FACTOR MIGRATION AND INCOME REDISTRIBUTION IN INTERNATIONAL TRADE*

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Abstract: Changing factor endowments can typically be expected to redistribute income in general equilibrium economics. These effects are examined in a small, open economy, where migration leaves the terms of trade unchanged. To acquire meaningful results, it is assumed there are more types of factors than goods. New redistribution results are presented in this general case, and where there is one more type of factor than good. The three and four factor models are examined in some detail.

I. INTRODUCTION

Changing factor endowments redistribute income in most general equilibrium situations. One factor's endowment and another's payment may be negatively or positively correlated. The two factors can be called enemies in the first instance, friends in the second. Prices of goods remain unaffected by endowment changes in a small, open economy, so a factor's real income moves in the same direction as its payment. Strict factor price equalization is found in even models, those with the same number of factors and goods. Endowment changes within the production cone leave factor payments unchanged, as can be seen in Samuelson [7]. General equilibrium marginal returns remain constant, so the textbook economy with two factors and goods exhibits trivial redistribution effects.

If there are more factors than goods, changing endowments do effect factor payments. Technical substitution between factors would seem to play a role in determining factor friendship. When there is one more factor than type of good, however, the factor mix is sufficient to determine relative strengths of friendship. Attention has recently turned to the three factor, two good model, in Batra and Casas [1], Ruffin [6], Suzuki [8], Jones and Easton [4], Takayama [9], Thompson [10], and Thompson and Clark [11]. The point is emphasized in Ruffin [6] that there is only one possible way changing endowments affect factor payments. Extreme factors in the factor intensity ordering must be enemies, while the other factor is a friend of each extreme.

The present paper examines and extends what is known about this sort of general equilibrium redistribution. New results are presented for the general case

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of more types of factors than goods. The possibilities in models with three and four factors are explored. Further results are presented where there is one more factor than good, a situation of practical interest because of its ready applicability.

II. THE BASIC MODEL

General properties of trade models are developed clearly in Jones and Scheinkman [5] and Chang [2]. Their structure is summarized by full employment,

$$\sum_j a_{ij}x_j = v_i \qquad (i=1, \cdots, r),$$

and competitive pricing,

$$\sum_i a_{ij}w_i = p_j \qquad (j=1, \cdots, n).$$

Prices of goods (p) and factor endowments (v) are exogenous, while the system determines factor payments (w) and outputs (x). Production functions exhibit constant returns to scale, firms adjusting factor mixes to minimize cost. The amount of factor i used to produce one unit of good j (a_{ij}) depends only upon relative factor payments.

Differentiate the national income equation, $Y = \sum_i w_i v_i$, and utilize Shephard's lemma to find $\partial Y/\partial v_k = w_k$. Where $v_{hk} \equiv \partial w_h/\partial v_k$, Sameulson's factor reciprocity, $v_{hk} = v_{kh}$, is found. Attention is restricted to models where the number of factors is greater than the number of goods, i.e., $r > n$. Genral equilibrium diminishing marginal returns, or downward sloping factor demand, occur if v_{kk} is negative. It can be proven that $\partial^2 Y/\partial v_k^2 < 0$ from cost minimization. The signs of the matrix of v_{hk} terms is known, if we can discover the signs of its upper triangle

$$\begin{bmatrix} v_{12} & v_{13} & \cdots & v_{1r} \\ & v_{23} & \cdots & v_{2r} \\ & & \ddots & \vdots \\ & & & v_{r-1,r} \end{bmatrix}.$$

III. REDISTRIBUTION RESULTS

Two preliminary results can be proven, namely (i) friendship is intransitive, and (ii) its negation transitive. Where $w_{hk} \equiv \partial w_h/\partial w_k$, v_{hk} and w_{hk} will have opposite signs, since $v_{hk} = w_{hk}v_{kk}$. Suppose that factor k is a friend of both factors h and i, i.e., $v_{hk} > 0$ and $v_{ki} > 0$. This means that w_{hk} and w_{ki} are negative. Expanding through payment changes, $v_{hk} = w_{hi}w_{ik}v_{kk}$ and $v_{ki} = w_{kh}w_{hi}v_{ii}$. Thus w_{hi} must be positive, so friendship is intransitive. If factor k is an enemy of both factors h and i, again $w_{hi}>0$. The relation "is enemies with" must be transitive. If two factors are friends, then one and only one must be the friend of every other factor.

Considering the effect of changing factor payments upon prices, from competitive pricing, $\sum_i a_{im}dw_i = dp_m$. By the small country assumption, $dp_m=0$. With w_k changing by a small increment, $\sum_i a_{im}w_{ik}=0$. Every factor must have at least one friend, as is known, since for every k there must be an i such that $w_{ik}<0$. Since $w_{kk}=1$,

$$-a_{kj}=\sum_{i\neq k} a_{ij}w_{ik}, \qquad \text{for all} \quad k \tag{1}$$

There are r times n of these equations. The goal is to solve for the unknown w_{hk} terms, of which there are r^2-r. In models with one more type of factor than good, there are the same number of equations and unknowns. Solutions for the w_{hk} terms can then be found in this straightforward way, without regard for interfactor substitution.

Consider the situation where $r=n+1$, with one more factor than good. Letting ${}_iA$ denote the determinant of the technology matrix with the ith row deleted, it can be shown from (1) that

$$w_{hk}=(-1)^{h+k}{}_hA/{}_kA\ . \tag{2}$$

Where $k=1$, for instance, (1) becomes

$$\begin{bmatrix} a_{21} & \cdots & a_{r1} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{2n} & \cdots & a_{rn} \end{bmatrix} \begin{bmatrix} w_{21} \\ \cdot \\ \cdot \\ \cdot \\ w_{r1} \end{bmatrix} = \begin{bmatrix} -a_{11} \\ \cdot \\ \cdot \\ \cdot \\ -a_{1n} \end{bmatrix}.$$

Cramer's rule leads to a solution for the w_{i1} terms. For instance, $w_{21}={}_2A'/{}_1A'={}_2A/{}_1A$, where primes indicate transposes. The signs of the ${}_iA$'s determine the signs of the w_{hk}'s, and thus also of the v_{hk}'s. Equation (2) can be used to demonstrate two further results where $r=n+1$.

First, if a factor is the friend of every other factor, then all other pairs of factors are enemies. Suppose factor k is every other factor's friend, i.e., $w_{ik}<0$, where $i\neq k$. Where k is even and ${}_kA$ positive, or k odd and ${}_kA$ negative, consider equation (2). The signs in the vector $({}_1A, {}_2A, \cdots)$ must alternative $(+\ -\ \cdots)$, except for the kth factor. If k is even and ${}_kA$ negative, or k odd and ${}_kA$ positive, the signs alternate $(-\ +\ \cdots)$. In any case, $w_{ih}>0$, $i\neq k$ and $h\neq k$, i.e., all other factor pairs are enemies.

Another result can be stated after introducing the factor intensity ordering (FIO) between industries m and n,

$$a_{1m}/a_{1n}>a_{2m}/a_{2n}>\cdots>a_{rm}/a_{rn}\ .$$

Factors can be renumbered in the manner indicated. Suppose all factors adjacent in a FIO are friends. It can be proven that if r is even (odd), the extreme factors must be friends (enemies). For any factor k,

$$0 > w_{k,k+1} = -{}_kA/{}_{k+1}A \ .$$

The ${}_iA$ must be all positive or all negative, since factors k and $k+1$ are friends. If r is even (odd), $(-1)^{1+r}$ is negative (positive), so the extreme factors 1 and r must be friends (enemies).

Jones (1984) shows that given a FIO in any economy where $r > n$, some pair $(w_{i1}, w_{i+1,1})$, $1 < i < r$, must have the signs $(-+)$. There must, in other words, be at least two sign reversals in the vector of terms $(w_{11}, w_{21}, \cdots, w_{r1})$. Suppose there is only one sign reversal, i.e., there is a k such that $w_{i1} > 0$ where $i < k$, and $w_{i1} < 0$ where $i \geq k$. Then

$$\sum_{i=1}^{k-1} \begin{pmatrix} a_{im} \\ a_{in} \end{pmatrix} w_{i1} \qquad \text{and} \qquad \sum_{i=k}^{r} \begin{pmatrix} a_{im} \\ a_{in} \end{pmatrix} w_{i1}$$

cannot span the origin in (a_{im}, a_{in}) space as required by competitive pricing. An implication of this result is that an extreme factor cannot be the friend of all other factors.

Relative strengths of migration elasticities, relating percentage changes in endowments and factor payments, can also be uncovered from the w_{hk} terms and the factor payment levels. Where such an elasticity $e_{hk} \equiv (v_k/w_h)v_{hk}$, it is straightforward to show that

$$e_{hk}/e_{ik} = (w_i/w_h)(v_{hk}/v_{ik}) \ ,$$

and

$$e_{hk}/e_{ki} = (Y_k/Y_i)(e_{hk}/e_{ik}) \ ,$$

where Y_h is the income of factor h. Absolute elasticities would require either a complete specification of the model, including substitution terms, or an econometric estimate of any particular elasticity, which conceivably would be much simpler.

IV. THREE AND FOUR FACTOR MODELS

Now consider results in the three factor model. With two goods, there is one FIO. Then ${}_iA > 0$, $i = 1, 2, 3$, so that w_{12} and w_{23} are negative while w_{13} is positive. The signs in the upper triangle of the v_{hk} matrix, as shown by Ruffin (1981), must be

$$\begin{bmatrix} v_{12} & v_{13} \\ & v_{23} \end{bmatrix} = \begin{bmatrix} + & - \\ & + \end{bmatrix}$$

The two extreme factors are enemies, while the middle is a friend of each extreme.

The specific factors model is a special case, where $a_{12}=a_{31}=0$. The two specific factors are enemies, while the common factor is a friend of each specific.

Next consider the four factor model. The result in Jones (1984) implies the three terms $(v_{12}\, v_{13}\, v_{14})$ of the v_{hk} matrix can have four possible signs. Then using the above transitivity results, the signs of the four factor case can be derived:

$$\begin{bmatrix} +-+ & ++- & -+- & -++ \\ \;\;+- & \;\;-+ & \;\;+- & \;\;++ \\ \;\;\;\;+ & \;\;\;\;+ & \;\;\;\;+ & \;\;\;\;- \end{bmatrix}.$$
$$\text{(a)} \qquad \text{(b)} \qquad \text{(c)} \qquad \text{(d)}$$

This technique can be used in the three factor case as well. If $a_{im}/a_{in}>a_{2m}/a_{2n}>a_{3m}/a_{3n}>a_{4m}/a_{4n}$, pattern (a) illustrates the result that if all factors adjacent in the FIO are friends, the extreme factors must be friends.. More generally, patterns (a) and (b) do the same for eight different FIO's apiece. Patterns (c) and (d) illustrate that if a factor is the friend of every other factor, all other pairs of factors are enemies. These two patterns are analogous to the specific factors model, where a middle factor behaves like the common factor, being the friend of all other factors. Sign pattern (c) would result if factor 3 were the common factor, pattern (d) from a common factor 2. Consider the specific factors model with three or more factors. These redistribution results are known, but provide an application of the above approach. Suppose each factor is specific to the sector of the same number, i.e., $a_{ij}=0$ for $i\neq j$, $i<r$. The technology matrix can be written

$$A=\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 0 & 0 & \cdots & a_{nn} \\ a_{r1} & a_{r2} & \cdots & a_{rn} \end{bmatrix}$$

It can be shown that

$${}_kA=(-1)^{r+k-1}\prod_{i\neq k} a_{ii}a_{rk}\,.$$

Where $k=r$, the expression is positive. If r is even, the ${}_iA$ vector alternates in signs $(+-\cdots)$, while if r is odd, $(-+\cdots)$. The last column of common factor terms in the v_{hk} matrix will be all positive. The common factor is thus a friend with each specific factor. The rest of the v_{hk} matrix is negative, indicating that specific factors are all enemies.

V. CONCLUSION

Practical experience suggests consensus on policy affecting international factor mobility is difficult to attain. Modern international economic theory needs models with meaningful results from factor migration in order to address these issues. This analysis suggests that factor owners are concerned with the effects migration creates upon their income. An advantage of trade models with more types of factors than goods is that these distributional effects have some content.

A further advantage of models with one more type of factor than good is that factor friendship can be uncovered once one knows the factor mix terms, without having to estimate factor substitution relationships. The required general equilibrium substitution terms have never been estimated, while factor mix terms are more readily calculated. Relative magnitudes of the friendship elasticities can also be calculated given factor payments and incomes, which are generally available. These general equilibrium trade models afford a practical and fruitful approach to the study of the distributional effects of factor migration.

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