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INTERNATIONAL DIFFERENCES IN PRODUCTION FUNCTIONS AND FACTOR PRICE EQUALIZATION

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Abstract: This is a note on the implication of relaxing the assumption of international trade theory that production functions are identical across countries. A Euclidean measure of the international difference between exponential production functions is used to examine properties of the mapping to implied international differences in factor prices across freely trading countries. For anticipated differences in estimated production functions, factor prices would be similar across countries if factor price equalization would otherwise hold.

1. INTRODUCTION

The factor price equalization (FPE) theorem in international trade theory has a curious history. It was discovered without fanfare in the 1930s by Lerner (1952), then independently formalized by Samuelson (1949). Chipman (1966) presents a history of its logic and historical development. While FPE has stirred some controversy over the years, it remains useful as pedagogy and point of reference.

The proof of FPE depends on a number of assumptions:

(a) free trade and free transport between countries
(b) cost minimizing firms in a competitive economy
(c) an identical number of productive factors and international markets
(d) international factor endowments inside the production cone
(e) identical neoclassical nonjoint production functions.

If any one of these assumptions is relaxed, FPE loses is logical necessity. Strands in the international trade literature examine the implications of relaxing various assumptions. This paper concentrates on relaxing the assumption of identical production functions.

Consider each assumption in turn. While trade is never entirely free, for many

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traded goods protection and transport costs are small percentages of price. The move toward global free trade is making this assumption more appropriate for many goods.

Cost minimization provides the basis for the theory of the firm, and competitive pricing is a reasonable assumption for the long run in many industries. Factor markets, for the most part, are competitive.

To the extent that factors or goods can be aggregated, their exact numbers should not grossly affect the quantitative nature of the comparative static results of general equilibrium models. Theoretical properties of models with many factors and many goods are developed by Chang (1979), Ethier (1979), Thompson (1987), and others.

With only commonly produced goods entering the argument, factor endowments across many trading countries would likely lie within common production cones, at least for a large portion of observed international trade.

The assumption of identical nonjoint production functions across countries stands out for those with any experience in applied production analysis. Implications of joint production are explored by Samuelson (1992) and Jones (1992). The step from general neoclassical "blackboard" production functions to functional forms which could be estimated and applied is a large one. Identical production functions would first imply that specified production functions for a particular good would have the same functional form (Cobb-Douglas, CES, translog, and so on). Further, estimated technical coefficients would in practice have to be identical across countries.

Estimates of production or cost functions vary for the same industry over time and for different industries in the same sector. On the other hand, similarity of production functions is one criterion for aggregating goods. It is worthwhile to investigate the theoretical implications of allowing some difference in production functions across countries.

The idea that production functions may differ across countries is hardly new to trade theory. The classical Ricardian constant cost model is implicitly built on the assumption of different, if simple, production functions. The technology transfer literature concentrates on technology shift parameters in production functions and the dynamic international transmission of production techniques. Amano (1964) distinguishes between comparative cost differences based on endowment differences and technology shift parameters. Bardhan (1965) uses technology shift parameters in the production functions of the Heckscher–Ohlin model to illustrate that a country with better (Hicks neutral) technology in an industry will have a higher price of the factor used intensively in that industry. Ruffin (1988) develops a Ricardian factor endowment trade model with different production functions across countries in the form of fixed unit input proportions for the different factors of production.

The present study begins to address the impact on factor proportions trade theory of international differences in production functions in the form of different exponential production coefficients. A measure of the distance between exponential production functions is specified. The focus is on how this measure relates to the implied differences between factor prices across each country's static general equilibrium.

If FPE would otherwise hold, international differences between factor prices go to zero as the distance between production functions goes to zero. More similar production functions between trading partners would generally lead to more similar sets of factor prices. The important underlying empirical issue is the extent to which observed international differences in factor prices are explained by observed differences in production functions.

Empirical tests of the FPE theorem and the Heckscher-Ohlin theorem, starting with Leontief (1953), extending through Leamer (1984), Dollar, Wolff, and Baumol (1988) and Brecher and Choudhri (1993), and surveyed by Deardorff (1984), assume identical production functions everywhere. Direct evidence of international differences in production functions is found, however, by Arrow, Chenery, Minhas, and Solow (1961) and Minhas (1962). Maskus (1990) argues that observed differences in cost minimizing input mixes and the direction of trade together effectively imply different production functions across countries. In spite of the famous classic argument of Pearce (1970) that the laws of physics are the same everywhere, trade theory should in practice be able to proceed under the working assumption that estimated production functions at any point in time would be different across countries.

The foundation of FPE has not been implemented in the fundamental sense of a systematic international comparison of production functions. Intuition from applied production analysis suggests that production functions would not be identical across countries. Indeed, the entire issue can be developed across countries in terms of efficiency frontier analysis. The present paper aims to widen the scope of factor proportions trade theory by explicitly allowing different international production functions.

2. THE MAPPING BETWEEN DIFFERENCES IN PRODUCTION FUNCTIONS AND FACTOR PRICES

Consider the set F of exponential production functions from the vector v of inputs to a particular output level x_0:

$$F=\{f: f(v)=\{\prod v_i^{\alpha_i}=x_0\}\ . \qquad (1)$$

A particular production function in F is characterized by its positive exponents α_i. For simplicity, concentrate on the set of unit isoquants where $x_0=1$. The unit level of output x_0 can be produced by any of the production functions in F. Various combinations of inputs would lead to $x_0=1$ along any particular unit isoquant in F.

With exponential production functions, the unit isoquants all intersect at the unit vector. For a given nonunit vector v of inputs, the various production functions in F would lead to different levels of output. For any particular production function $f(v)$, various input vectors would lead to the same output along an isoquant.

The distance $d(f,f^*)$ between any two production functions f and f^* in F is a real number defined by some functional. Following Rudin (1976), $d(f, f^*)$ would qualify as a functional metric if

(i) $d(f,f^*)>0$ when $f\neq f^*$, and $d(f,f^*)=0$ when $f=f^*$
(ii) $d(f,f^*)=d(f^*,f)$
(iii) $d(f,f^*)\leq d(f,f')+d(f',f^*)$ for any f' in F. (2)

An example of a metric on function spaces would be

$$d(f,f^*)\equiv \max |f(v)-f^*(v)|\,, \tag{3}$$

where the vector v is limited to a closed set. The functional in (3), however, is not differentiable and not useful for the study at hand.

An intuitive metric involves integrating across differences in values of the function. Let v be a scalar as with Ricardian labor inputs, and consider the metric

$$d(f,f^*)\equiv \int_{\alpha}^{\beta} |f(v)-f^*(v)|\,dv\,, \tag{4}$$

where α and β are limiting elements of v. The neoclassical Inada conditions imply that $\alpha>0$ and β is finite. The choice of the limits of integration in an applied situation would depend on characteristics of the data.

When v is a vector of inputs, the metric in (4) can be expressed

$$d(f,f^*)\equiv \sum_i \int_{\alpha_i}^{\beta_i} |f(v)-f^*(v)|\,dv_i\,. \tag{5}$$

The metric in (5) has geometric and intuitive appeal. The distance between two production functions is essentially defined as the space between the unit isoquants, up to the limits of integration.

Setting these limits of integration is necessary with exponential production functions given that some of every factor is required in production. Isoquants are asymptotic to each axis and the distance between unit isoquants accumulates as they approach an axis. Limits of integration cut off the measure at a relevant range of inputs.

When there are two inputs, the unit isoquant can simply be taken as a function from one input to the other $v_1=h(v_2)$ where v_i represents the input of factor i. The distance measure in (5) can then be expressed as the simple integral

$$d(f,f^*)=\int_{\alpha_2}^{\beta_2} |h-h^*|\,dv_2\,. \tag{6}$$

The international difference in a factor price is measured

$$dw_i = | w_i - w_i^* | \tag{7}$$

where * represents the foreign variable. For simplicity, consider only cases with $w_i > w_i^*$.

Let f represent the production function for a particular good at home, and f^* the production function for the same good in the foreign country. If $d(f,f^*)=0$ and FPE would otherwise hold, $dw_i=0$ for every factor i. When $d(f,f^*)=0$, the mapping from the vector p of international prices to the vector w of factor prices is locally one to one and invertible, at least under other sufficient conditions laid out by Chipman (1966). This is the FPE result. Furthermore, for any given $d(f,f^*)$ the mapping from p to w would also be one to one and invertible.

The relation ϕ_i between $df \equiv d(f,f^*)$ and the international difference in a particular factor price w_i is the focus of this study:

$$\phi_i(df) = dw_i\,. \tag{8}$$

When FPE holds, $\phi_i(0)=0$.

3. CHARACTERISTICS OF THE DISTANCE MAPPING

In the Lerner–Pearce diagram of Fig. 1, dotted lines represent a range of differences between exponential production functions for good 1: $df_1 \equiv d(f_1, f_1^*)$. Let c_j represent the cost of a unit of good j, $c_j = w_i a_{1j} + w_2 a_{2j}$, where a_{ij} is the cost

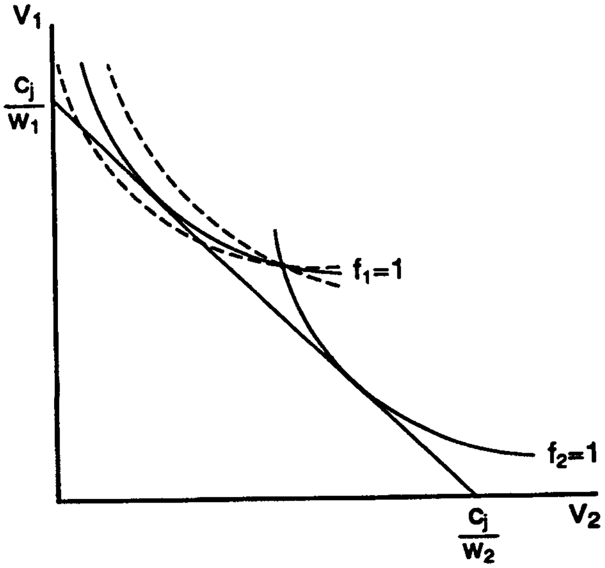

Fig. 1.

minimizing amount of factor i per unit of good j. Consider unit value isoquants, where $p_j = c_j = 1$. The unit isocost line would then intersect either factor's axis at $1/w_i$. If price (and cost) are held constant as a production function varies across countries, the distance between intersections of the isocost line along each axis would reflect the international difference in a factor price.

Assume the position of unit isoquant f_2 in Fig. 1 is the same across countries. Increasing differences in unit isoquant f_1 would create increasing differences in the same factor price across countries with the isocost line adjusting to the cost minimization.

Consider the mapping ϕ_i in (8). With $w_i > w_i^*$ by construction, $dw_i = w_i - w_i^*$. Any particular df will lead to unique dw_i, which implies that the two ϕ_i mappings are one to one. Further, ϕ_i is continuous at zero, since

$$\lim_{df \to 0} \phi_i(df) = \phi_i(0) = 0\,, \tag{9}$$

a restatement of the FPE result. Continuity of ϕ_i would also follow if for any nonnegative N

$$\lim_{df \to N} \phi_i(df) = \phi_i(N)\,. \tag{10}$$

Given smooth convex isoquants in both sectors, the ϕ_i mapping would apparently be continuous. Without specifying particular productions functions, however, a formal proof that ϕ_i is continuous may be unattainable.

Consider two dw_i which are arbitrarily close together: $|dw_i'' - dw_i'| < \varepsilon$, for any $\varepsilon > 0$. In other words, dw_i' lies in the open interval $W \equiv (dw_i'' - \varepsilon, dw_i'' + \varepsilon)$. Let dw_i' correspond to df' and dw_i'' to df''. There should be $\delta > 0$ such that the set D defined as $(df'' - \delta, df'' + \delta)$ is a subset of $\phi_i^{-1}(W)$. If df' were in the open set D, it would follow that $\phi_i(df')$ would be in W, or $|\phi_i(df'') - \phi_i(df')| < \varepsilon$. Since ε is arbitrarily small, ϕ_i would be continuous at the arbitrary point df'', and thus continuous over the domain.

To be more concrete, consider the two factor, two good model with Leontief technology in Fig. 2. The isoquants are right angles and the isocost line connects the minimum points of the isoquants. For simplicity, shift the origin up along the v_1 axis to the level of the f_2 isoquant. Rescale inputs so that $v_{22} = 1$ and $w_1 = 1$, as indicated in Fig. 2. The intersection of the f_1 unit isoquant with the isocost line then occurs at a point $(v_1, 1 - v_1)$, $0 < v_1 < 1$. The "foreign" unit isoquant f_1^* is a linear expansion from the new origin to $(\Psi v_1, \Psi(1 - v_1))$, $\Psi > 1$. Define the distance between the home and foreign production functions for good 1 as the distance between minimum points on their isoquants. It follows that $df = [(\Psi v_1 - v_1)^2 + (\Psi(1 - v_1) - (1 - v_1))^2]^{1/2} = v_1(\Psi - 1)\sqrt{2}$. The w_1^* implied by the f_1^* isoquant is found by considering the similar triangles $((1, 0), (\Psi v_1, 0), (\Psi v_1, \Psi(1 - v_1))$ and $((\Psi v_1, \Psi(1 - v_1)), (0, (\Psi(1 - v_1)), (0, 1/w_1^*))$. It follows directly that $w_1^* = \Psi(1 - v_1)/(1 - \Psi v_1)$. The international distance be-

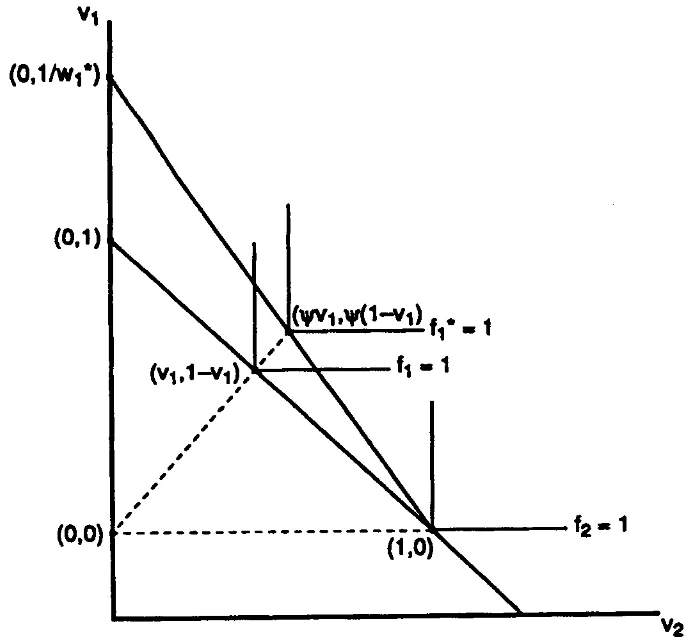

Fig. 2.

tween w_1 and w_1^* is then $dw_1 = 1 - w_1^* = (\Psi - 1)/\Psi(1-v_1)$. It follows that $df = (v_1\sqrt{2})dw_1$. Let dw_1 be arbitrarily small: $dw_1 = df/(v_1\sqrt{2}) < \varepsilon$, for any $\varepsilon > 0$. It follows that $df = (v_1\sqrt{2})dw < v_1\varepsilon\sqrt{2} \equiv \delta$. Such a δ can always be found for any arbitrarily small ε, and the mapping from this df to dw_1 is continuous.

With exponential production functions, ϕ_i must be monotonically increasing in df. If the assumption of exponential production functions is dropped, ϕ_i would no longer necessarily be monotonic. Unit isoquants for good 1 can be sketched which result in dw_i falling as df rises. The results in this study are limited to exponential production functions, but would hold across other functional forms.

The slope of ϕ_i indicates the sensitivity of dw_i to df. Compare ϕ_i and ϕ_i' in Fig. 3. The steeper ϕ_i' implies that a larger factor price difference is created for the same $df = \alpha$: $\phi_i'(\alpha) > \phi_i(\alpha)$. Less flexibility in the production structure, reflecting more convex isoquants, is represented by ϕ_i'. With ϕ_i', there are greater isocost adjustments and factor price differences as df increases. Steeper ϕ_i occur as isoquants become more convex. At the other extreme, ϕ_i would lie flat on the df axis if inputs were perfect substitutes.

4. A SPECIFICATION OF THE DISTANCE MAPPING

This section presents a specification of the production model with two factors and two goods, the simplest general equilibrium model where FPE would hold if

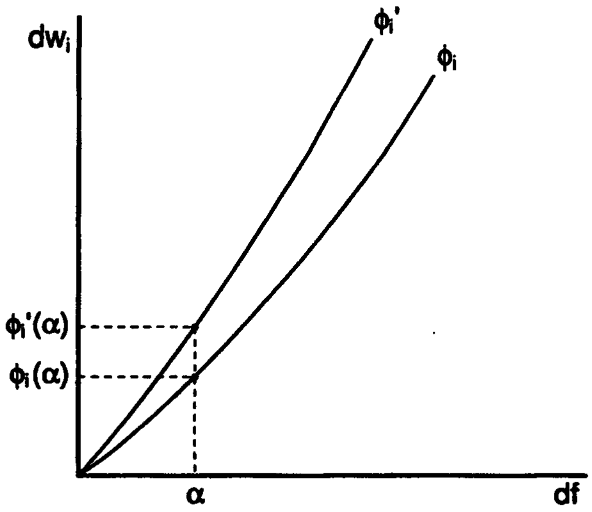

Fig. 3.

production functions were identical and the other sufficient conditions were met. Cobb–Douglas production functions are reported to provide a frame of reference. Specification of CES production functions, however, leads to ϕ_i functions with quantitative properties similar to these reported in the Cobb–Douglas model. Other functional forms might lead to different quantitative insights.

A constant returns to scale production function for good 2 is assumed to be identical in the home and foreign countries:

$$x_2 = x_2^* = v_{12}^{0.25} v_{22}^{0.75} . \tag{11}$$

Production functions for good 1 in the home and foreign countries are:

$$x_1 = v_{11}^{\gamma} v_{21}^{1-\gamma} \quad \text{and} \quad x_1^* = v_{11}^{\gamma^*} v_{21}^{1-\gamma^*} . \tag{12}$$

Different coefficients γ and γ^* would imply different unit isoquants and different factor prices with free trade between the two countries.

Concentrate on the unit isoquants where $1 = p_1 = p_2 = x_1^* = x_1 = x_2^* = x_2$. Where a_{ij} is the amount of factor i used per unit of good j, $a_{12} = a_{22}^{-3}$ from (12) along the unit isoquant for good 2. For good 1, $a_{11} = a_{21}^{(\gamma-1)/\gamma}$ at home and $a_{11} = a_{21}^{(\gamma^*-1)/\gamma^*}$ abroad. Using these unit isoquants along with the isocost lines and the condition of cost minimization, the factor mix terms can be written as functions of the relative price of factors:

$$a_{21} = [((1-\gamma)/\gamma)(w_1/w_2)]^{\gamma} \quad \text{and} \quad a_{22} = [3(w_1/w_2)]^{0.25} \tag{13}$$

In the foreign country, a_{22}^* is similarly expressed with γ^*. The implied factor prices are