielding constant marginal utility. If a physical measure X' of quality were employed, utility would be represented by $Y +$

Let $f(\theta)$ represent the marginal distribution of willingness to pay among consumers.[2] Suppose there are sufficient consumers that the support of $f(\cdot)$ is the interval $[\underline{\theta}, \bar{\theta}] \in R^{+}$. Moreover, suppose $f(\cdot)$ is continuous throughout the interval and differentiable at all interior points.

Assume a firm's selection of quality involves fixed costs which are sufficiently large to make each firm sell units of only one quality in the neighborhood of the equilibrium.

A marginal value of quality function $H(Q)$ is specified by

$$\int_{H(Q)}^{\bar{\theta}} f(\theta)d\theta = Q. \tag{1}$$

If customers were lined up starting with those who value quality most, $H(Q)$ would describe the marginal value of quality to customer Q.

If two firms offer products of the same quality X and produce a total output Q, market clearing would require both firms to charge the price P where total quantity demanded by all consumers who value the product at least P would equal Q. This equilibrium would occur if and only if $P = XH(Q)$.

Suppose, however, the two firms offer products of low and high qualities $X_1 < X_h$, at prices P_1 and P_h respectively. Then a consumer of type θ would either:

(i) buy the good with quality X_h if $P_h \leq \theta X_h$ and $(X_h - X_1)\theta \geq P_h - P_1$;
(ii) buy the good with quality X_1 if $P_1 \leq \theta X_1$ and $(X_h - X_1)\theta \leq P_h - P_1$; or
(iii) buy nothing if $P_h > \theta X_h$ and $P_1 > \theta X_1$.

Market clearing requires that (P_1, X_1, Q_1) and (P_h, X_h, Q_h) satisfy

$$P_1 = X_1 H(Q_1 + Q_h) \tag{2i}$$

$$P_h = X_1 H(Q_1 + Q_h) + (X_h - X_1) H(Q_h), \tag{2ii}$$

where Q_1 and Q_h are quantities of the low and high quality products.

$\theta U(\hat{X}')$, $U' \geq 0$ and $U'' \leq 0$. The two measures of quality are monotonic transformations of each other, $X = U(\hat{X}')$.

2. If the distribution function $f(\theta)$ is reinterpreted, the assumption that each consumer buys either one or zero units can be relaxed. Assume that consumer taste can be characterized by a vector $(\theta_1, \theta_2, \theta_3, \ldots)$ where θ_i represents the consumer's willingness to pay for the quality of unit i.

The demand system specified by (2) is utilized. Note that a product of higher quality commands a higher price, $P_h > P_1$. The assumption that $f(\theta)$ is differentiable implies $H(Q)$ is twice continuously differentiable. Therefore, (1) implies

$$dH(Q)/dQ = -\{f[H(Q)]\}^{-1} \leq 0 \tag{3a}$$

$$d^2H(Q)/dQ^2 = -\{f'[H(Q)]\}\{f[H(Q)]\}^{-2}. \tag{3b}$$

Differentiation of (2) implies that the inverse demand functions $P_h(Q_h, X_h, Q_1, X_1)$ and $P_1(Q_h, X_h, Q_1, X_1)$ are continuous in both quantities and qualities, differentiable everywhere in quantities, and piecewise linear in qualities.[3] Further, $\partial P_h/\partial Q_h < 0$, $\partial P_h/\partial X_h > 0$, $\partial P_h/\partial Q_1 < 0$, and $\partial P_h/\partial X_1 < 0$. Similarly, $\partial P_1/\partial Q_h < 0$, $\partial P_1/\partial X_h < 0$, $\partial P_1/\partial Q_1 < 0$, and $\partial P_1/\partial X_1 > 0$. The demand system in (2) exhibits intuitive responses. Some second and higher order derivatives of $P_1(\cdot)$ and $P_h(\cdot)$ depend on derivatives of the marginal density function $f(\theta)$.

Suppose sales of the foreign firm are subject to a binding quota, while sales of the domestic firm are unrestricted. Assume production technologies of the two firms differ, but both exhibit constant returns to scale with respect to output and decreasing returns with respect to quality. Total production cost of the domestic firm is $[c_d(X_d)]Q_d$ and total cost of the foreign firm is $[c_f(X_f)]Q_f$, where d and f refer to the domestic and foreign firm. Each unit cost function $c_i(\cdot)$ is strictly convex and twice continuously differentiable.[4]

Profit functions for the domestic and foreign firm are given by

$$\pi_d(Q_d, X_d, Q_f, X_f) = [p_d(\cdot) - c_d(X_d)]Q_d \tag{4i}$$

$$\pi_f(Q_d, X_d, Q_f, X_f) = [p_f(\cdot) - c_f(X_f)]Q_f. \tag{4ii}$$

As the demand system (2) indicates, the manner in which quality enters a firm's inverse demand and profit functions depends on whether the firm

3. Inverse demands are piecewise linear in qualities because the derivative of one firm's inverse demand with respect to either its own quality or its rival's quality changes abruptly at the point where both qualities are equal.

4. Measuring quality in physical units rather than utility units would a require a transformation of the unit cost functions $c_i(X_i)$ and their derivatives. See note 1.

produces the high or low quality product. Properties of the quota constrained Nash equilibrium must, therefore, depend on the relative quality of imports. This paper examines the case in which the foreign firm initially produces the low quality product. This choice is motivated by a desire to apply this model to industries in which the domestic firm competes with a low quality or less opulent foreign substitute, as in the automobile industry of the early 1980s.

III. Simultaneous Choice of Quality and Quantity

In the simultaneous model, firms select outputs and qualities contemporaneously. In the absence of a strictly binding quota, any interior Nash equilibrium $(Q_d^*, X_d^*, Q_f^*, X_f^*)$ must satisfy the first order condition

$$0 = \partial\pi^d/\partial Q_d = \partial\pi^d/\partial X_d = \partial\pi^f/\partial Q_f = \partial\pi^f/\partial X_f. \tag{5}$$

Suppose a quota of the form $Q_f \leq \bar{Q}_f < Q_f^*$ is imposed. If the quota $\bar{Q}_f$ is sufficiently close to Q_f^*, a new Nash equilibrium emerges in which the quota is strictly binding in the sense that $\partial\pi^f/\partial Q_f > 0$. Using (2) and (4), such a quota constrained Nash equilibrium must satisfy the first order conditions

$$\partial\pi^d/\partial Q_d = X_f H(Q_d + Q_f) + (X_d - X_f)H(Q_d) - c_d$$

$$+ Q_d[X_f H'(Q_d + Q_f) + (X_d - X_f)H'(Q_d)] = 0 \tag{6i}$$

$$\partial\pi^d/\partial X_d = [H(Q_d) - c_d']Q_d = 0 \tag{6ii}$$

$$\partial\pi^f/\partial X_f = [H(Q_d + Q_f) - c_f']Q_f = 0, \tag{6iii}$$

where the derivative of each firm's cost function with respect to quality is written $c_i' \equiv dc_i/dX_i$. Sufficient second order conditions for the Nash equilibrium are

$$\begin{vmatrix} \partial^2\pi^d/\partial Q_d^2 & \partial^2\pi^d/\partial Q_d\partial X_d \\ \partial^2\pi^d/\partial X_d\partial Q_d & \partial^2\pi^d/\partial X_d^2 \end{vmatrix} > 0 \tag{7i}$$

$$\partial^2\pi^d/\partial Q_d^2 < 0,\ \partial^2\pi^d/\partial X_d^2 < 0, \text{ and } \partial^2\pi^f/\partial X_f^2 < 0. \tag{7ii}$$

Interest focuses on the derivatives of X_d^* and X_f^* with respect to $\bar{Q}_f$. Differentiating the system (6) and accounting for all derivatives that are identically equal to zero implies the desired derivatives must satisfy the linear system

$$\begin{bmatrix} \partial^2\pi^d/\partial Q_d^2 & \partial^2\pi^d/\partial Q_d\partial X_d & d^2\pi^d/\partial Q_d\partial X_f \\ \partial^2\pi^d/\partial X_d\partial Q_d & \partial^2\pi^d/\partial X_d^2 & 0 \\ \partial^2\pi^f/\partial X_f\partial Q_d & 0 & \partial^2\pi^f/\partial X_f^2 \end{bmatrix} \begin{bmatrix} dQ_d^*/\partial\overline{Q}_f \\ dX_d^*/\partial\overline{Q}_f \\ dX_f^*/\partial\overline{Q}_f \end{bmatrix} = -\begin{bmatrix} \partial^2\pi^d/\partial Q_d\partial Q_f \\ 0 \\ \partial^2\pi^f/\partial X_f\partial Q_f \end{bmatrix}. \quad (8)$$

Cramer's rule yields

$$dQ_d^*/d\overline{Q}_f = -(\partial^2\pi^d/\partial X_d^2)I_d/\Delta \quad (9i)$$

$$dX_d^*/d\overline{Q}_f = -(\partial^2\pi^d/\partial Q_d\partial X_d)I_d/\Delta \quad (9ii)$$

$$dX_f^*/d\overline{Q}_f = -(\partial^2\pi^f/\partial Q_d\partial X_f)I_f/\Delta, \quad (9iii)$$

where:

$$I_d \equiv \frac{\partial^2\pi^d}{\partial Q_d\partial Q_f}\frac{\partial^2\pi^f}{\partial X_f^2} - \frac{\partial^2\pi^f}{\partial Q_f\partial X_f}\frac{\partial^2\pi^d}{\partial Q_d\partial X_f} \quad (10i)$$

$$I_f \equiv \frac{\partial^2\pi^d}{\partial X_d^2}\frac{\partial^2\pi^d}{\partial Q_d\partial Q_f} + \frac{\partial^2\pi^d}{\partial Q_d^2}\frac{\partial^2\pi^d}{\partial X_d^2} - \left(\frac{\partial^2\pi^d}{\partial Q_d\partial X_d}\right)^2, \quad (10ii)$$

and Δ is the determinant of the 3×3 matrix in (8).

Comparison of (9i) and (9ii), combined with (6) yields

Proposition 1: *A change in a quota restriction induces changes in the equilibrium domestic quantity and quality in opposite directions:*

$$sgn\,[dX_d/dQ_f] = -sgn\,[dQ_d/dQ_f].$$

Consider the last row of (8). Whenever (6iii) is satisfied, $\partial^2\pi^f/\partial Q_f\partial X_f = Q_fH' + H - c_f' = Q_fH'(Q_d+Q_f) = \partial^2\pi^f/\partial Q_d\partial X_f$, which implies

$$\frac{dQ_d^*}{d\overline{Q}_f} + 1 = -\frac{\partial^2\pi^f/\partial X_f^2}{\partial^2\pi^f/\partial Q_d\partial X_f}\frac{dX_f^*}{d\overline{Q}_f} \quad (11)$$

By (7ii), $\partial^2\pi^f/\partial X_f^2 < 0$. Hence, the term in front of dX_f^*/dQ_f in (11) must be negative. If $dQ_d^*/d\bar{Q}_f > 0$, then $dX_f^*/dQ_f < 0$. Proposition 1 and (11) together imply

Proposition 2: *Regardless of the distribution of tastes, the comparative statics of any interior Nash equilibrium with $X_f^* < X_d^*$ must follow one of three patterns:*

	$dQ_d^*/d\bar{Q}_f$	$dX_d^*/d\bar{Q}_f$	$dX_f^*/d\bar{Q}_f$
I	−	+	+
II	−	+	−
III	+	−	−

This array presents the possible comparative statics when the domestic firm produces the higher quality product. Mathematically, there are eight potential qualitative effects that a quota change could have on domestic quantity, domestic quality, and foreign quality, but only three are possible. If domestic output falls with a quota, both domestic quality and foreign quality must rise. Domestic upgrading cannot occur unless foreign quality also improves. Examples in the following section show that equilibria of each of the three types can be obtained using a simple distribution of tastes.

These results are more general than those of Ries [1993], where foreign multiproduct oligopolistic firms in Cournot competition would not upgrade quality, while domestic firms necessarily increase output and might downgrade. The model of Ries leads to sign pattern I of Proposition 2.

A little reflection on the structure of the model makes the results in Proposition 2 transparent. As Spence [1975] and others have argued, profit maximization requires that a firm choose the quality at which the marginal cost of quality equals its marginal benefit to the marginal customer. This principle holds in any static model with the firm free to choose quality. Determining how a quota change affects a firm's choice of quality is equivalent to determining the change in the value of quality to the marginal consumers. If the marginal cost of quality increases with quality ($c_i'' > 0$), a firm would increase (decrease) quality if its new marginal consumers value quality more (less) than its previous marginal consumers.

The value of quality to the domestic firm's marginal consumers is $H(Q_d)$. Since $H'(Q_d) < 0$, domestic quantity and the value of quality to its marginal customers move in opposite directions. The domestic firm's profit maximizing quantity and quality thus move in opposite directions, as reflected by Proposition 1. The value of quality to the foreign firm's marginal consumers is $H(Q_d+Q_f)$. When a tighter quota leads to a reduction in Q_d, the sum Q_d+Q_f falls and $H(Q_d+Q_f)$ rises. When the foreign firm's new marginal consumers value quality more, foreign quality rises as in case III.

When a tighter quota leads to an increase in Q_d, there are two possibilities. If the increase in Q_d is smaller than the reduction in the quota ($dQ_d^*/d\bar{Q}_f > -1$), $Q_d + Q_f$ falls and the new marginal consumers of the foreign firm value quality more than its previous marginal consumers. Foreign quality then rises as in case II. Alternatively, if the increase in Q_d exceeds the reduction in the quota ($dQ_d^*/d\bar{Q}_f < -1$), $Q_d + Q_f$ rises and the foreign firm's new marginal consumers value quality less. Foreign quality then falls as in case I.

IV. Simultaneous Choice with Uniform Distribution of Consumer Types

The example of this section is based on the assumption of a uniform distribution of consumer marginal valuation of quality, $f(\theta)$. Analysis of this special case is motivated by two considerations. First, the uniform distribution can be used to construct examples that yield the three equilibria in Proposition 2, which suggests that each is equally plausible. Second, this analysis will facilitate study of the model of sequential choice.

When $f(\theta)$ is uniform, $H(Q)$ is linear,

$$H(Q) = \bar{\theta} - hQ,$$

where h is a positive constant. Calculating the first order conditions and performing the static exercise yields a special case of system (8),

$$\begin{bmatrix} -2hX_d & -hQ_d & -h\overline{Q}_f \\ -hQ_d & -Q_d c_d'' & 0 \\ -h\overline{Q}_f & 0 & -\overline{Q}_f c_f'' \end{bmatrix} \begin{bmatrix} dQ_d^* / d\overline{Q}_f \\ dX_d^* / d\overline{Q}_f \\ dX_f^* / d\overline{Q}_f \end{bmatrix} = \begin{bmatrix} hX_f \\ 0 \\ h\overline{Q}_f \end{bmatrix} \qquad (8')$$

Solving this system, or equivalently, substituting into equations (9) and (10) implies

$$dQ_d^*/d\bar{Q}_f = c_d'' Q_d I_d / \Delta \qquad (13i)$$

$$dX_d^*/d\bar{Q}_f = -hQ_d I_d / \Delta \qquad (13ii)$$

$$dX_f^*/d\bar{Q}_f = h\bar{Q}_f I_f / \Delta, \qquad (13iii)$$

where:

$$I_d = (h\bar{Q}_f)\ [X_f c_f'' - h\bar{Q}_f] \tag{14i}$$

$$I_f = (hQ_d)\ [(2X_d - X_f)c_d'' - hQ_d] \tag{14ii}$$

$$\Delta = h\bar{Q}_f Q_d [h\bar{Q}_f c_d'' - c_f''\ (2X_d c_d'' - hQ_d)]. \tag{14iii}$$

The relative locations of the three types of equilibria that arise in $(\bar{Q}_f, X_f)$ space are illustrated in Figure 1. The boundary $\Delta=0$ separates regions of negative and positive, and has slope equal to the expression in brackets in (14iii). When $\Delta = 0$, $h\ \bar{Q}_f c_d'' = c_f''(2X_d c_d'' - hQ_d)$, which implies $2X_d c_d'' > hQ_d$. The slope of the locus $\Delta = 0$ thus depends on the sign of $c_f'''(X_f)$. Figure 1 is drawn under the assumption of a positive slope. Regardless of its slope, $\Delta < 0$ to the left and $\Delta > 0$ to the right.

The boundary defined by $I_d=0$ passes through the origin and has slope $h_f/(X_f c_f + c_f'')$. This slope is positive, provided that c_f'' does not diminish too rapidly, or specifically that $X_f c_f''$ is an increasing function of X_f. If this slope is never positive, there cannot be an equilibrium with positive $\bar{Q}_f$ and X_f. I_d is positive (negative) at any point to the left (right) of the $I_d=0$ boundary.

The boundary defined by $I_f=0$ is horizontal. I_f is negative at any point above and positive at any point below. Note that all three boundaries intersect at the equilibrium point.[5]

An assumption commonly employed to reduce the set of potential Nash equilibria is stability to a tatonment adjustment process in which firms in each round make optimal choices given their rival's choice in the previous round. This stability requires a negative Δ. A proof is in Appendix B.

Figure 1 illustrates the three qualitatively different equilibria. Restricting attention to stable equilibria, regions I, II, and III correspond loosely to high, intermediate, and low import quality.[6] The three regions correspond directly to the sign patterns in Proposition 2. Sign patterns I, II, or III occur when the original combinations of Q_f and X_f are in respectively regions I, II, or III.

A tighter quota leads to import upgrading $(dX_f^*/d\bar{Q}_f < 0)$ when the equilib-

5. Employing the fact that $\partial^2\pi^f/\partial Q_f\partial X_f = \partial^2\pi^f/\partial Q_d\partial X_f$ in equilibrium, straightforward manipulation of (10) and the definition of Δ, implicit in (8), implies $[\partial^2\pi^d/\partial X_d^2]I_d + [\partial^2\pi^f/\partial X_f^2]I_f = \Delta$, regardless of the distribution of tastes. When any two of I_f, I_d, and Δ are zero, the third must also be zero.

6. Recall that the foreign product always has lower quality. Hence the terms "high", "intermediate", and "low" refer to some absolute measure.