FREE TRADE AND INCOME REDISTRIBUTION ACROSS LABOR GROUPS: COMPARATIVE STATICS FOR THE U.S. ECONOMY

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ABSTRACT

The move toward free trade will redistribute income, modelled in the present paper by a general equilibrium model of production with eight groups of labor and three sectors, namely agriculture, manufacturing, and business services. With constant elasticity production, comparative static elasticities relating differences in factor endowments to factor prices are nearly zero. Stolper-Samuelson price elasticities of factor prices and Rybczynski endowment elasticities of outputs are relatively large and tied closely to the pattern of factor intensity. Aggregating blue collar workers leads to misleading results. Characterizing free trade by a falling relative price of manufac tures, relatively unskilled labor groups suffer falling wages.

I. INTRODUCTION

Income redistribution is a recurring theme in international trade theory and a major topic in the political debate over free trade. Intuition among international economists about trade and income redistribution is based largely on the factor proportions model with two productive factors. The use of models with many factors is rare since the mathematics is diffi-

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cult and results are not readily related to the basic concepts of factor abundance and factor intensity. The present paper examines the comparative statics of a general equilibrium model of production with nine factors and three goods.

Simulations in the present paper are based on observed factor shares and industry shares for agriculture, manufactures, and business services, the three broad aggregates of output. Labor is aggregated according to the eight skill groups in the Census data. Various unskilled groups of labor are also aggregated up to blue collar workers. Simulations with Cobb-Douglas production functions are compared with those from constant elasticity of substitution production functions.

The factor price equalization theorem from trade theory depends on the relative numbers of productive factors and international markets in an open economy. The numbers of factors and goods in a model of an economy is ultimately a matter of available data and the purpose of the model. Conditions for factor price equalization do not hold in the present model, which means differences in factor prices would persist between freely trading economies. The range of the mapping from the endowments of factors to their prices describes how different the prices of each factor would be between freely trading economies.

An insight from the present study is the overwhelming influence of factor intensity. The direction and relative magnitudes of the effects of changing prices on factor prices (Stolper-Samuelson elasticities) and changing factor endowments on outputs (Rybczynski elasticities) can be predicted according to the most apparent degrees of factor intensity. Across a range of assumptions about production technology, these comparative static results depend almost entirely on factor shares and industry shares.

Stolper-Samuelson comparative static elasticities are generally elastic in absolute value in the present model, indicating that changes in the prices of goods will have large effects on factor prices. The current move toward free trade promises to shift relative output prices and substantially redistribute income across labor groups.

Different labor groups may correctly sense there is a lot at stake in the ongoing political debate over free trade. Rybczynski partial derivatives elasticities relate differences in endowments to outputs. In the present model, these comparative static terms are elastic. International factor supply differences would be associated with large international differences in the pattern of production. By implication, trade volumes would be large across freely trading countries.

Allen (1938) points out that aggregating any two factors in a production function will alter the substitution elasticities between the other pairs of factors. Comparative static results in a general equilibrium model would change. In the present paper, aggregation of blue collar workers leads to a sign change in the impact of changing prices of manufactured goods on blue collar wages. Also, technical separability of individual production functions is shown to be an irrelevant criterion for aggregation when the focus is the comparative static properties of a general equilibrium model of production.

II. FACTOR PRICE EQUALIZATION AND AGGREGATION

The work of Jones and Scheinkman (1977), Ethier (1974), Chang (1979), and others extends the 2×2 HOLS model to higher dimensional models with many factors. In any even model with the same number of domestic factors and internationally traded goods, factor supply or endowment changes within the production cone have no effect on factor prices. Where w represents factor prices and v factor endowments, $\partial w/\partial v = 0$. Freely trading economics experience factor price equalization (FPE).

The comparative static general equilibrium model is based on the structural assumptions of full employment of the factors of production and competitive pricing of outputs. Full employment occurs because of the perfect flexibility of factor prices. Competitive pricing occurs with cost minimization and zero profits. Full employment of factor i is written $v_i = \Sigma_j a_{ij} x_j$, where a_{ij} is the cost minimizing amount of factor i used per unit of good j and x_j is the output of good j. Competitive pricing of good j means price equals cost of production: $p_j = \Sigma_i a_{ij} w_i$.

The algebraic statement of the comparative static model is summarized by

$$\begin{bmatrix} \sigma & \lambda \\ \theta' & 0 \end{bmatrix} \begin{bmatrix} \hat{w} \\ \hat{x} \end{bmatrix} = \begin{bmatrix} \hat{v} \\ \hat{p} \end{bmatrix}. \tag{1}$$

where ^ represents percentage change. This model is explicitly developed by Jones and Sheinkman (1977), Chang (1979), and others. The top equation in (1) refers to the full employment of each factor, and the second equation to competitive pricing of each output. Factor prices in the vector w and outputs in the vector x are endogenous, while factor endowments in the vector v and prices in p are exogenous. The industry share matrix λ and the factor share matrix θ' both sum to one across rows. Rows in the factor demand substitution matrix σ sum to zero due to the assumption of linear homogeneity in production.

If the number of factors is at least as great as the number of traded goods $(r \geq n)$ and all goods are internationally traded, the system in (1) can be inverted. If $r < n$, the system is overdetermined, but Samuelson (1949) stresses that FPE would occur across the factors used to produce traded goods. In the multifactor models of this paper, $r \varepsilon \{9, 8, 7\}$ and $n = 3$.

When $r > n$, FPE does not hold. Nevertheless, the $\partial w/\partial v$ elasticities across a wide range of specifications are inelastic, a property called near factor price equalization (NFPE). If factor prices are not much affected by changing or different endowments, factor prices across trading partners would be nearly equal. The issue of the relative number of goods and factors loses much of its importance due to NFPE.

Diewart (1974) shows that own substitution elasticities, which describe substitution away from an input when its price rises, decrease as factors are aggregated. Also, aggregation alters substitution elasticities between other pairs of factors, changing the system's comparative statics. The effect of a change in a factor endowment on the price of factors not involved in the aggregation may even change sign. Comparative static results of the aggregated factors are not necessarily weighted averages of the comparative static results of the disaggregated factors.

These potential distortions are unsettling. Simulations in the present paper, however, are found to be fairly stable across aggregation schemes. Factor shares and industry shares carry some weight in determining the comparative static elasticities.

III. FACTOR SHARES AND INDUSTRY SHARES

Observed factor shares and industry shares are the basis of the present model specifications. Figures on employment by occupation and industry for all sectors are contained in the 1980 Census of Population. The eight types of labor in the Census data are

1. management, professional
2. technical, sales, secretarial
3. service
4. farm, forestry, fishing
5. craft, precision, repair
6. operators, fabricators, laborers
7. transport, moving
8. handlers, cleaners, helpers.

The factor share of each type of labor in each sector's income are directly calculated from national income data, and capital receives the residual. A unit of output is defined as one dollar.

The derived factor share matrix θ' in (1) is in Table 1. The eight types of labor are labelled 1 through 8 and capital K. The sectors are agriculture A, manufactures M, and services S.

Factor intensity can be described by ratios of factor shares, since $\theta_{im}/\theta_{in} = (w_i a_{im}/p_m)/(w_i a_{in}/p_n) = a_{im}/a_{in}$ for any of the three pairs of goods m and n. The concept of factor intensity in the present model is troublesome, since each pair of factors would have to be ranked across each pair of industries. On a simpler level, some obvious generalizations emerge. Capital is generally the most intensive input in agriculture, receiving more than half of the

Table 1. Factor Share Matrix θ'

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*K*
A	.059	.033	.003	.139	.086	.045	.030	.009	.596
M	.148	.116	.008	.002	.167	.286	.031	.028	.214
S	.264	.208	.058	.001	.094	.065	.035	.018	.257

Table 2. Industry Share Matrix λ

	A	*M*	*S*
1	.012	.175	.813
2	.008	.174	.818
3	.003	.049	.948
4	.791	.079	.130
5	.034	.389	.577
6	.016	.615	.369
7	.041	.245	.714
8	.019	.364	.617
K	.103	.217	.680

ncome (59.6%) in agriculture. Farmworkers receive 13.9% of the income in agriculture. Skilled labor is used intensively in services, with relatively skilled groups 1 and 2 together receiving almost half of the income (47.2%) in services. The more unskilled labor groups 5 and 6 are used intensively in manufacturing, together receiving almost half (45.3%) of that sector's income.

The distributions of inputs across industries, the industry shares λ, are in Table 2. While agriculture is the most capital intensive industry, the service sector employs 68% of the capital stock. Skilled labor groups 1 and 2, as well as unskilled groups 3, 5, 7, and 8, are employed mostly in services. Manufacturing employs more than half of all operators (group 6) and almost a third of all craft workers (group 5). Agriculture employs almost all farmworkers (group 4) and less than 5% of other labor groups.

IV. COMPARATIVE STATICS

Substitution elasticities are first derived directly from factor shares and industry shares in a Cobb-Douglas specification. Let S_{ih} represent the Allen elasticity of substitution between the price of factor h and the input of factor i in sector j. Sato and Koizumi (1973) show that the factor price elasticity in sector j $(\hat{a}_{ij}/\hat{w}_h)$ can be written $E^j_{ih} = \theta_{ij}S^j_{ih}$. Jones and Easton (1983) develop properties of the aggregate factor demand elasticities in matrix σ. $\sigma_{ih} \equiv \Sigma_j \lambda_{hj} E^j_{ih}$. In the Cobb-Douglas model, Allen elasticities equal one and $\sigma_{ih} = \Sigma_j \lambda_{hj} \theta_{ij}$. Table 3 presents the Cobb-Douglas substitution matrix σ in (1).

Own factor substitution elasticities in Table 3 lie between −1 and −0.7. All pairs of factors are weak substitutes. The largest elasticities appear in the last column, describing how capital can be substituted for the various types of labor when wages change. Industries are less inclined to substitute labor for capital when the price of capital changes.

Comparative statics in the model report the effects of changing endowments v and prices p on factor prices w and outputs x. Premultiply (1) by the inverse of the system matrix to find

$$\begin{bmatrix} \hat{w} \\ \hat{x} \end{bmatrix} = \begin{bmatrix} \sigma & \lambda \\ \theta' & 0 \end{bmatrix}^{-1} \begin{bmatrix} \hat{v} \\ \hat{p} \end{bmatrix} \equiv \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \hat{v} \\ \hat{p} \end{bmatrix}. \tag{2}$$

Table 3. Substitute Elasticities σ

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_K
w_1	−.758	.190	.049	.003	.106	.103	.034	.020	.253
w_2	.242	−.739	.049	.002	.106	.034	.034	.020	.252
w_3	.258	.203	−.943	.001	.097	.075	.035	.018	.256
w_4	.093	.062	.010	−.890	.093	.067	.031	.012	.522
w_5	.212	.166	.037	.006	−.878	.150	.033	.022	.252
w_6	.189	.149	.026	.004	.139	−.832	.032	.057	.236
w_7	.227	.178	.043	.007	.115	.120	−.968	.020	.260
w_8	.218	.171	.039	.004	.120	.145	.033	−.978	.248
w_K	.218	.170	.041	.015	.109	.111	.034	[illegible]	−.717

The array of comparative static elasticities then equals the inverse of the system matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \hat{w}/\hat{v} & \hat{w}/\hat{p} \\ \hat{x}/\hat{v} & \hat{x}/\hat{p} \end{bmatrix}. \tag{3}$$

Percentage changes in factor prices and outputs due to 1% exogenous changes in endowments and output prices are displayed in the inverse matrix.

V. COBB-DOUGLAS COMPARATIVE STATICS WITH 9 FACTORS

Table 4 presents the comparative static results for the 9×3 model in (1) using Tables 1, 2, and 3. The $\partial w/\partial v$ own elasticities lie between −0.96 and −0.25. The $\partial w/\partial v$ cross elasticities are generally very small, a property called near factor price equalization by Thompson (1990). The only elastic result is $\partial w_4/\partial v_K$, which reflects the heavy reliance of agriculture and farmworkers (group 4) on capital input.

Every 1% increase in the supply of professional labor group 1 would result in a decline in the wages of that group by 0.65%, an increase in service output by 0.54%, and decreases in the wage of farmworkers (group 4) by 0.70% and operators (group 6) by 0.07%. All other labor groups would benefit from this increase of professional workers, with gains ranging from 0.49% for service workers (group 3) to 0.11% for craft workers (group 5). Professionals are a factor "friend" with capital and every type of labor except farmworkers and operators. Professional organizations which artifically constrain growth in their profession thus lower the prices of most other factors.

Changes in the supplies of other types of labor and capital are analyzed reading down each column in Table 4. Capital and transport workers (group 7) are friends of every other factor. Farmworkers (group 4) are very weak enemies of most other labor groups.

The $\partial w/\partial p$ results in Table 4 closely follow a pattern suggested by factor intensity. There are positive links between each sector and its most intensively used factors. The service

Table 4. 9×3 Comparative Static Elasticities

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_K	p_A	p_M	p_S
w_1	−.65	.30	.09	−.03	.07	−.04	.04	.01	.21	−.19	−.42	1.6
w_2	.39	−.74	.10	−.03	.05	−.03	.04	.01	.21	−.22	−1.69	1.9
w_3	.49	.42	−.86	−.03	−.01	−.27	.04	−.00	.23	−.25	−1.5	2.7
w_4	−.70	−.67	−.18	−.25	−.04	−.27	.01	−.05	2.1	5.4	−1.3	−3.1
w_5	.11	.10	−.01	−.00	−.81	.35	.03	.04	.19	−.03	1.4	−.33
w_6	−.07	−.06	−.09	−.02	.33	−.22	.02	.04	.07	−.16	3.2	−2.0
w_7	.24	.20	.05	.00	.10	.08	−.96	.02	.26	.02	.17	.81
w_8	.16	.14	.01	−.02	.18	.30	.03	−.96	.16	−.14	1.2	−.01
w_K	.18	14	.04	.07	.08	.03	.03	.01	−.58	.47	−.07	.61
x_A	−.93	−.89	−.23	.95	−.10	−.43	.01	−.08	2.7	5.9	−2.0	−4.0
x_M	−.43	−.36	−.25	−.03	.57	1.5	.01	.13	.12	−.29	5.4	−5.2
x_S	.54	.46	.17	−.04	−.04	−.36	.04	−.01	.25	−.27	−1.9	2.1

ector and relatively skilled labor groups 1 and 2 are linked. The manufacturing sector and elatively unskilled labor groups 6, 7, and 8, as well as craft group 5 are linked. Finally, the agriculture sector, capital, and farmworkers (group 4) are linked. Farmworkers are especially sensitive to agriculture prices.

Every 1% decline in the price of manufactures would lower the wage of craft workers (5) by 1.4%, operators (group 6) by 3.2%, transport workers (group 7) by 0.17%, and handlers group 8) by 1.2%. Wages of professionals (group 1) would rise by 0.42%, technical workers (group 2) by 0.69%, service workers (group 3) by 1.5%, and farmworkers (group 4) by 1.3%. The price of capital would fall 0.07%.

Brown (1986) points out that the average price of manufactures relative to services and agriculture fell by 26% between 1969 and 1985. Increased global specialization and trade suggests that the price of manufactures relative to business services will continue to fall in the US. The wages of skilled groups 1 and 2 can thus be expected to rise in the future, while the wages of relatively unskilled labor used intensively in manufacturing will fall.

VI. CES COMPARATIVE STATICS WITH 9 FACTORS

The degree of technical substitution between factors affects the comparative static elasticities of the model. In the Cobb-Douglas model, the Allen elasticity of substitution between factor prices and factor inputs equals one. Results with constant elasticity of substitution (CES) can also be examined.

Changing the CES substitution elasticities by a positive multiple ϕ changes the $\partial x/\partial p$ results in Table 4 by the same multiple, and changes the $\partial w/\partial v$ results by $1/\phi$. The $\partial w/\partial p$ and $\partial x/\partial v$ results are left unchanged with any degree of CES. These convenient properties have not been noted in the literature. A proof is in the Appendix.

Increasing the degree of CES substitution reduces the $\partial w/\partial v$ comparative static partial elasticities in submatrix A proportionately. When there is more ability to substitute as factor prices changes, exogenous endowment changes impose milder adjustments on factor markets. A higher degree of substitution would mean flatter production isoquants, less factor price adjustment if endowments change, and more output adjustment if prices of goods change. The production frontier is less concave, and the $\partial x/\partial p$ elasticities larger.

The $\partial w/\partial v$ results become more elastic and the $\partial x/\partial p$ results become inelastic if the CES is reduced. Less elastic technical substitution means more convex production isoquants and larger adjustments in the underlying isocost hyperplane if factor endowments change. The production frontier is more concave. Price changes would thus lead to smaller output adjustments. The large own $\partial w/\partial v$ elasticities for transport workers (7) and handlers (8) stand out. With a CES of 0.5, for instance, these two own elasticities would both be -1.92. These two labor groups are relatively small and mostly sector specific, which explains these large effects.

Very small CES substitution may be interpreted as capturing the short run in the sense that firms do not have the opportunity to drastically alter their factor mix. With increasing substitution over time, the $\partial w/\partial v$ elasticities would decline and the $\partial x/\partial p$ elasticities would increase. NFPE would become the rule with increased substitution over time. In a model with rigid wages, short run unemployment would be the implication.

Increased substitution leads to less concavity in the production frontier and larger output adjustments for any exogenous price change. Every 1% decline in agriculture prices leads to a 5.9% decline in agricultural output in the Cobb-Douglas model. Such output sensitivity overstates what would be found in a model in which each of these three sectors is decomposed into numerous industries. Diewart (1974) points out that aggregating goods leads to larger $\partial x/\partial p$ elasticities.

VI. RESULTS OF THE MODELS WITH 9 FACTORS

With near factor price equalization, international capital flows and labor migration have minimal long run impact on income distribution. The underlying assumptions of complete output adjustment, full employment, and free factor mobility across sectors understate regional and transitory effects on factor prices. The influence of factor shares and industry shares predominate in the Stolper-Samuelson ($\partial w/\partial p$) and Rybczynski ($\partial x/\partial v$) results.

Farmworkers receive especially large benefits from protection in agriculture. Lower tariffs and lower price supports on agricultural goods would lower the capital payment and raise the wages of labor groups 1, 2, 3, and 6.

Relatively unskilled labor groups 5, 6, 7, and 8 lose when protection is lowered in manufacturing. Every 1% decrease in the price of manufactures due to increased import competition would lower the wages of labor groups 5 and 6 in Table 4 by 1.4% and 3.2%, respectively. Relatively unskilled labor groups 7 and 8 would lose 0.17% and 1.2%, while the capital payment would rise just under 0.1%. Skilled groups 1 and 2, used more intensively in services, would enjoy 0.42% and 0.69% wage increases. Servce workers (group 3) are used almost exclusively in services, and would enjoy a 1.5% wage increase.

Skilled groups 1 and 2, as well as service workers (group 3) benefit from higher prices of services. An issue the US is currently pursuing in negotiations is the high levels of protection other nations have for their domestic business services (telecommunications, finance, construction, banking, and so on). If other countries lower this protection, the demand for business services in the US will rise due to increasing exports and the US will specialize more in service production.

In the Cobb-Douglas model, every 1% increase in the price of services increases the wages of professionals (group 1), technical workers (group 2), and service workers (group 3) by 1.6%, 1.9% and 2.7%. Farmworkers (group 4) would lose 3.1% as agricultural output declines. Crafts (group 5) and operators (group 6) would lose 0.33% and 2% respectively as manufacturing output declines. Such shifts are inevitable given the evolving pattern of global specialization.

VII. AGGREGATING BLUE COLLAR WORKERS

The motivation for aggregating factors is to arrive at a simpler model. An informal approach is taken to aggregating blue collar workers, since the chi-square tests of Clark, Hofler, and Thompson (1988) indicate there is no technical separability among any of the inputs in this data set. Transport workers (group 7) and handlers (group 8), similar in factor

Table 5. 8 × 3 Comparative Static Elasticities

	v_1	v_2	v_3	v_4	v_5	v_6	v_B	v_K	p_A	p_M	p_S
	−.65	.30	.09	−.03	.07	−.04	.05	.21	−.18	−.42	1.6
	.39	−.74	.11	−.03	.05	−.03	.05	.21	−.22	−.70	1.9
	.48	.42	−.86	−.03	−.01	−.28	.05	.23	−.25	−1.5	2.7
	−.70	−.67	−.18	−.25	−.04	−.28	−.03	2.2	5.4	−1.3	−3.1
	.12	.08	−.01	−.00	−.81	.35	.08	.19	−.04	1.3	−.29
	−.08	−.06	−.09	−.02	.33	−.22	.07	.06	−.17	3.2	−2.1
	.21	.18	.03	−.00	.13	.17	−.94	.22	−.04	.52	.51
	.18	.15	.04	.07	.08	.03	.04	−.58	.47	−.08	.61
	−.92	−.89	−.22	.95	−.11	−.45	−.05	2.7	5.9	−2.0	−3.9
	−.42	−.34	−.25	−.03	.57	1.5	.08	−.12	−.29	5.4	−5.1
	.54	.46	.17	−.04	−.04	−.37	.05	.25	−.27	−1.9	2.1

Table 6. 7 × 3 Comparative Static Elasticities

	v_1	v_2	v_3	v_4	v_5	v_B	v_K	p_A	p_M	p_S
w_1	−.70	.28	.10	−.02	.01	.14	.20	−.15	−.91	1.8
w_2	.36	−.77	.10	−.03	.00	.14	.18	−.18	−.94	1.9
w_3	.54	.42	−.77	−.01	−.49	.20	.11	−.02	−5.4	5.0
w_4	−.61	−.57	−.04	−.24	−.56	−.19	2.2	5.6	−5.8	−.15
w_5	.01	.00	−.18	−.04	−.20	.03	.36	−.34	7.4	−4.0
w_B	.33	.26	.09	−.02	.04	−.93	.22	−.10	−.62	1.6
w_K	.16	.13	.02	.07	.16	.08	−.61	.45	.52	.20
x_A	−.74	−.70	−.01	.99	−.88	−.23	2.6	5.8	−8.7	.67
x_M	−.82	−.58	−.87	−.17	3.2	−.23	.49	−1.5	27	−18
x_S	.60	.46	.30	−.00	−.66	.22	.09	.04	−7.0	5.1

shares and industry shares, are aggregated into "blue collar workers" (labor group B) to produce the 8 × 3 model in Table 5.

The $\partial w/\partial v$ Cobb-Douglas results remain very inelastic. All of the comparative static elasticities in Tables 4 and 5 are nearly identical. The $\partial w/\partial p$ and $\partial x/\partial v$ elasticities for the aggregated labor group B in Table 5 are weighted averages of elasticities for groups 7 and 8 in Table 4.

Operators (group 6) are added into the blue collar group for the 7 × 3 model reported in Table 6. The comparative static results in Table 6 are misleading. Note that $\partial w_B/\partial_{pM} < 0$, which indicates that a loss of protection in manufactures would raise the wage of this blue collar group. The disaggregated models tells us, however, that the wage of each of the three groups in B would fall with a lower price of manufactures. In this instance, aggregation leads to a misleading result.

Eliminating a tariff on manufactures in the nine factor model lowers the wages of labor groups 6, 7, and 8, but the same policy in the 7 × 3 factor model would raise the blue collar wage. The lesson is that labor should be disaggregated by skill levels, and any model with aggregated labor might be qualitatively incorrect.

The signs of $\partial w_K/\partial p_M$ ($\partial x_M/\partial v_K$) and $\partial x_A/\partial p_S$ ($\partial x_S/\partial p_A$) also switch under the aggregation in the 7 x 3 model. There is, nevertheless, a high degree of similarity across Table 4, 5, and 6.

Technical separability in production may be too strict a general criterion for aggregation when the focus is on comparative statics. A wiser approach is simply to analyze the comparative statics as aggregation proceeds on a step-by-step basis. If only highly aggregated data is available, any empirical results must be taken with a grain of salt. Not only is a great deal of detail lost with high aggregation, but quantitative and even qualitative distortions occur.

VIII. CONCLUSION

The methodology of the present paper builds directly on a well known general equilibrium framework, and derives a set of comparative static elasticities. Results offer a quantitative feel for an important theoretical model in international trade theory. At the heart of the matter are the slopes and curvature of higher dimensional production frontiers, contract curves, and production isoquants. While any move toward free trade is known to increase welfare, the present analysis focuses on the type of income redistribution which occurs. Using such comparative static models, the redistributive results due to trade policies and a range of price changes can be examined.

As competitive economies trade more freely, their factor prices will adjust. The move toward free trade in the US continues to lower the price of manufactures relative to business services, altering the distribution of income across groups of labor. Specifically, unskilled labor groups, generally used intensively in manufacturing, can expect to suffer lower wages. Batra and Slottje (1993) find some empirical evidence supporting this hypothesis.

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APPENDIX

With CES production, the Allen elasticity is a positive number ϕ: $s_{ih} = \phi$. The factor price elasticity in sector j is $E_{ih} = \phi\theta_{ij}$, and the aggretate factor elasticity is $\sigma_{ih} = \phi\Sigma_j\lambda_{hj}\,\theta_{ij}$. In (1), the matrix of substitution elasticities with CES production is $\phi\sigma$.

The system matrix is then

$$\begin{bmatrix} \phi\sigma & \lambda \\ \theta' & 0 \end{bmatrix}. \tag{1A}$$

From (2), it follows that

$$\begin{bmatrix} \sigma & \lambda \\ \theta' & 0 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{2A}$$

hich implies

$$\begin{aligned} \sigma A + \lambda C &= 1 \\ \sigma B + \lambda D &= 0 \\ \theta' A &= 0 \\ \theta' B &= 1. \end{aligned} \tag{3A}$$

The properties referred to in the text for the inverse of (1A) can be written

$$\begin{bmatrix} \phi\sigma & \lambda \\ \theta' & 0 \end{bmatrix} \begin{bmatrix} A/\phi & B \\ C & \phi D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{4A}$$

To prove (4A), note that it implies four properties analogous to (3A):

$$\begin{aligned} \phi\sigma A/\phi + \lambda B &= \sigma A + \lambda C = 1 \\ \phi\sigma B + \phi\lambda D &= \phi(\sigma B + \lambda D) = 0 \\ \theta' A/\phi &= 0 \\ \theta' B &= 1. \end{aligned} \tag{5A}$$

The conditions in (5A) are identical to (3A), and (4A) is proven. Inverting the CES model as in (2),

$$\begin{bmatrix} \phi\sigma & \lambda \\ \theta' & 0 \end{bmatrix}^{-1} = \begin{bmatrix} A/\phi & B \\ C & \phi D \end{bmatrix}. \tag{6A}$$

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