A model of the international oil market model with optimal depletion and offer curves suggests importers face a backward bending offer curve. An oil tariff would then raise oil imports and lower the price of oil including the tariff. Simulations of price and extraction paths for the coming century provide insight into the future of oil depletion and terms of trade.

The present model of the international oil market combines optimal depletion and offer curves to provide some long term perspective. Oil suppliers set production according to the transversality condition pegging the growth rate of price to the real interest rate. Market conditions suggest a backward bending exporter offer curve. An oil tariff would then not only raise imports but also lower the price including the tariff in the Metzler (1949) effect.

The present model of trade between oil exporter OPEC and manufactures exporter OECD is based on the neoclassical offer curves of Marshall (1879) and Meade (1952) as derived by Mundell (1960) and Johnson and Bhagwati (1960) from a production frontier and utility maximization in general functional form. Dixit and Norman (1980) and Wong (1995) develop links between production and consumption with general expenditure and revenue functions. The comparative static factor proportions model of Samuelson (1953) and Jones (1965) includes factor markets but not
utility maximization or trade levels. Bhagwati and Srinivasan (1983) derive offer curves and trade levels with complete specialization and constant cost production.

The present paper simulates price and depletion paths for the coming century under various assumptions. The exporter offer curve is determined by optimal depletion, and sensitivity to the oil import elasticity is examined.

**The exporter offer curve**

For reference, the oil exporter is called OPEC and the oil importer OECD. In the backward bending portion of the OPEC offer curve in Figure 1 a lower relative price of oil raises OPEC oil exports and lowers OECD manufactured exports. The income effect of the falling terms of trade erodes the purchasing power of OPEC. The OECD tariff in Figure 1 lowers the international relative price of oil and raises oil imports in the Metzler (1949) effect.

![Figure 1. OECD Tariff](image)
Metzler (1949), Kemp (1964), and Minabe (1974) develop the conditions of a backward bending offer curve in terms of the marginal propensity to import and the exporter offer curve elasticity. The four conditions favoring a backward bending offer curve are

1. a high level of trade,
2. a low marginal utility of imports,
3. large difference in factor intensity, and
4. limited substitution in production.

The high level of OPEC trade implies a larger income effect. Satiated OPEC consumption implies low marginal utility of imports and little substitution in consumption. The high capital intensity and lack of substitution in oil production imply a relatively concave OPEC production frontier and little adjustment in oil output with a change in the relative price of oil. These conditions suggest the backward bending OPEC offer curve and Metzler effect pictured in Figure 1.

**Optimal depletion and price**

Optimal depletion is based on the principle that owners of oil in the ground consider it an asset and extract to equalize its rate of return to that from other assets. Assumptions include given property rights, known reserves, and perfect foresight. The dynamic inter-temporal condition for optimal profit is a constant marginal profit per barrel,

$$
\pi_t = \rho \pi_{t+1},
$$

(1)

where

- $$\pi_t = (p_t - c_t)$$ is marginal profit at time $$t$$
- $$p_t$$ is the price of oil
- $$c_t$$ is marginal extraction cost
- $$q_t$$ is extraction
- $$\rho \equiv 1/(1 + r)$$ is the discount factor
- $$r$$ is the real interest rate.
The present simulations assume constant marginal extraction cost and a constant real interest rate but these assumptions can be relaxed without dramatically altering the derived depletion and price paths.

The output path in (1) implies the transversality condition

\[ r = \frac{\pi_{t+1} - \pi_t}{\pi_t} = \frac{\Delta \pi_{t+1}}{\pi_t}. \]  

(2)

Marginal profit increases at a rate equal to the real interest rate with the marginal barrel earning its opportunity cost, the real interest rate.

Extraction shrinks the OPEC offer curve and raises the international relative price of oil. The present simulations report results with constant import demand elasticities of -0.5 and -1 to gauge sensitivity.

**Simulated terms of trade**

Simulations begin with the current world production of 22 billion barrels. Assume a constant extraction cost of $20/barrel, approximately the extraction cost in West Texas. Extraction costs are lower in most producing areas but will begin a gradual climb. Improved technology and new discoveries lower extraction cost but involve sizeable upfront costs. Simulated extraction and price paths are reasonably robust to increasing extraction cost. The real rate of interest is assumed to be \( r = 2\% \) and sensitivity to \( r = 4\% \) is examined.

The price is set to $25 or 25% above extraction cost. Recent higher oil prices are a result of supply disruptions and production bottlenecks, and price based on extraction cost seems more realistic for the long term economic analysis. The price path would shift upward from any higher starting point resulting in very similar extraction and depletion paths.

Simulations assume constant elasticity OECD oil import demand, \( q_t = \alpha p_t^\varepsilon \) where \( \alpha \) is a constant and \( \varepsilon \) is the import demand elasticity. An import elasticity of -1 results in the inverse demand function \( p_t = 5.5 \times 10^{11}/q_t \) scaled to the present consumption of \( q_0 = 22 \) billion and the current
price $p_0 = $25. Simulations for an import elasticity of -0.5 and the implied inverse demand function $p_t = 1.21 \times 10^{22}/q_t^2$ are also reported.

The price paths in Figure 2 are independent of the OECD oil import elasticity due to (2). Depletion shrinks the OPEC offer curve in Figure 1. The price of oil takes about a century to double with a 2% real interest rate and about half a century at 4%. The 2% price path in Figure 2 is consistent with the price projections of the Energy Information Agency (2007) for 25 years. Rehrl and Frederich (2006) predict a somewhat steeper price increase in an oligopoly model with rising extraction cost.

**Figure 2. Price Paths**

- $r=2\%$
- $r=4\%$

**Simulated extraction**

The extraction paths in Figure 3 are derived from the 2% price schedule in Figure 2. Extraction falls as the OPEC offer curve shrinks with depletion. With an oil import elasticity of -0.5 extraction is 68% of its present level after a century. Higher import elasticity implies slower extraction and an import elasticity of -1 leads to extraction at 45% of the present level after a century.
Investment in oil substitutes would increase oil import elasticity and the present extraction paths might be considered upper limits.

Price and depletion paths are reasonably robust to demand growth of a few percent. There will be more consumers and industries around the world demanding oil but environmental externalities and a growing array of energy alternatives make it reasonable to assume stable demand over the coming century. With demand growth of 1% extraction may rise slightly before beginning to decline. With an import elasticity of -1 and 1% demand growth, extraction peaks after 75 years before declining. With an import elasticity of -0.5 and 1% demand growth, extraction continuously increases as the growing demand overpowers the oil import elasticity. Wood, Long, and Moorehouse (2004) predict extraction will peak after 30 years due to growing demand before beginning a long gradual decline.
Simulated depletion

To examine depletion, global recoverable oil reserves are set at the EPA estimate of proven reserves, 2.978 trillion barrels. Estimates of proven, economic, recoverable, and endowments of various grades of petroleum and related products such as shale oil vary a good deal. Declining reserves in Figure 4 are calculated by deducting extraction each year from beginning reserves. Reserves fall to 37.5% (46%) of their present level after a century with an elasticity of -0.5 (-1). With 1% demand growth and an elasticity of -0.5 (-1) reserves are depleted within 100 (120) years.

The economizing effect of a rising price is often neglected but is apparent in Figures 3 and 4 as depletion slows due to the rising price.

Projected OPEC terms of trade and imports

Table 1 summarizes the future of price, extraction, and oil export revenue with an oil import elasticity of -0.5. Holding the price of OECD manufactured exports constant, the rising price of oil in Figure 2 is the relative price of oil. OPEC oil export revenue in the last column of Table 1 then equals OPEC imports given a constant unit price of manufactured imports.
Table 1. Oil Revenue, Import Elasticity -0.5

<table>
<thead>
<tr>
<th>year</th>
<th>$/bbl</th>
<th>bil bbls</th>
<th>$bil revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$25.00</td>
<td>22.00</td>
<td>$550.00</td>
</tr>
<tr>
<td>25</td>
<td>$28.04</td>
<td>20.79</td>
<td>$582.95</td>
</tr>
<tr>
<td>50</td>
<td>$33.19</td>
<td>19.09</td>
<td>$633.60</td>
</tr>
<tr>
<td>100</td>
<td>$55.51</td>
<td>14.76</td>
<td>$819.33</td>
</tr>
</tbody>
</table>

With the improved terms of trade $/bbl, OPEC exports bil bbl fall as imports rise. After 50 years OPEC terms of trade improve 32%, extraction falls 14%, and imports rise 14%. After 100 years OPEC terms of trade improve 124%, extraction falls 32%, and imports rise 53%.

An OECD oil import elasticity of -1 implies constant OPEC oil revenue, a flat OECD offer curve, and slower extraction. Extraction would fall to 10 billion barrels after 100 years compared to 150 years to reach that level with the -0.5 elasticity in Table 1.

As the price of oil rises over the decades, demand may become more elastic leading to slower extraction. The import elasticity can also be expected to increase with improved technology and backstop energy sources. More elastic demand would imply slower extraction.

**Conclusion**

The present model integrates offer curves and optimal depletion, simulating the long term international oil market over the coming century. Optimal depletion implies an increase in the relative price of oil of 32% after 50 years and 124% after 100 years. There is a sizeable increase in oil export revenue over the coming century under any set of reasonable assumptions. Theory suggests a backward bending OPEC offer curve implying the Metzler effect that a tariff would lower the price of oil including the tariff, and increase depletion.
References


